PRODUCT DIFFERENTIATION AND DEMAND ELASTICITY

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ABSTRACT

This paper shows that product differentiation is compatible with perfect competition under free entry and exit and small firm size relative to size of market. Despite Chamberlin's view to the contrary, his monopolistic competitors are price takers. Even though no product has a perfect substitute, the presence of many close but imperfect substitutes is enough to bring price taking about. Advertising can pay off under perfect competition with product differentiation, however, since products have separate identities and price depends on quality, even though firms are price takers for any given quality.

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PRODUCT DIFFERENTIATION AND DEMAND ELASTICITY

This paper shows that product differentiation is compatible with perfect competition under free entry and exit and small firm size relative to size of market. Under the conditions given by Chamberlin [1965] in his classic treatise on monopolistic competition, firms will be price takers and perfect competition will prevail. Despite the widespread view in economics that monopolistic competitors face downward-sloping demand and produce with excess capacity and sub-optimal firm size, the existence of many imperfect substitutes for a product is enough to turn its supplier into a price taker. Other economists have derived similar results—for example, Fradera [1986] and Rosen [1974]—using approaches that are different from the one taken here. However, the derivation here is simpler, shorter, more accessible, and freer of restrictive assumptions. It focuses on the key issue of demand elasticity.

Monopolistic competition implies many firms in an industry, with each firm supplying an infinitesimal share of industry output value, the result of free entry and exit and large market size relative to the output that minimizes average cost for any firm. In addition, firms maximize profit, and buyers maximize utility. In these respects, monopolistic competition resembles perfect competition with homogeneous products. Under monopolistic competition, however, firms supply products that are close but not perfect substitutes. Nevertheless, I will show that if its share of industry output value is small enough, a firm's own-price elasticity of demand will be as large as desired. At the end of the paper, I will ask how small such a share must be if a firm is to be a *de facto* price taker.

With the above in mind, let X be a differentiated product that survives in long-run equilibrium in an industry called the X industry that operates under monopolistic competition. Let P_x and x be the price and quantity demanded of X, and let ε_x be the own-price elasticity of demand for X, given by $\varepsilon_x = -(P_x x_P)/x$, where x_P is the change in x per unit of a small increase in P_x, with other prices in the economy held constant. Let I^* be the economy's total income, which is assumed to be independent of the price set by any single firm, and let E_x be the expenditure on all products that are neither substitutes for nor complements with X. If $I = I^* - E_x$, I equals the sum of expenditures on X and on its substitutes and complements. Since changes in P_x do not change I^* or E_x, they do not change I.

Suppose that ε_x has a finite upper bound, say $\varepsilon_x \leq C$ for some positive C that holds regardless of how small the share of X in X-industry output value is. I will show that this assumption leads to a contradiction. Let products $(Y_1, Y_2 \ldots)$ be all the substitutes for and complements with X that survive in long-run equilibrium, with prices $(P_1, P_2 \ldots)$ and quantities demanded $(y_1, y_2 \ldots)$. These quantities are assumed to be second-order continuous functions of prices and buyers' incomes. We have:

$$I = P_x x + \Sigma_k P_k y_k, \qquad (1).$$

where the summation is over all products in I except X. From the expenditure side, I is a sum of prices times output quantities. Let P_x increase by a small amount, dP_x, with all other prices held constant. Since I does not change, we have the following when dP_x tends to zero:

$$0 = dI/dP_x = x + P_x x_P + \Sigma_k P_k y_{kP}, \qquad (2).$$

where dI is the change in I, and y_{kP} is the change in y_k per unit of dP_x. Here y_{kP} is positive when Y_k is a substitute for X and negative when Y_k is complementary with X.

Let $S_x = P_x x/I$ be the equilibrium share of X in I. Since $\varepsilon_x = -(P_x x_P)/x$, we have $(P_x/I)(x + P_x x_P) = S_x(1 - \varepsilon_x)$. An own-price elasticity of demand reflects the availability of substitutes for a product. Thus there should be a link between ε_x and the cross-price elasticities that arise when a change in P_x alters the demands for goods that are substitutes for or complements with X. Note from (1) that $(\Sigma_k P_k y_k)/I = (1 - S_x)$, and let ε_{Ax} be the share-weighted average cross-price elasticity of demand over all products in I other than X when P_x changes. That is, $\varepsilon_{Ax} = \Sigma_k[(P_k y_k/I)(\varepsilon_{kP})]/(1 - S_x)$, where this sum is over all products in I except X.

Because the sum of these weights equals $(1 - S_x)$, we can write $(1 - S_x)\varepsilon_{Ax}$ as the sum of product shares times cross-price elasticities over all Y_k in I. That is:

$$(1 - S_x)\varepsilon_{Ax} = \Sigma_k[(P_k y_k/I)(\varepsilon_{kP})] = \Sigma_k[(P_k y_k/I)(P_x y_{kP}/y_k)] = (P_x/I)[\Sigma_k P_k y_{kP}], \qquad (3).$$

where ε_{kP} is the cross-price elasticity of demand between X and Y_k when P_x changes. Equation (2) then becomes $S_x(1 - \varepsilon_x) + (1 - S_x)\varepsilon_{Ax} = 0$, when we multiply both sides by P_x/I. Re-arranging these terms gives:

$$\varepsilon_x = 1 + [(1 - S_x)/S_x]\varepsilon_{Ax}. \qquad (4).$$

If S_x is small enough, $(1 - S_x)/S_x$ will be as large as desired. Thus if ε_{Ax} remains greater than or equal to some positive lower bound, ε_x will be as large as desired if S_x is small enough. A profit-maximizing firm with a positive marginal cost will never produce where ε_{Ax} is negative, since $\varepsilon_x > 1$ must hold for such a firm, which implies $\varepsilon_{Ax} > 0$. However, ε_{Ax} could be quite small when S_x is quite small, so that the two offset in their effects on ε_x. This is what downward-sloping demand for X requires, but with infinitesimal S_x, demand is not downward-sloping.

To show this, we divide all products in I into those in the X industry and those outside this industry. Let I_x be total expenditure on products in the X industry and I_{nx} be total expenditure on products outside, with $I = I_x + I_{nx}$. If $S^x{}_x = P_x x/I_x$ is the equilibrium share of X in industry output value, $S^x{}_x = S_x(I/I_x) \geq S_x$. Thus, if $S^x{}_x$ is infinitesimal in equilibrium, the same will be true of S_x. From equation (3), $\varepsilon_{Ax} = \varepsilon^x{}_{Ax} + \varepsilon^{nx}{}_{Ax}$, where $\varepsilon^x{}_{Ax} = \Sigma_k[(P_k y_k/I)(\varepsilon_{kP})]/(1 - S_x)$, with summation over all products in the X industry, and $\varepsilon^{nx}_{Ax} = \Sigma_k[(P_k y_k/I)(\varepsilon_{kP})]/(1 - S_x)$, with summation over all substitutes for and complements with X that are outside the X industry.

One product is a substitute for another when an increase in the first product's price raises the quantity demanded of the second and when a fall in the first product's quantity demanded raises the demand price of the second. Thus, we say that X is a 'close' substitute for Y_k when two conditions are met. First, the cross-price elasticity, ε_{kP}, must be no less than some positive lower bound, B. The value of B must be small enough that product shares of X-industry output value are quite small, although this value is arbitrary to a degree. Since $\varepsilon_{kP} \geq B$ for any product, Y_k, in the X industry, $(I/I_x)\varepsilon^x_{Ax} \geq B[(1 - S^x_x)/(1 - S_x)]$, which is bounded away from zero as long as S^x_x is bounded away from one—that is, as long as the X industry is not a monopoly. I shall also require that I_x/I be bounded below. That is, there exists an A such that $I_x/I \geq A > 0$, where A can be any positive value. This prevents the structure of the X industry from being irrelevant to the value of ε_x.

Second, a given percentage increase in x—or a given dx/x—must cause at least some minimal percentage decrease in P_k at any given y_k if P_k is the price of a 'close' substitute. A product, Y_k, other than X is in the X industry if and only if X is a 'close' substitute for Y_k. In this context, I shall also assume symmetry. That is, if X is a 'close' substitute for Y_k, then Y_k is a 'close' substitute for X. Likewise, if X is complementary with any product in I_{nx}, then that product is complementary with X—a fall in its price raises the demand for X and a rise in its quantity raises the demand price of X.

Starting from equilibrium, let P_x again rise by a small amount, dP_x, with all other prices held constant, resulting in changes of dI_x and dI_{nx} in I_x and I_{nx} with $dI_x = -dI_{nx}$. Fix dP_x and note that $(1 - S_x)\varepsilon^{nx}_{Ax} = (dI_{nx}/I)/(dP_x/P_x) = (P_x/I)(dI_{nx}/dP_x)$ to a close approximation, provided dP_x is small enough. I shall interpret the survival of X to imply that its output is no less than some minimum, regardless of S^x_x or S_x, and scale this output in such a way that $x \geq 1$, which implies $(P_x/I) \leq S_x$. Thus, if dI_{nx}/dP_x is bounded over all S_x, ε^{nx}_{Ax} will be as small as desired for S_x small enough, and ε_{Ax} will be as close to ε^x_{Ax} as desired.

What if dI_{nx}/dP_x is unbounded? When dI_x is negative, a bounded ε_x implies that dI_x is bounded below, and $dI_{nx} = -dI_x$ is then bounded above. It follows that dI_{nx} can only be unbounded for given dP_x if dI_x is positive and dI_{nx} is negative. In this sense, complements with X predominate in I_{nx}. An unbounded $-dI_{nx}$ implies an unbounded dI_x—and thus an unbounded increase in the value of 'close' substitutes for X, owing to increases in the quantities demanded of these 'close' substitutes when P_x increases. Because of symmetry, a unit increase in the quantity demanded of a 'close' substitute, starting from equilibrium, will cause some minimal decrease in the demand price of X at any given x. If the increase in value of 'close' substitutes for X is large enough, the quantity demanded of X at $P_x + dP_x$ will be zero. Since dP_x can have any small positive value, this contradicts the assumption that ε_x is bounded above.

Thus, unless ε_{Ax} can be made as close as desired to ε^x_{Ax} by making S^x_x small enough, ε_x can be made as large as desired in the same way. But if ε_{Ax} can be made as close as desired to ε^x_{Ax}, ε_x can also be made as large as desired. Recall that $(I/I_x)\varepsilon^x_{Ax} \geq B[(1 - S^x_x)/(1 - S_x)]$, which is bounded above zero as long as the X industry is not a monopoly. This can be rewritten as $\varepsilon^x_{Ax} \geq AB[(1 - S^x_x)/(1 - S_x)]$, which is also bounded above zero. Therefore, when ε_{Ax} can be made as close to ε^x_{Ax} as desired, ε_{Ax} will have a positive lower bound. From (4), ε_x will then be as large as desired when S^x_x is small enough.

In the above, the values of ε_x, S_x, and ε_{Ax} are independent of B as long as this lower bound meets the requirements given above. However, for any selected value of B, suppose that when X

is a ‘close’ substitute for Y_k and Y_k is a ‘close’ substitute for Y_j, then X is also a ‘close’ substitute for Y_j. Then the X industry will consist of all firms whose products are ‘close’ substitutes for X, and these products will all be ‘close’ substitutes for one another. Each product in the X industry, so defined, has an own-price elasticity that will be as large as desired when its share of industry output value is small enough.

Finally, how small does S_x have to be for the supplier of X to be a *de facto* price taker? Suppose that $S^x{}_x$ is an average share for the X industry so that $n_x = 1/S^x{}_x$, where n_x is the number of firms in this industry. Then we can as well ask how large n_x has to be for the supplier of X to be a *de facto* price taker. To this end, let $\varepsilon^A{}_{kP} = I/I_x(\varepsilon^x{}_{Ax})$. When n_x is large, $\varepsilon^A{}_{kP}$ is approximately the share-weighted average of ε_{kP} values across the X industry—in general, this average equals $\varepsilon^A{}_{kP}[(1 - S_x)/(1 - S^x{}_x)]$. We can rewrite (4) as:

$$n_x = (\varepsilon_x - 1)/\varepsilon^A{}_{kP} + I_x/I, \qquad (4a).$$

assuming that $\varepsilon^{nx}{}_{Ax}$ is small enough to ignore.

Suppose that $I_x/I = .7$ and that the supplier of X is a *de facto* price taker if $\varepsilon_x \geq 9$, in which case a 5% cut in P_x will lower x by 45%. If $\varepsilon^A{}_{kP} = .3$, n_x will be between 27 and 28, $S^x{}_x$ will be about .036, and S_x about .025, so that $(1 - S_x)/(1 - S^x{}_x)$ is about .99. If $\varepsilon^A{}_{kP}$ is lower for these given values of ε_x and I_x/I, n_x will be larger, and if $\varepsilon^A{}_{kP}$ is higher, n_x will be smaller. Suppose that $\varepsilon^A{}_{kP}$ again equals .3, but that $n_x = 9$. Then ε_x is just under 4, and the supplier of X might well be a price maker instead of a price taker. However, with only nine firms in the industry, this is oligopoly. If $\varepsilon_x = 1.3$, $n_x = 2$, and we have duopoly.

Under oligopoly, firms may behave like Chamberlin’s monopolistic competitors when cross-price elasticities within the industry are not too high—so that firms do not behave strategically—and the number of competitors is not so large that each firm is a *de facto* price taker, but not so small that firms earn positive economic profit in equilibrium. Of course, this number may be east of the sun and west of the moon.

It follows that Chamberlin's monopolistic competition with many competitors is a form of perfect competition in which firms are price takers, even though no firm's product has a perfect substitute. However, in perfect competition with differentiated products, firms and products have separate identities and can be distinguished from one another. It is therefore possible to advertise a specific firm's product successfully if the advertising leads potential customers to believe that it has a higher quality than they had previously perceived. For that quality, the firm is still a price taker, however. While market failure can always result from too few competitors and entry barriers, it does not result from product differentiation with many competitors, provided buyers are well informed.

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