Changes in Confidence and Asset Returns*

Guihai Zhao†

September 4, 2013

Abstract

I show that asset price movements are driven by changes in confidence (size of ambiguity), whereas changes in confidence are driven by past aggregate consumption growth and unusual events like major economic and political shocks. The calibrated model matches well two facts that fail classic asset pricing models: the weak correlation of price-dividend ratios with lagged consumption growth, and the weak correlation between equity returns and consumption growth. It also explains the pro-cyclical variation of price-dividend ratios, the predictability of excess returns, among others. Investors fear stocks because they do poorly in "bad times," or states of low confidence.

1 Introduction

"*Uncertainty is worse than knowing the truth, no matter how bad*" (The Magazine of Wall Street, November 30, 1929, p. 177).

*I am grateful to Jerome Detemple, Larry Epstein, Simon Gilchrist, Francois Gourio, and Jianjun Miao for their continuing advice and support on this project. I appreciate the helpful comments of John Campbell, Jeffrey Fuhrer, Michael Gechter, Ali Ozdagli, Michael Nowotny, seminar participants at Boston University, FRB Boston, the discussant, David Chapman, and participants at BC/BU Green Line Macro Workshop. I would also like to thank John Campbell for providing interest rate data from Beeler and Campbell (2012). All errors are my own.

†The Department of Economics, Boston University, maxzhao@bu.edu

When making decisions, investors face both risk and ambiguity. Risk refers to the situation where there is a probability measure to guide choice, while ambiguity refers to the situation where the decision maker is uncertain about the data-generating process itself due to cognitive or informational constraints. In this paper, investor confidence, or the size of ambiguity, is represented by a set of one-step-ahead measures regarding aggregate consumption growth. I argue that asset price movements are driven by changes in confidence, which induce changes in the stochastic discount factor in equilibrium. This specific endogenous time-varying feature allows a general equilibrium model to capture a wide range of asset pricing phenomena.

Many consumption-based asset pricing models can match the equity premium, return volatility, and risk-free rate. However, the behavior of the price-dividend ratio in the data poses a serious challenge. On the one hand, price-dividend ratios predict returns but not consumption and dividend growth (Campbell and Shiller 1988a,1988b; Fama and French 1988; Hodrick 1992). If model-implied price variation comes from future changes in the consumption/dividend growth process, then the price-dividend ratio counterfactually predicts consumption and dividend growth. On the other hand, the price-dividend ratio has a weak correlation with lagged consumption in the short run (Bansal, Kiku and Yaron 2012 (henceforth BKY 2012)), but, as documented here, lagged consumption growth predicts the future price-dividend ratio in the long run. If the only source of price variation is past consumption, this will counterfactually imply too strong a correlation between the price-dividend ratio and past consumption, especially for the short run. Similarly, if price variation has nothing to do with past consumption, the correlation between the price-dividend ratio and past consumption will be too weak compared to the data, especially in the long run.

Another related fact that fails classic asset pricing models is the weak correlation between equity returns and consumption growth documented by Cochrane and Hansen

(1992), Campbell and Cochrane (1999), and Lettau and Ludvigson (2011). The key reason why models cannot account for these two facts is that price variation comes only from uncertainty in consumption/dividend growth. Thus, the price-dividend ratio has a strong correlation with consumption growth, which implies a strong correlation between equity returns and consumption growth.

I develop a consumption-based asset pricing model that helps to explain the preceding features of asset market data. There are two main ingredients in the model. First, departing from the rational expectations hypothesis, I assume that all identical investors are ambiguity averse and have the recursive multiple-priors preference axiomatized by Epstein and Schneider (2003) and Hayashi (2005). This model of preference permits a three-way separation of intertemporal elasticity of substitution (IES), risk aversion, and ambiguity aversion. Investors in this economy have in mind a benchmark or reference measure of the economy's dynamics that represents the best estimate of the data-generating process. They are concerned that the reference measure is misspecified and that the true measure is actually in a set of alternative measures that are statistically close to the reference measure. The level of confidence is represented by the size of the set of alternative measures at a given time. Second, under the reference measure, consumption and dividend growth are independently and identically distributed log normal processes, with the same mean and standard deviation as in the data. The model can accommodate more complex consumption processes, including processes with predictability, conditional heteroskedasticity, and non-normality. However, those are not salient features of the consumption data. Most importantly, I want to emphasize that even with the independent and identically distributed (i.i.d.) consumption growth assumption the model generates interesting asset price behavior internally. The set of alternative measures is generated by a set of different mean values around the reference mean value. Confidence changes correspond to changes in the size of this set of different mean values. Specifically, the persistent

fluctuations in confidence are driven by (i) recent aggregate consumption growth realizations; low aggregate consumption realizations have negative effects on investors' sense of predictability and perceived control, make them more concerned about model misspecification, and, as a result, less confident (bigger set of alternative measures) and high aggregate consumption realizations have positive effects on investors' sense of predictability and perceived control, make them less concerned about model misspecification, and, as a result, more confident and (ii) an exogenous shock (unusual events); similar to past aggregate consumption growth, 'bad' shocks make the set bigger and 'good' shocks make the set smaller. Exogenous shocks could be major economic and political shocks like the Cuban missile crisis, the assassination of JFK, the OPEC I oil price shock, and the 9/11 terrorist attacks or they could come from daily economic news. Investors' confidence level partially depends on the history of aggregate consumption growth rather than on an individual's own past consumption growth. This specification, which is similar to Campbell and Cochrane's (1999) ***external*** habit formation, simplifies the analysis. It eliminates terms in marginal utility by which extra consumption today raises confidence tomorrow.

In this paper, price variations are driven by changes in confidence. A low aggregate consumption realization, or a negative exogenous shock,[1] will make investors less confident and lower their worst expected growth rate of consumption, which in turn lowers the price-dividend ratio (pro-cyclical variation of price-dividend ratios) and increases expected returns (counter-cyclical variation of expected returns). This model is both "forward-looking" and "backward-looking" in the sense that (i) a drop in the price-dividend ratio reflects a decline in worst-case future expected growth and (ii) the

[1] Bansal and Shaliastovich (2010) documented that some large stock price movements are uncorrelated with concurrent or future movements in macro variables. This fact can be explained in my model by the exogenous component of confidence. Other than consumption growth, major economic and political shocks can change individuals' confidence dramatically. Even without large movement in lagged, current, or future macro fundamentals, there could still be large movements in returns due to those exogenous shocks.

decline in worst future expected growth is ***partially*** driven by lagged aggregate consumption growth. This feature distinguishes this model from many "backward-looking" models, where the only source of price variation comes from consumption dynamics. Those models tend to generate too much predictability in price-dividend ratios by lagged consumption. While short-run lagged consumption has very weak predictive power for forecasting price-dividend ratios, long-horizon lagged consumption growth predicts price-dividend ratios with a fairly high R^2 - 3.14%, 12.05%, 18.09% for 5-, 8-, and 10-years horizons. In addition the R^2 is increasing in the length of horizon. With the current price determined by changes in the future state of the economy, "forward-looking" models cannot incorporate the fact that price-dividend ratios can be predicted by long horizon lagged consumption growth. Using simulated data, my model provides a good match to the prediction of price-dividend ratios by lagged consumption growth, and the correlation between equity returns and consumption growth is low, as suggested by the data. Although variations in the price-dividend ratios reflect changes in worst-case future expected growth, the expected growth rate is constant under the reference model. Thus, the model will not incorrectly imply that dividend yields predict consumption and dividend growth. At the same time, both dividend yields and stock returns are driven by changes in confidence and, thus, dividend yields predict excess returns as in the data. The simulation result confirms this implication. The calibrated model can also match the first moments of market return, risk-free rate, and price-dividend ratio and the second moments of the market return and price-dividend ratio observed in the data. To see the model's performance on historical returns, I assume that the unusual stock return movements are triggered by exogenous shocks and construct the measure of exogenous shocks from unusual stock return movements.[2] Then, feeding into the model actual consumption data and the measure of exogenous shocks, I find that the returns predicted by the model

[2]Stock returns that are larger than 2 standard deviations.

provide a surprisingly good account of fluctuations in historical stock returns.

The specification for changes in confidence is consistent with psychological insights in Bracha and Weber (2012) and supported by empirical evidence. Bracha and Weber argued that, for models of risk and uncertainty, confidence results when investors believe they understand how things work. However, in unfavorable environments, the sense of predictability and perceived control is destroyed and panics result. Perceived control is influenced by learning from recent personal experience. In this paper, I assume that low aggregate consumption realizations have negative effects on investors' sense of predictability and perceived control, make them more concerned about model misspecification, and, as a result, less confident. To support the confidence specification empirically, I use three different measures for ambiguity that appear in the literature: dispersion in forecasts from the Philadelphia Fed's Survey of Professional Forecasters (SPF), the Chicago Board Options Exchange Volatility Index (VIX), and the absolute changes in VIX. For all these measures, demeaned lagged consumption growth and unusual events have significantly negative effects on size of ambiguity and these effects are persistent. This robust result confirms the psychological insights regarding how confidence changes.

This paper is related to a number of papers that have studied the implications of ambiguity and robustness for finance and macroeconomics.[3] This paper contributes to the literature by introducing a recursive multiple-priors model that can account for the weak correlation between price-dividend ratios and lagged consumption growth, the weak correlation between equity returns and consumption growth, predictability of equity returns by price-dividend ratios (but no predictabilities of consumption and dividend growth),

[3]Papers study the multiple-priors preference and its application include Cao, Wang, and Zhang (2005), Chen and Epstein (2002), Epstein and Miao (2003), Epstein and Wang (1994), Garlappi, Uppal, and Wang (2007), Ilut (2012), Leippold, Trojani, and Vanini (2008), and Routledge and Zin (2009). Epstein and Ji (2013) propose a new continuous time framework that captures ambiguity about both volatility and drift. Papers of robustness applications include Anderson, Hansen, and Sargent (2003), Cagetti, Hansen, Sargent, and Williams (2002), Hansen (2007), Hansen and Sargent (2001, 2010). For a survey on robustness, see Backus, Routledge, and Zin (2004) and Hansen and Sargent (2008).

and many other asset-pricing puzzles. The model uses a specific confidence process that is consistent with psychological insights and empirical evidence.

Several studies are closely related to this paper. Under the recursive multiple-priors framework of Epstein and Schneider (2003), Epstein and Schneider (2007) modeled learning under ambiguity using a set of priors and a set of likelihoods. The set of priors is updated by a generalized Bayes rule. Epstein and Schneider (2008) analyzed asset pricing implications using this learning framework. Ju and Miao (2012) proposed a smooth ambiguity model with learning and studied the asset return implications. Those models generate dynamics in ambiguity size, or confidence, by Bayesian learning. While Ju and Miao (2012)'s model can also match the procyclical variation of price-dividend ratios, the countercyclical variation of equity premia and equity volatility, I use a different preference and consumption growth process and address three more important puzzles: the two weak correlations and predictability of equity returns (but not consumption and dividend growth). Drechsler (2012) built a model with time-varying Knightian uncertainty to explain the volatile variance premium. There are two significant differences between his model and mine: (1) his reference measure is the long-run risk model and (2) the size of ambiguity in Drechsler's (2012) model follows an exogenous autoregressive process, which also drives the volatility of the long-run risk components. Since the model is simulated under the reference model, Drechsler's (2012) model is potentially subject to all the criticisms of the long-run risk model (e.g., predictability of consumption and dividend growth by the price-dividend ratio). Long-run risk plays an important role in price variation in this specification. The model in this paper differs from Drechsler (2012) in that (1) the confidence level depends on lagged consumption growth and one exogenous factor and (2) the consumption and dividend growth are independently and identically distributed. Thus, price dynamics are generated endogenously even with constant volatility.

Ilut and Schneider (2012) showed how time-varying confidence about productivity

generates business cycle fluctuations. Confidence follows an exogenous AR(1) process in their model. In this paper, confidence also follows a process of AR(1). However, I argue that the fluctuations in confidence are driven by two specific factors: past realizations of the underlying fundamental (consumption growth here) and unusual events. This small difference is important in that the performance of the model depends crucially on this feature. With an exogenous AR(1) confidence process, the model will not be able to generate the pro-cyclical variation of price-dividend ratios or counter-cyclical variations of excess returns; the correlation between price-dividend ratios and lagged consumption growth will be too small and the model will not generate sufficient predictability of excess returns.

This paper is also related to the distorted belief literature. In Cecchetti, Lam, and Mark (2000), the consumption growth follows a two-state Markov process and the representative agent has distorted beliefs about persistence of the state-transition probabilities. They studied the asset pricing implications under a specific belief distortion. Recently, under the long-run risk framework, Bansal and Shaliastovich (2010) presented a model of distorted belief about expected consumption growth. The distortion comes from recency-biased learning using signals with time-varying volatility. They found that the disconnect of significant moves in asset prices and the real economy and the predictability of excess returns can be explained by this distorted belief model. Adam, Marcet, and Nicolini (2012) examined the asset pricing implications when the agent has distorted beliefs about price in a standard consumption-based asset pricing model, and the beliefs about price change by learning from past price observations. This paper differs from distorted belief models in that I start with ambiguity-averse investors who are concerned about model misspecification. There is a set of alternative measures that are hard to distinguish from each other and investors are not sure which are the true measures. They pick the worst-case measure in equilibrium, but adapting this measure, induced by changes in confidence, as

a unique prior would seem contrived.

The paper continues as follows. Section 2 outlines the model and solves it analytically. Section 3 discusses the results of empirical analysis. Section 4 provides concluding comments.

2 The Model

In a pure exchange economy, identical ambiguity-averse investors maximize their utility over consumption processes. Individual consumption in period t is denoted by C_t, and C_t^a is the average consumption by all individuals in the economy. In equilibrium, identical individuals choose the same level of consumption, so $C_t = C_t^a$. In section 2.3, I will specify how each individual's confidence level responds to the history of aggregate consumption growth. Therefore, except in section 2.3, I drop the superscripts in what follows where they are not essential for clarity. Two assets are available for trade, a risk-free bond in zero net supply and equity that pays dividend D_t every period.

2.1 Consumption and Dividend Growth Process

Under reference measure P, consumption and dividends have the joint dynamics described below:

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + \sigma \eta_{t+1} \\ \Delta d_{t+1} &= \zeta \Delta c_{t+1} + \mu_d + \sigma_d u_{t+1} \end{aligned} \tag{1}$$

where $c_t = LogC_t$, $d_t = LogD_t$, Δc_{t+1} and Δd_{t+1} are the growth rate of consumption and dividends respectively. I assume that all shocks are i.i.d normal and orthogonal to

each other. Consumption and dividend growth are i.i.d. log normal with conditional expectation μ_c and $\zeta\mu_c + \mu_d$, respectively.

The literature reports several ways to model dividends and consumption separately,[4] and I follow Ju and Miao (2012) in this paper. The parameter $\zeta > 0$ can be interpreted as the leverage ratio on expected consumption growth, as in Abel (1999); together with the parameter σ_d, this allows one to calibrate the correlation of dividend growth with consumption growth. The parameter μ_d helps match the expected growth rate of dividends.

The reference measure P represents investors' best estimate from data. However, investors are concerned that this reference measure is misspecified and that the true measure is actually in a set of alternative measures that are statistically 'close' to the reference measure. The ambiguity-averse agent acts pessimistically and evaluates future prospects under the worst-case measure.

2.2 Ambiguity about Expected Consumption Growth

One requirement for the alternative measures is that they must be equivalent to the reference measure P (i.e., they put positive probabilities on the same events as P). The set of alternative measures is generated by a set of different mean consumption growth rates around the reference mean value μ_c. Specifically, under alternative measure $p^{\tilde{\mu}}$, consumption growth follows:[5]

$$\Delta c_{t+1} = \mu_c + \tilde{\mu}_t + \sigma\eta_{t+1}$$

where $\tilde{\mu}_t \in A_t = [-a_t, a_t]$ with $a_t > 0$. Each trajectory of $\tilde{\mu}_t$ will yield an alternative measure $p^{\tilde{\mu}}$ for the consumption growth process. The set of measures generated by A_t is a compact set, and this set of beliefs represents the agent's confidence regarding expected

[4]See Campbell (1999), Cecchetti, Lam, and Mark (1993), BY (2004), and BKY (2007).
[5]For simplicity, I assume there is no ambiguity about μ_d.

consumption growth. The a_t represents investors' confidence about future consumption growth. A larger a_t implies that the agent is more ambiguity averse, or less confident; likewise, a smaller a_t means the agent is more confident about the consumption process, or less ambiguity averse. In the following section, I specify how a_t changes over time.

2.3 Changes in Confidence

Bracha and Weber (2012) argued that the human need for predictability and control is central to psychological accounts of mania and panics. Perceived control is influenced by learning from recent personal experience. In models of risk and uncertainty, confidence results when investors believe they understand how things work. However, in unfavorable environments, the sense of predictability and perceived control is destroyed and panics are triggered. In this paper, I make use of these psychological insights using an external confidence specification where low average consumption realizations, or negative exogenous shocks, have negative effects on investors' sense of predictability and perceived control, make them more concerned about model misspecification, and, as a result, less confident. Likewise, high average consumption realizations, or positive exogenous shocks, have positive effects on investors' sense of predictability and perceived control, make them less concerned about model misspecification, and, as a result, more confident. Specifically, confidence changes in the following way,

$$a_t = a + \lambda(a_{t-1} - a) + \kappa \cdot (\Delta c_t^a - \mu_c) + \sigma_e e_t \tag{2}$$

where the persistence of the shocks is captured by λ and parameter a is the long-run mean confidence level. $\Delta c_t^a - \mu_c$ captures how "surprising" the average consumption realization is given investor's reference measure. When it is lower than the reference mean,

investors will become more concerned about model misspecification and therefore less confident, while a high average consumption realization ($\Delta c_t^a \geq \mu_c$) will make them less concerned about model misspecification and more confident about future consumption process. Thus $\kappa < 0$ here.

I assume that average consumption is not the only factor that influences investors' confidence; confidence is also driven by an exogenous factor e_t with volatility of σ_e. The exogenous shocks could be major economic and political shocks like the Cuban missile crisis, the assassination of JFK, the OPEC I oil-price shock, and the 9/11 terrorist attacks or they could also be shocks from daily economic news. The confidence level is propagated by the exogenous factor when it moves confidence in the same direction as consumption growth. One example is the Great Depression; as Romer (1990) demonstrated, production declined 1.8% between August 1929 and October 1929 and dropped dramatically afterward because of the banking collapse. The initial production decline lowered individuals' confidence, but the banking collapse propagated this effect and caused extreme pessimism about the future.[6]

Individuals' loss of confidence in the future can be caused by a decline in consumption, which could come from negative TFP shocks in the RBC model or from, for example, negative news shocks, monetary shocks, or investment-specific shocks captured by the exogenous factor. If there is no consumption decline, and the negative exogenous shock lasts only for a short time, confidence will recover quickly through observation of continued positive feedback from consumption. Otherwise, if the exogenous shocks propagate the effect of bad consumption growth on confidence, individuals will become extremely pessimistic.

In addition to the psychological insights, the confidence process in equation (2) is

[6]In this paper, there is no production and thus reductions in confidence do not have any real effect on consumption. In a production economy, for example, as in Ilut and Schneider (2012) and Zhao (2013), reductions in confidence about future productivity cause firms to hire less and reduce production and, thus, result in a recession.

supported by empirical evidence. To see this, I first find two measures: one for the size of ambiguity and the other for the exogenous confidence shock e. There are some commonly used measures for ambiguity: the dispersion of market participants' forecasts, the VIX, and the absolute changes in VIX. I used all three measures to show that the results are robust to different measures. The exogenous confidence shock e is new in this paper and there is no measure for it in the literature. However, shock e represents unusual events and economic news, and when such shocks occur, historically there have been big jumps in stock returns. Thus, one natural measure of those shocks is the size of the jumps. Given these measures for ambiguity and exogenous confidence shock, I ran an ordinary least squares (OLS) base on equation (2) and checked the signs of coefficients and their significance.[7][8]

For forecasts dispersion, I follow Anderson, Ghysels, and Juergens (2007), Ilut and Schneider (2012), and Drechsler (2012) and measure the size of ambiguity using the dispersion in forecasts of next quarter's consumption growth from the Philadelphia Federal Reserve's (Fed's) SPF. The dispersion is calculated as the difference between the 75th percentile and the 25th percentile of the individual forecasts in levels. Since all forecasters can access the same public information, the difference in their forecasts should reflect differences in their beliefs. When making decisions, individual investors think that everyone else's forecasts could be the true model. Note that individuals' forecasts may differ from the beliefs they use for decision making. Without concern for the consequences, individuals will give their best estimates when they perform the forecasting, while to make a decision that is going to affect their future utility, they are ambiguity averse and take the

[7]OLS is consistent if there is no serial correlation in error terms. Serial correlation is tested for all regressions in Table 1 using the Durbin-Watson test. The p-values for panels A, B, C, and D are 0.923, 0.367, 0.044, and 0.122. So, the null hypothesis that the residuals are uncorrelated cannot be rejected for panel A, B, and D. For panel C, the null hypothesis cannot be rejected at the 1% significance level.

[8]Since the exogenous confidence shock and demeaned consumption are orthogonal, the OLS coefficients and standard errors do not change much when regressors include only one of them. And p-values for Durbin-Watson tests are all above 5%.

worst-case scenario instead. If we replace an investor's set of alternative measures with the set of forecasts, the dispersion in forecasts should capture the size of ambiguity.

The VIX is another measure of ambiguity used in the literature (Williams 2009). The VIX is constructed using the implied volatility of S&P 500 index options, and it measures the expected stock market volatility for the next 30 days. High expected volatility corresponds to a high level of ambiguity in the market.[9] One gap here is that VIX measures the ambiguity in the stock market, while equation (2) is concerned with the ambiguity level of consumption growth. When the model is solved using log linearization, the expected equity return is a linear function of confidence level a_t. Thus, the size of ambiguity for the stock market is also a linear function of consumption growth ambiguity with the same coefficient in this model. To determine the appropriate measure for consumption growth, I divide the VIX measure by this calibrated coefficient value $A_{1,E}$ (=2.13). The VIX, as a measure of implied volatility, has the disadvantage of being interpreted as a measure of expected risk instead of ambiguity. Nevertheless, studies have suggested that implied volatility derived from options is too large to serve as a reasonable forecast of future return variance (Eraker 2004, Carr and Wu 2009). This might indicate that the VIX reflects both ambiguity and expected risk.

Amado and Teräsvirta (2013) proposed absolute changes in the VIX as a measure of ambiguity of the stock market. They argued that "if the level of the VIX represents expected risk, then the variation in the VIX will reflect the uncertainty related to the expectation of risk, i.e. the uncertainty of uncertainty, and hence can be interpreted as a measure of uncertainty." To obtain the measure of ambiguity for consumption growth, I divide the absolute changes in the VIX by $A_{1,E}$.

To obtain the measure for the exogenous confidence shock e, I first collect daily stock returns that have big movements, bigger than 2 standard deviations, and aggregate to

[9]Drechsler (2012) showed that the level of the VIX and the SPF forecasts dispersion are highly correlated.

β_0	β_1	β_2	β_3	R^2
Panel A: $Disp = \beta_0 + \beta_1 Disp(-1) + \beta_2(\triangle c - \mu_c) + \beta_3 e + \varepsilon$				
0.0012	0.5516	-0.0334	-0.0039	0.3940
(0.0000)	(0.0065)	(0.0003)	(0.0015)	
Panel B: $WorstB = \beta_0 + \beta_1 WorstB(-1) + \beta_2(\triangle c - \mu_c) + \beta_3 e + \varepsilon$				
0.0027	0.4140	0.1414	0.2192	0.3581
(0.0000)	(0.0152)	(0.0092)	(0.0290)	
Panel C: $VIX = \beta_0 + \beta_1 VIX(-1) + \beta_2(\triangle c - \mu_c) + \beta_3 e + \varepsilon$				
0.0045	0.8384	-0.0361	-2.6199	0.7984
(0.0000)	(0.0040)	(0.0065)	(0.1132)	
Panel D: $abs(\Delta VIX) = \beta_0 + \beta_1 abs(\Delta VIX)(-1) + \beta_2(\triangle c - \mu_c) + \beta_3 e + \varepsilon$				
0.0024	0.3909	-0.1785	-0.4417	0.2221
(0.0000)	(0.0144)	(0.0058)	(0.1353)	

Table 1: Predictability of confidence

In panel A and B, the forecast dispersion and implied worst case forecast are from 1981 Q3 to 2012 Q4, quarterly U.S. real consumption growth is from 1981 Q2 to 2012 Q3 from the Bureau of Economic Analysis. For panel C and D, values of the VIX are from the CBOE at the beginning of each month from 1990 Jan to 2012 Dec, monthly U.S. real consumption growth is from 1989 Dec to 2012 Nov from the Bureau of Economic Analysis.

monthly/quarterly frequency. To eliminate the potential effect of consumption growth on those big return movements, and also to be consistent with the model's assumption that the exogenous shocks are uncorrelated with consumption growth shocks, I then run a regression of the measure of big return movements on consumption growth, and take the residual as the adjusted measure. Finally, I rescale and demean the adjusted series such that the resulting measure has the same mean and standard deviation as the model's assumptions for the exogenous shocks.

Table 1 provides the OLS regression results using different ambiguity measures (panel A: forecasts dispersion, panel C: VIX, panel D: absolute changes in the VIX). Since what matters in equilibrium is the worst-case belief, panel B also provides the OLS regression results using the worst forecast in SPF for an additional check. In panels A and B, the

forecast dispersion and implied worst-case forecast are from 1981 Q3 through 2012 Q4.[10] Accordingly, quarterly U.S. real consumption growth is from 1981 Q2 through 2012 Q3 from the Bureau of Economic Analysis, and quarterly exogenous confidence shock is from 1981 Q2 through 2012 Q3. For panels C and D, I use closing values of the VIX from the Chicago Board of Exchange (CBOE) at the beginning of each month from January 1990 through December 2012.[11] The VIX index from the CBOE is reported in annualized volatility terms; I square it to obtain variance and divide by 12 to obtain a monthly quantity, and then take the square root to put it in monthly volatility terms. The monthly U.S. real consumption growth is from December 1989 through November 2012 from the Bureau of Economic Analysis, and monthly exogenous confidence shock is from December 1989 through November 2012. The results suggest that, for all three measures, both consumption growth and exogenous confidence shocks have negative effects on size of ambiguity (β_2 and β_3 are all negative values with very small standard errors). Bad consumption growth, or bad unusual events, will make the agent less confident and increase the size of ambiguity. Furthermore, all three panels suggest that the size of ambiguity is persistent and confidence shocks have persistent effects (β_1 is big and significant). Panel B uses the worst forecast in SPF as the worst-case belief and provides additional support for the above findings. Positive β_2 and β_3 means that bad consumption growth, or bad unusual events, will make the worst-case belief worse, thus lowering the confidence level. Similarly, β_1 is big and significant and, thus, confidence shocks have persistent effects.

The persistence feature of confidence in this paper is consistent with those of several other papers. Bracha and Weber (2012) suggested that confidence is influenced by learning from recent personal experience and that confidence moves slowly with sig-

[10]The earliest consumption growth forecast available in SPF is from 1981 Q3, and the survey was conducted near the beginning of the quarter.

[11]More data are available from 1986, but starting from 1990, CBOE has calculated VIX using a new methodology. Higher frequency VIX is also available, but the highest frequency consumption from BEA is monthly.

nals. Epstein and Schneider (2007, 2008) presented a model of learning under ambiguity that shows how updating affects confidence. Ilut and Schneider (2012) used a persistent confidence process to show how ambiguity about productivity generates business cycle fluctuations.

While Table 1 provides robust supporting evidence of the confidence process in equation (2), the estimation coefficients are different for different measures of ambiguity. Instead of picking one measure from those three and estimating the coefficients, I follow BY (2004) and Campbell and Cochrane (1999) and calibrate the parameters in the confidence process using consumption, dividend, and stock market data.

2.4 Preference: Epstein-Zin Preference with Ambiguity Aversion

Epstein-Schneider (2003) axiomatized an intertemporal model of multiple-priors utility, and Hayashi (2005) extended that model to allow for the recursive preference of Kreps and Porteus (1978) and Epstein and Zin (1989). This model of preference permits a three-way separation of intertemporal substitution, risk aversion, and ambiguity aversion. Investors' utility over consumption is represented by the following model,

$$V_t(C_t) = [(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta\{\mathcal{R}_t(V_{t+1}(C_{t+1}))\}^{\frac{1-\gamma}{\theta}}]^{\frac{\theta}{1-\gamma}}$$

$$\mathcal{R}_t(V_{t+1}(C_{t+1})) = \{\min_{p_t \in \mathcal{P}_t} \mathbb{E}_{p_t}(V_{t+1}^{1-\gamma}(C_{t+1}))\}^{\frac{1}{1-\gamma}}$$

This is a standard Epstein-Zin preference, except that we have a "min" operator within the aggregator. $0 < \beta < 1$ reflects the agent's time preference, γ is the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ is the IES. Utility maximization is subject to the budget constraint,

$$W_{t+1} = (W_t - C_t)R_{c,t+1}$$

where W_t is the wealth of the agent, and $R_{c,t}$ is the unobservable return on all invested wealth, or the consumption claim.

The set of one-step-ahead beliefs $\mathcal{P}_t$ consists of the measures $p_t^{\tilde{\mu}}$ generated in section 2.2. I show in the appendix that the worst-case measure p_t^o that gives the minimum continuation value is $p_t^{-a_t}$, which is generated by likelihood with the worst mean $-a_t$ each period.[12] Thus, the "min" operator in the preference can be replaced by $p_t^o = p_t^{-a_t}$, which is generated by the worst mean.

2.5 Asset Pricing

Since the representative agent evaluates expectations under the worst-case measure when making portfolio choices, the Euler equation holds under the worst-case measure. Therefore, assets can be priced using the Euler equation under the worst-case measure. Given the worst-case measure, as in Epstein and Zin (1989), the pricing kernel or the stochastic discount factor can be written as,

$$M_{t,t+1} = \beta^\theta (\tfrac{C_{t+1}}{C_t})^{-\frac{\theta}{\psi}} R_{c,t+1}^{\theta-1},$$

or

$$m_{t,t+1} = \theta log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{3}$$

where $m_{t,t+1} = log(M_{t,t+1})$, $r_{c,t+1} = log(R_{c,t+1})$. For any asset j, the first-order condition yields the following asset pricing Euler condition,

$$E_{p_t^o}[exp(m_{t,t+1} + r_{j,t+1})] = 1, \tag{4}$$

where $E_{p_t^o}$ is the expectation operator for the worst-case measure, and $r_{j,t+1}$ is the log of the gross return on asset j.

[12] See also Epstein and Wang (1994) for a proof.

To solve the model, I assume that the log price-consumption ratio for a consumption claim, z_t, is linear in the confidence level, a_t, the state variable of the model:

$$z_t = A_0 + A_1 a_t, \tag{5}$$

and that the log price-dividend ratio for a dividend claim, $z_{t,m}$, is similarly linear:

$$z_{t,m} = A_{0,m} + A_{1,m} a_t. \tag{6}$$

The log return on consumption claim is given by the Campbell and Shiller (1988b) approximation,

$$r_{c,t+1} = k_0 + k_1 z_{t+1} + \Delta c_{t+1} - z_t. \tag{7}$$

where $k's$ are log linearization constants, which are discussed in more detail below. In order to solve A_0, A_1, one need to substitute (7), (5),(3) into Euler equation (4). By approximating log market return in a similar way,[13] $A_{0,m}, A_{1,m}$ can be found. Details of both derivations are provided in the Appendix. As noticed by previous studies,[14] the parameters A_0 and A_1 determine the mean of the price-consumption ratio, $\bar{z}$, and the parameters k_0 and k_1 are nonlinear functions of $\bar{z}$,

$$\bar{z} = A_0(\bar{z}) + A_1(\bar{z})a$$

and

$$k_0 = log(1 + exp(\bar{z})) - \bar{z}k_1,\ k_1 = \frac{exp(\bar{z})}{1+exp(\bar{z})}.$$

To get a highly accurate approximation, one needs to iterate numerically until a fixed point for $\bar{z}$ is found.

[13] $r_{m,t+1} = k_{0,m} + k_{1,m} z_{t+1,m} + \Delta d_{t+1} - z_{t,m}$.

[14] Campbell (1993), Campbell and Koo (1997), BKY (2007), and Beeler and Campbell (2012)

The solution coefficients for the effect of confidence a_t on the price-consumption ratio, A_1, and on the price-dividend ratio, $A_{1,m}$, respectively, are

$$A_1 = \frac{1-\frac{1}{\psi}}{k_1(\lambda-\kappa)-1},\ A_{1,m} = \frac{\zeta-\frac{1}{\psi}}{k_{1,m}(\lambda-\kappa)-1}.$$

Since κ is small, both $k_1$and $k_{1,m}$ are smaller than 1 under the fixed point value of $\bar{z}$, A_1 is negative if the IES, ψ, is greater than 1; and $A_{1,m}$is negative if $\psi > 1$ and $\zeta > 1$. In this case, the intertemporal substitution effect dominates the wealth effect. In response to the high confidence level, or low a_t , investors buy more assets and prices increase. In addition, $A_{1,m} > A_1$ when $\zeta > 1$, which means that a confidence shock leads to a stronger reaction in the price of the dividend claim than in the price of the consumption claim. Note that an increase in persistence of confidence shocks, λ, will increase the magnitude of the response of both valuation ratios to confidence fluctuations. Similarly, an increase in the absolute value of κ, or an increase in the magnitude of the effect of consumption growth on confidence, will imply a stronger response of both valuation ratios to confidence fluctuations.

Given the solution for the return on consumption claim, $r_{c,t+1}$, the innovation to the pricing kernel can be written as (shown in the appendix),

$$m_{t,t+1} - E_{p_t^o}(m_{t,t+1}) = ((\theta-1)k_1A_1\kappa - \gamma)\sigma\eta_{t+1} + (\theta-1)k_1A_1\sigma_e e_{t+1},$$

with $((\theta-1)k_1A_1\kappa - \gamma)$, and $(\theta-1)k_1A_1$ capturing the pricing kernel's exposure to consumption shocks, and exogenous confidence shocks respectively. The exposure to both consumption shocks and exogenous confidence shocks increase as λ increases, and the exposure to consumption shocks also increases as $|\kappa|$ increases.

Although the ambiguity-averse agent acts pessimistically and prices assets under the worst-case measure, we are interested in expected returns under the reference model because it is supposed to be the best estimate of the data generating process based on

historical data. Given $A_{0,m}, A_{1,m}, k_{0,m}, k_{1,m}$, the solution coefficients of expected market return on confidence level is,

$$A_{1,E} = A_{1,m}(k_{1,m}\lambda - 1).$$

Since $\zeta > 1$, if ψ is greater than one, we have $A_{1,m} < 0$, which implies $A_{1,E} > 0$. Thus, ambiguity increases expected market return. The expected return is time-varying because a_t is time-varying.

The risk-free rate can be derived by substituting $r_{c,t+1}$ into the Euler equation (4). As in the appendix, the coefficient on confidence a_t is negative, implying that a low confidence level (high a_t) corresponds to a low interest rate.

2.6 Magnitude of Ambiguity

Given our definition for the set of alternative measures, one natural question is what a reasonable size of ambiguity is, or what the reasonable value for a_t is. I use the error detection probability approach suggested by Anderson, Hansen, and Sargent (2003) to provide a sense of the reasonable magnitude of ambiguity.

This approach quantifies the statistical closeness of two measures by calculating the average error probability in a Bayesian likelihood ratio test of two competing models. Intuitively, measures that are statistically close will be associated with large error probabilities, but measures that are easy to distinguish imply low error probabilities. Formally, let l be the log likelihood function of the worst-case measure relative to the reference measure and P^a be the alternative worst-case measure. Then, the average probability of a model detection error in the corresponding likelihood ratio test is

$$\epsilon = 0.5 \cdot P(l > 0) + 0.5 \cdot P^a(l < 0),$$

where ϵ is just a simple equally weighted average of the probability of rejecting the reference model when it is true ($P(l > 0)$) and the probability of accepting the reference

model when the worst case model is true ($P^a(l < 0)$). To obtain a closed-form solution, assume the size of ambiguity is a constant a; it follows that

$$\epsilon = \Phi(-\tfrac{a}{2}\sqrt{\tfrac{N}{\sigma^2}}),$$

where N is length of consumption data, Φ is the cumulative distribution function of the standard normal, and ϵ is decreasing in N and a and increasing in σ.[15]

In general, a closed-form expression for the detection error probability is not available because the size of ambiguity is not constant over time. However, the linear Gaussian framework for the consumption growth and confidence process in this paper allows one to calculate the exact likelihood function values using the Kalman filter. The state space representation encompasses exactly these two processes: consumption growth process as the measurement equation and confidence process as the state transition equation. The reference measure is i.i.d normal and, thus, the likelihood function is simple; the likelihood function value of the worst-case measure is obtained recursively using the Kalman filter given the simulated data. Then, the error probability is calculated using simulated data. Anderson, Hansen, and Sargent (2003) considered 10% to 20% as a reasonable bound on the detection-error probability. In this paper, parameters are picked such that the detection-error probability is 26%, which means that investors would not be able to identify the correct model 26% of the time.

3 Empirical Findings

Given the analytical solution, in this section, I simulate data by drawing shocks randomly, and show how the simulated data replicate many interesting behaviors and statistics in

[15]This is very intuitive in that, when time period N is long enough, or when the worst case model is different enough (big a), agent can use the consumption data to statistically separate the reference model from the worst case belief. When the variance is big, its hard for agent to distinguish the reference model from alternative models.

the data. Then, I measure the historical exogenous shocks by unusual jumps in stock returns, and feed the model with historical consumption growth and exogenous shocks to see what it tell us about historical movements in stock prices and returns.

3.1 Data

To make the empirical results comparable to the LLR model, I use annual data on consumption and asset prices from 1930 through 2008 in this paper. Consumption data are based on U.S. real nondurables and services consumption per capita from the Bureau of Economic Analysis. Stock return and dividend data are from CRSP and converted to real terms using the consumer price index (CPI). For the real risk-free rate, Beeler and Campbell (2012) created a proxy for the ex-ante risk-free rate by forecasting the ex-post quarterly real return on three-month Treasury bills with past one-year inflation and the most recent available three-month nominal bill yield. I use the same ex-ante risk-free rate.

3.2 Simulation and Calibration

In calibration and simulations, I assume an annual decision interval[16] and then generate three sets of i.i.d. standard normal random variables; I then use these to construct the annual series for consumption, dividends, and the state variable confidence. It is important to note that consumption and dividends growth is constructed under the reference measure, which offers the right dynamics to use for reporting simulation moments in that the it is the one used by agents and provides a good fit to the historical data. Negative realizations of ambiguity size, a_t, will be replaced with zero, in which case investors fully trust the reference model. I set size of ambiguity to the long-run mean values to initialize

[16]The model can also be simulated under a monthly decision interval, but reliable monthly dividend growth data are not available, and thus not able to see the historical performance of the model.

each simulation and discard the first 10 years before using the output.

For statistical inference, I report the median moments from 10,000 simulations run over the sample with 79 observations that match the length of the actual data. I also report the tail percentiles of the Monte-Carlo distributions (5% and 95%).

First, the parameters in the consumption and dividend growth process are calibrated to match the moments of annual per capita U.S. consumption and dividend data. $\mu_c = 0.0193$ is the annual mean consumption growth rate from the data, and the standard deviation of the consumption is chosen to match the standard deviation in the consumption growth data $\sigma = 0.0216$. For the given leverage parameter, ζ, μ_d is chosen such that the average rate of dividend growth is equal to the mean growth rate of dividends in the data. Similarly, for the given leverage ratio, ζ, σ_d can be calibrated to match the standard deviation of dividend growth in the data. Note that the parameters ζ and σ_d together determine the correlation between consumption and dividends growth, and thus the choice of the leverage ratio corresponding to the correlation of consumption and dividend growth. In the literature, the leverage ratio is generally believed to be between 2 and 3. For example, Abel (1999) and Ju and Miao (2012) set the value to be 2.74, BKY (2007) calibrated the value as 2.5. In this setting, $\zeta = 3$ implies that the correlation between consumption and dividend growth is 0.59. In the data (1930 through 2008), the correlation is 0.55, and the correlation could be much lower if using different data sets as reported in Campbell (2003). In this paper, the leverage ratio is set to be $\zeta = 2$ to show that the model works well even with a low correlation.

In terms of preference parameters, I use a risk aversion of 2 and IES of 2. The literature examining the IES magnitude in the data leads to estimates that are both well above and below 1. BY (2004) argued that if consumption volatility is time varying, IES tends to be greater than 1. Epstein, Farhi and Strzalecki (2012) suggested that when using recursive utility and calibrating its parameters, one should make a quantitative

assessment of how much temporal resolution of risk matters. They calculated a timing premium for BY's (2004) calibration of $RRA = 10$ and $IES = 1.5$ and found that the representative agent would give up 20% of lifetime consumption to have all risk resolved next month. The general pattern is that the timing premium is increasing with the product of RRA and IES. My specification for RRA and IES is more reasonable in this sense.

Finally, the time preference β is calibrated to match the level of risk-free rate. The persistence parameter for confidence, λ, is chosen to match the first-order autocorrelation of the price-dividend ratio. One important feature of the model is that the confidence level partially depends on past consumption growth and, therefore, the model can match the correlation of the price-dividend ratio with lagged consumption growth in the data. This feature is controlled by the ratio of κ/σ_e, which is calibrated to match the R^2 of the regression of the price-dividend ratios on one-year lagged consumption growth. At the same time, given the ratio of κ/σ_e, κ or σ_e can be calibrated to match the level of equity return. The parameter a is calibrated to match the error detection probability of 26%; investors would not be able to identify the correct model 26% of the time, which is above the level suggested by Anderson, Hansen, and Sargent (2003). Thus, the size of ambiguity is reasonable. Table 2 provides the parameter configuration used to calibrate the model.

3.3 Basic Moments

Table 3 displays the model implications for the unconditional moments of five variables: the log consumption and dividends growth rates, log stock return, log risk-free interest rate, and log price-dividend ratio. The model is calibrated to match the first and second moments of log consumption and dividends growth rates, the level of log equity return, level of risk-free rate, and the first-order autocorrelation of the log price-dividend ratio,

Preference	β	γ	ψ	a	λ	κ	σ_e
	0.98645	2	2	0.007	0.97	-0.02	0.0044
Consumption	μ_c	σ					
	0.0193	0.0216					
Dividends	μ_d	ζ	σ_d				
	-0.0271	2	0.1017				

Table 2: Configuration of model parameters.

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + \sigma\eta_{t+1} \\
\Delta d_{t+1} &= \zeta\Delta c_{t+1} + \mu_d + \sigma_d u_{t+1} \\
a_{t+1} &= a + \lambda(a_t - a) + \kappa \cdot (\Delta c_{t+1} - \mu_c) + \sigma_e e_{t+1}
\end{aligned}$$

Table 2 reports investors' preferences and endowment process parameters. All parameters are given in annual terms.

with all those moments labeled with a star. The correlation of consumption and dividend growth using the simulation data is 40%.

As shown in Table 3, the model does a good job of matching the key asset price moments. Specifically, the model replicates the level of the log equity returns, log risk-free rate, and the autocorrelation of the price-dividend ratio by choosing parameter values. The remaining moments were not used to pick parameters, so I can use them to check the model's predictions. The median level volatility of log equity return is 17.77, which is very close to the data counterparts. The level of the log price-dividend ratio is also consistent with the data, but with a lower volatility of 0.29. As mentioned in Beeler and Campbell (2012), the persistently high stock price at the end of the sample period partially contributes to the high standard deviation of the log price-dividend ratio; the estimate would be 36% if the sample ended in 1998, which is much closer to this model's median value.

Note that the contribution to total equity return from the exogenous shock to confidence is 34%; from consumption shocks of confidence it is 27%. When there is only constant size in ambiguity, the equity return drops to 2.9%; thus, the stochastic feature

	Data	Model		
Moment	Estimate	Median	5%	95%
$E(\Delta c)$	1.93	1.93*	1.53	2.33
$\sigma(\Delta c)$	2.16	2.15*	1.88	2.44
$E(\Delta d)$	1.15	1.15*	-0.9	3.19
$\sigma(\Delta d)$	11.05	11.01*	9.61	16.71
$E(r_e)$	5.47	5.47*	3.09	8.11
$\sigma(r_e)$	20.17	17.77	15.27	20.33
$AC1(r_e)$	0.02	-0.03	-0.21	0.16
$E(r_f)$	0.56	0.56*	-0.19	0.93
$\sigma(r_f)$	2.89	0.44	0.26	0.74
$AC1(r_f)$	0.65	0.87	0.71	0.95
$E(p-d)$	3.36	3.08	2.58	3.32
$\sigma(p-d)$	0.45	0.29	0.17	0.48
$AC1(p-d)$	0.87	0.87*	0.71	0.95

Table 3: Model implied moments

Table 3 presents moments for the model and data from the annual datasets. The model implied moments displayed is the median, 5%, and 95% percentiles from 10,000 finite sample simulations of equivalent length to the dataset. Column 2 display data statistics that measured in real terms, with data sampled on an annual frequency and cover the period from 1930 to 2008. Means and volatilities of returns and growth rates are expressed in percentage terms. Note the * statistics used to calibrate model parameters.

	$E(r_e)$	$\sigma(r_e)$	$E(r_f)$	$\sigma(r_f)$	$E(p-d)$	$\sigma(p-d)$
Data	5.47	20.17	0.56	2.89	3.36	0.45
Fully Specified Model	5.47	17.77	0.56	0.44	3.08	0.29
$\sigma_e = 0$	3.6	11.88	1.84	0.05	3.7	0.05
$\kappa = 0$	4.01	15.86	1.17	0.44	3.49	0.24
$\sigma_e = 0,\ \kappa = 0$	2.9	11	1.92	0.00	4.04	0.00

Table 4: Basic moments for different settings

Table 4 presents moments for different model settings and data from the annual datasets. The model implied moments displayed are the median values from 10,000 finite sample simulations of equivalent length to the dataset (from 1930 to 2008). Means and volatilities of returns and growth rates are expressed in percentage terms.

of confidence accounts for 47% of total equity return. The first and second moments of equity return, risk-free rate, and price-dividend ratios implied by different model settings are provided in Table 4. The exogenous shocks play the most important role in determining the moments of the simulated data. When $\sigma_e = 0$, equity return becomes 3.6 compared to 5.47 of the fully specified model; the standard deviation of equity return also drops significantly, and the standard deviation of price-dividend ratio becomes 0.05 versus 0.29 in the fully specified model. In the case that $\kappa = 0$, where lagged consumption has no effect on the confidence level, the equity return drops more than 50%; the volatilities of equity return and price-dividend ratio are also significantly lower than before. The consumption growth shock is less important than the exogenous confidence shock, but it also plays an important role in explaining equity return and volatility of equity return. The level and standard deviation drop from 5.47 and 17.77 to 4.01 and 15.86 when shutting down the channel of past consumption growth on confidence.

	Data			Model				
	$\hat{b}$	t	$\hat{R}^2$	50%	50%	50%	5%	95%
				(b)	(t)	(R^2)	(R^2)	(R^2)
Panel A: $\sum_{j=1}^{J}(r_{m,t+j} - r_{f,t+j}) = a + b(p_t - d_t) + \varepsilon_{t+j}$								
1 Y	-0.093	-1.803	0.044	-0.161	-2.565	0.073	0.019	0.161
3 Y	-0.264	-3.231	0.170	-0.440	-3.202	0.190	0.046	0.378
5 Y	-0.413	-3.781	0.269	-0.671	-3.598	0.275	0.058	0.521
Panel B: $\sum_{j=1}^{J}(\Delta c_{t+j}) = a + b(p_t - d_t) + \varepsilon_{t+j}$								
1 Y	0.011	1.586	0.060	-0.001	-0.100	0.006	0.000	0.050
3 Y	0.010	0.588	0.013	-0.002	-0.127	0.017	0.000	0.132
5 Y	-0.001	-0.060	0.000	-0.004	-0.135	0.027	0.000	0.201
Panel C: $\sum_{j=1}^{J}(\Delta d_{t+j}) = a + b(p_t - d_t) + \varepsilon_{t+j}$								
1 Y	0.074	1.977	0.092	-0.001	-0.033	0.006	0.000	0.050
3 Y	0.107	1.330	0.059	-0.005	-0.050	0.017	0.000	0.133
5 Y	0.089	1.214	0.039	-0.008	-0.063	0.028	0.000	0.209

Table 5: Predictability of excess return, consumption, and dividend by dividend yield
Columns 2-4 of Table 5 display coefficients, T-statistics, and R-squared statistics from predictive regressions of excess returns, consumption growth, and dividend growth on log price-dividend ratios using historical data. The data employed in the estimation are real, sampled on an annual frequency and cover the period from 1930 to 2008. Columns 7-9 present the model implied median, 5%, and 95% percentiles R-squared of the predictive regressions from 10,000 finite sample simulations of equivalent length to the dataset. Columns 5-6 display the model implied median of coefficients and T-statistics. Standard errors are Newey–West with 2*(horizon-1) lags.

3.4 Consumption, Dividends, and Return Predictability

In this model, the price-dividend ratio variability is driven by persistent variation in confidence, which creates similar variation in equity returns. Consumption and dividend growth have constant expectations under the reference model; thus, theoretically, the price-dividend ratio will not predict consumption and dividend growth. At the same time, consistent with the model implications, many empirical studies have argued that the log price-dividend ratio predicts excess stock returns and not dividend growth (Campbell and Shiller 1988b; Fama and French1988; Hodrick 1992).

Table 5 provides the model's implied predictability results. I regress excess returns,

consumption growth, and dividends growth, measured over horizons of one, three, and five years, onto the log price-dividend ratio at the start of the measurement period. The reported results include both model and data statistics. Panels B and C show that for both the data and the model there is relatively little predictability in consumption and dividend growth. All coefficients are insignificant in the data and for the model. The R-squares for one-year horizon in the data could come from time aggregation.[17] Beeler and Campbell (2012) ran the same regression for quarterly data over the period 1947 Q2 through 2008 Q4, and found no predictability in consumption and dividend growth for all horizons.

For excess returns, the model-implied median finite-sample coefficients, t-statistics, and R^2s match the data well. Consistent with the data, the model-implied median R^2s rise with maturity, from 7.3% at the one-year horizon to 27.5% at the five-year horizon, and the magnitudes of both coefficients and t-statistics increase with maturity. Because of the well-known Stambaugh (1999) bias in predictive regressions with persistent regressors whose innovations are correlated with innovations in the dependent variable, the coefficients, t-statistics, and R^2 statistics of predictive regressions may not be biased.

BKY (2012) suggested that to reduce the potential spurious regression problem in predictive regressions, one can replace the price-dividend ratio regressor with the price-dividend ratio minus the risk-free rate. The adjusted regressor is not extremely persistent and therefore can reduce the Stambaugh bias. At the same time, the adjusted regressor should make virtually no difference to the price-dividend ratio's predictive power, as only short-horizon risks embodied in the risk-free rate are subtracted from the dividend yield. I report the results of the predictive regression of excess return using the adjusted price-dividend ratio in Table 6. Different from BKY (2012), where they found

[17] When simulating the model under a monthly decision interval, and aggregating the consumption/dividend and price-dividend ratio into the annual frequency, there will be some short-run predictability as in the data.

	Data			Model				
	$\hat{b}$	t	$\hat{R}^2$	50%	50%	50%	5%	95%
				(b)	(t)	(R^2)	(R^2)	(R^2)
$\sum_{j=1}^{J}(r_{m,t+j} - r_{f,t+j}) = a + b(p_t - d_t - r_{f,t}) + \varepsilon_{t+j}$								
1 Y	-0.093	-1.743	0.042	-0.163	-2.563	0.073	0.019	0.161
3 Y	-0.267	-3.225	0.169	-0.447	-3.204	0.189	0.046	0.377
5 Y	-0.417	-3.794	0.267	-0.680	-3.600	0.275	0.059	0.520

Table 6: Predictability of excess return by adjusted dividend yield

Columns 2-4 of Table 6 display coefficients, T-statistics, and R-squared statistics from predictive regressions of excess returns log price-dividend ratios minus risk-free rates using historical data. The data employed in the estimation are real, sampled on an annual frequency and cover the period from 1930 to 2008. Columns 7-9 present the model implied median, 5%, and 95% percentiles R-squared of the predictive regressions from 10,000 finite sample simulations of equivalent length to the dataset. Columns 5-6 display the model implied median of coefficients and t-statistics. Standard errors are Newey–West with 2*(horizon-1) lags.

that the predictive ability of the adjusted dividend yield is much lower than that of the dividend yield,[18] Table 6 shows that the coefficients, t-statistics, and R-squared statistics are similar as before for both the data and the model. This seems to suggest that the predictive regression is stable after using Newey-West standard errors with suitable lags. In addition, the predictability in the model fit the predictability in the data very well.

3.5 Correlation of Price-Dividend Ratios with Lagged Consumption Growth

BKY (2012) argued that for forward-looking models, current price-dividend ratios are determined by changes in the future cash flow dynamic shile price-dividend ratios depend on past cash flow realizations in backward-looking models. In the LRR model, a drop in the current price-dividend ratios reflects a decline in future expected growth and/or a rise in future volatility; thus, it is forward-looking. The habit model (Campbell and Cochrane 1999) is backward-looking in the sense that lagged consumption growth drives the risk

[18] They construct the ex ante risk free rate in a different way.

aversion up and down, leading to changes in the current price-dividend ratio. Because of this distinction, it is important to look at the effect of lagged consumption growth on price-dividend ratios in actual data. BKY (2012) regressed the log of the price-dividend ratio on J lags of consumption growth, where J is from 1 to 5. The point estimate R^2s are relatively small for the short run, and R^2 has a pattern of increasing with J. They also constructed a 95% confidence band around the data estimates using block-bootstrap, from about $[0 - 10\%]$ for $J = 1$, to about $[0 - 20\%]$ for $J = 5$. In the LRR model, the price dividend ratio is predicted by lagged consumption with R^2s that are close to zero for all horizons and match the magnitude of R^2s in short run, but R^2s in the LRR model do not increase in horizon as in the data. At the same time, they found the habit model produces too much predictability, where the R^2s are outside the 95% confidence band for all horizons.

The model in this paper is both forward-looking and backward-looking in the sense that (i) a drop in price-dividend ratios reflects a decline in worst future expected growth and (ii) on the other hand, the decline in worst future expected growth is ***partially*** driven by lagged consumption growth. In addition to the past consumption growth, another exogenous factor influences confidence. Therefore, the model will have a weaker correlation with lagged consumption than backward-looking models that are driven purely by lagged consumption dynamics. To see the magnitude of this correlation, I follow BKY (2012) and run the following regression:

$$p_{t+1} - d_{t+1} = a + \sum_{j=1}^{J} b_j \Delta c_{t+1-j} + u_{t+1}.$$

In the actual data and in the simulated data, I regress the log of the price-dividend ratio on J lags of consumption growth, with $J = 1, 3, 5, 8, 10$. Table 7 reports the results. In the data, price-dividend ratios are predictable when including sufficiently long lagged consumption growth. R^2 changes from 3% to 18% for $J = 5$ and $J = 10$, respectively. This suggests that lagged consumption contributes stock price variation in long run. The

	Data	Model		
	R^2	50%	5%	95%
		R^2	R^2	R^2
$p_{t+1} - d_{t+1} = a + \sum_{j=1}^{J} b_j \Delta c_{t+1-j} + u_{t+1}$				
1 Y	0.007	0.007*	0.000	0.054
3 Y	0.005	0.025	0.002	0.144
5 Y	0.031	0.048	0.007	0.227
8 Y	0.121	0.091	0.016	0.341
10 Y	0.181	0.125	0.025	0.408

Table 7: PD ratio prediction by lagged consumption growth

Columns 2 of Table 7 display R-squared statistics from predictive regressions of PD ratio on lagged consumption growth using historical data.The data employed in the estimation are real, sampled on an annual frequency and cover the period from 1930 to 2008. Columns 3-5 present the model implied median, 5%, and 95% percentiles R-squared of the predictive regressions from 10,000 finite sample simulations of equivalent length to the dataset.

model matches well the correlation between the log of the price-dividend ratio and lagged consumption growth, with median R^2 changing from 0.7% to 12.5% for the 1- and 10-year horizons.[19] In addition, similar to the data, the model implies that R^2s are increasing in the horizon. While the median R^2s are somewhat larger than their data counterparts for the three-year horizon prediction, the confidence bands for these R^2s contain the data magnitudes. In addition, all the model-implied median R^2s fall into the 95% confidence band around the data estimates constructed by BKY (2012).

3.6 The Correlation of Consumption Growth with Stock Returns

Most consumption-based asset pricing models imply that equity returns are highly, if not perfectly, correlated with consumption growth. However, in the data, the correlation is low. As Cochrane and Hansen (1992) emphasized, the actual low correlation lies at the heart of many empirical failures of those models. This fact, together with the weak

[19]Note that the $R^2 = 0.7\%$ for $J = 1$ is from calibration.

correlation of price-dividend ratio with lagged consumption growth in section discussed 3.5, implies that stock prices and returns should not be driven only by the uncertainty in consumption.

In this model, stock prices and returns are driven by changes in confidence, which in turn depend partially on past consumption growth and partially on exogenous confidence shocks. As shown in section 3.3, the exogenous confidence shocks play the most important role in explaining stock prices and returns. Hence the model implies a low correlation between consumption growth and stock returns. Table 8 provides correlations between consumption growth and excess returns for different leads and lags. In the data, the contemporaneous correlation between consumption growth and stock returns is low, and returns are negatively correlated with past consumption growth and positively correlated with subsequent consumption growth. The highest correlation is with next year's consumption growth, 0.62 in the data.

Using simulated data, the contemporaneous correlation is 0.32; there is little correlation with previous consumption and no correlation with future consumption. Even though the contemporaneous correlation is higher for the model than in the data, it is much closer to the data than many other models. In addition, as Campbell and Cochrane (1999) argued, assuming a monthly decision interval and then aggregating the simulation data to annual frequencies will help generate a similar correlation pattern to that in the data: low contemporaneous correlation, negative correlation with previous consumption growth, and positive correlation with next period consumption growth. When simulating the model under a monthly decision interval, I find the same effect, and the correlation is much closer to the data.

The big positive correlation between returns and next year's consumption growth is typically interpreted as evidence that returns move on news of future cash flows. I would interpret this in a different way. In this model, returns are driven by confidence,

Correlation	Data	Model	
		Simulation Data	Historical Data
r_t^e,Δc_{t-2}	-0.14	-0.01	-0.18
r_t^e,Δc_{t-1}	-0.16	-0.01	-0.10
r_t^e,Δc_t	0.14	0.32	0.43
r_t^e,Δc_{t+1}	0.62	0.00	0.59
r_t^e,Δc_{t+2}	0.16	0.00	0.27

Table 8: Correlation between excess returns and consumption growth

Columns 2 of Table 8 displays correlation between historical consumption growth and returns.The data employed in the estimation are real, sampled on an annual frequency and cover the period from 1930 to 2008. Columns 3 presents the model implied median correlation between consumption growth and returns from 10,000 finite sample simulations of equivalent length to the dataset. Columns 4 presents correlation between historical consumption growth and the model implied returns by feeding the model with historical data.

which depends partially on past consumption growth and partially on the exogenous confidence shocks. For simplicity, the consumption growth is i.i.d. and uncorrelated with exogenous shocks in this model. While in a production economy, for example, Ilut and Schneider (2012) and Zhao (2013), exogenous confidence shocks have real effects and cause future consumption growth changes. Thus, confidence shocks are usually positively correlated with future consumption.[20] The positive correlation between returns and next year's consumption growth could be due to the causal relationship of confidence shocks and future consumption growth, which is not the focus of this paper. Even missing this causal relationship, when feeding the model with historical consumption growth and confidence shocks, the model implies returns correlated with the historical consumption growth similar to the data, and the match will be better if I use a monthly interval and aggregate into annual rates.

[20]When the exogenous confidence shock data are constructed from unusual stock return jumps as described in the following section, I find a correlation between confidence shocks and next year's consumption growth of 0.41.

3.7 Price-Dividend Ratio and Excess Return

In addition to the preceding facts, many empirical studies have documented puzzling links between aggregate asset markets and macroeconomics: Price–dividend ratios move pro-cyclically (Fama and French 1989) and conditional expected equity premiums move counter-cyclically (Campbell and Shiller 1988a, 1988b; Fama and French 1989).

In this model, the log price-dividend ratio is a linear function of confidence and consumption volatility, with a negative coefficient on size of ambiguity, $A_{1,m}$; a drop in consumption growth lowers the confidence level, increases the size of ambiguity, a_t, and thus lowers the price-dividend ratio. The model implies a pro-cyclical price-dividend ratio variation. Because aggregate consumption growth and output growth are positively correlated, I use consumption growth as a pro-cyclical economic indicator. The median correlation between the log price-dividend ratio and consumption growth in the model simulation, $Corr(\Delta c_t,\ p_t - d_t) = 0.04$, is lower than their correlation in the data, 0.19. This could be due to the causal relationship of confidence shocks and future consumption growth that is not modeled in this paper. When feeding the model with actual data, the correlation between the model implied log price-dividend ratio and real consumption growth is 0.31.

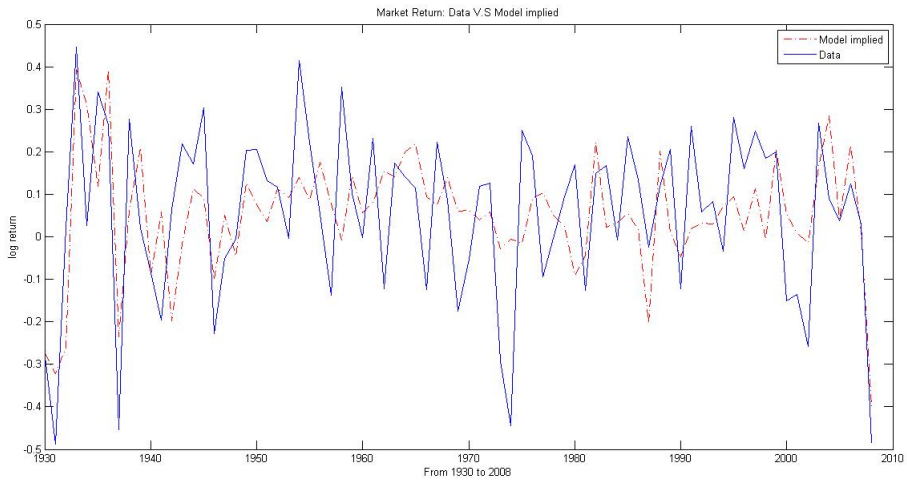
As shown in section 2.6, the coefficient on confidence, or the size of ambiguity for the expected return, $A_{1,E}$, is positive. A high consumption growth realization increases investors' confidence level, decreases a_t, and results in low expected return. Therefore, the model implies that conditional expected equity returns move counter-cyclically. In the data, the correlation between consumption growth and future excess return, $Corr(\Delta c_t,\ r_{e,t+1} - r_{f,t+1}) = -0.10$. The median correlation implied by the model is -0.01, which is lower than its data counterpart. Again, when feeding the model with actual data, the correlation between the model-implied excess return and real consumption growth is -0.10 and matches the data exactly.

3.8 Model-Implied Historical Equity Returns

Instead of simulated artificial data, in this section, I feed the model actual data and see the model's prediction for equity returns. To calculate historical confidence levels, I need a measure for the exogenous confidence shocks, which can be interpreted as major economic and political shocks like the Cuban missile crisis, the assassination of JFK, the OPEC I oil-price shock, and the 9/11 terrorist attacks. When such shocks occur, historically there are big jumps in stock returns. Thus, one natural measure of those shocks would be the size of the jumps. Using the same method as in section 2.3, I first collect daily stock returns that have big movements, bigger than 2 standard deviations, and aggregate them to annual frequency. To eliminate the potential effect of consumption growth on those big return movements, and also to be consistent with the model's assumption that the exogenous shocks are uncorrelated with consumption growth shocks, I then run a regression of the measure of big return movements on consumption growth, and take the residual as the adjusted measure. Finally, I rescale and demean the adjusted series such that the resulting measure has the same mean and standard deviation as the model's assumptions for the exogenous shocks. Figure 2 presents the model's prediction for equity returns by feeding real consumption growth and the measure for exogenous shocks into the model, together with actual market returns.

To my eyes, the model provides a good account of fluctuations in stock returns, with a correlation between the model-implied returns and the data of 0.56. In particular, the model matches well the data during the Great Depression, World War II, and the Great Recession. The worst performance occurs from early the 1950s to the late 1970s. One possible explanation is that an agent's belief in the Fed's ability to stabilize the economy may affect confidence. If an agent believes that the central bank can successfully stabilize the economy, he or she will not fear confidence shocks. However, if the agent doesn't believe that the central bank can stabilize the economy, even small confidence shocks

can have big effects on the confidence level. The persistently high inflation from the late 1960s to the early 1980s may indicates that the Fed didn't do a good job of stabilizing the economy, which implies that the confidence shocks should have had bigger effects. Based on Figure 2, it seems that the model would match the data better using the modified process. Thus, the poor performance could be due to this missing feature of confidence.[21]



4 Conclusion

Alternative asset pricing models generally are able to account for the equity premium, volatility and risk-free rate puzzles. However, the behavior of price-dividend ratios in the data pose a serious challenge to many models. On the one hand, price-dividend ratios predict returns but not consumption and dividend growth. If model-implied price variation comes from future changes in the consumption/dividend growth process, then

[21] Zhao (2013) argued that the agent's belief regarding the central bank's ability to stabilize the economy depends on its past performance. In addition, Zhao studied the asset return implications in a New Keynesian framework by incorporating this feature into the confidence process. Another way of including this fearture in the simple model of this paper is to use an exogenous inflation process that induces agent's belief about Fed's ability. Then confidence process will have heterogeneous volatility and thus for stock returns/price-dividend ratios/interest rates.

the price dividend ratio counterfactually predicts consumption and dividend growth. On the other hand, the price-dividend ratio has a weak correlation with lagged consumption in the short run, but, as documented in this paper, lagged consumption growth predicts the future price-dividend ratio in the long run. Consistent with the low correlation of the price-dividend ratio and past consumption growth, the contemporaneous correlation between equity returns and consumption growth is low. This suggests that cash flow dynamics cannot be the only source of price variation. Otherwise, the model would either generate too much contemporaneous correlation between equity returns and consumption growth or counterfactually imply too strong a correlation between the price-dividend ratio and past consumption. Similarly, if price variation has nothing to do with consumption, the correlation between the price-dividend ratio and past consumption would be too weak compared to the data, especially for the long run.

In this paper, departing from the rational expectation hypothesis that there is a single objective probability (coinciding with the investor's subjective belief) measure governing the state process, I assume the investor is ambiguity averse. Investors' confidence, or the size of ambiguity, is represented by a set of one-step-ahead measures regarding the consumption growth rate. Changes in confidence correspond to changes in the set of expected consumption growth rates.

I argue that aggregate asset prices variation is driven by changes in confidence about future consumption growth. When consumption growth is low, or there is a negative confidence shock, investors become less confident about future consumption growth, leading to a fall in the current asset price and a rise in expected return. When consumption growth improves, or positive shocks occur, investors become more confident, resulting in a rise in the current asset price, and a drop in expected return. Therefore, in this model, price-dividend ratios predict market returns as in the data. However, the expected consumption growth rate is i.i.d. under the reference model, which represents

the best estimate of the consumption growth process and is used for simulation. This implies that price-dividend ratios do not predict cash-flow as in the data. Since confidence is partially driven by consumption growth realizations, the model implies a weak correlation between price-dividend ratios and short-run lagged consumption growth, and the correlation becomes stronger when more lagged consumption growth is included in the regression. Similarly, the contemporaneous correlation between equity returns and consumption growth is low as in the data.

In addition to the preceding stylized facts, the model matches well the basic moments of equity return, risk-free rate, dividend yield, consumption growth, and dividend growth. The model also generates pro-cyclical variation of price-dividend ratios and counter-cyclical variation of excess return.

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Appendix

The endowment process in the model is

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + \sigma\eta_{t+1} \\ \Delta d_{t+1} &= \zeta\Delta c_{t+1} + \mu_d + \sigma_d u_{t+1} \end{aligned} \tag{8}$$

And the confidence process is

$$\begin{aligned} &a_t = a + \lambda(a_{t-1} - a) + \kappa \cdot (\Delta c_t - \mu_c) + \sigma_e e_t \\ &a_t = a + \lambda(a_{t-1} - a) - \kappa a_{t-1} + \kappa\sigma\eta_t + \sigma_e e_t \text{ - worst case measure} \\ &a_t = a + \lambda(a_{t-1} - a) + \kappa\sigma\eta_t + \sigma_e e_t \text{ - reference measure} \end{aligned} \tag{9}$$

with η_{t+1}, u_{t+1}, $e_{t+1} \sim i.i.d.\ N(0,1)$.

The Worst-Case Belief

First, I prove the worst-case belief is the one with lowest expected growth rate, $-a_t$. Given the endowment and confidence process, rewrite the utility over consumption as

$$\frac{V_t}{C_t} = \left(1 - \beta + \beta\left\{\min_{p_t \in \mathcal{P}_t} \mathbb{E}_{p_t}\left[\left(\frac{V_{t+1}}{C_{t+1}}\right)^{1-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right\}^{\frac{1}{\theta}}\right)^{\frac{\theta}{1-\gamma}}.$$

Let $v_t = \frac{V_t}{C_t}$, and the state variable is a_t. Now the utility can be rewritten as

$$v_t(a_t) = \left(1 - \beta + \beta\left\{\min_{\tilde{\mu}_t \in [-a_t, a_t]} \mathbb{E}_{\tilde{\mu}_t}\left[(v_{t+1}(a_{t+1}))^{1-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right\}^{\frac{1}{\theta}}\right)^{\frac{\theta}{1-\gamma}}.$$

Since the process for a_t is independent of the choice of $\tilde{\mu}_t$, thus choice of $\tilde{\mu}_t$ has no effect on a_{t+1} . Plus $v_t(a_t)$ is increasing function of $\tilde{\mu}_t$, therefore the worst-case belief

is the one with lowest expected growth rate, $\tilde{\mu}_t = -a_t$. Then the min operator can be replaced with the worst-case measure.

Solving the Model

The Euler equation for the economy is evaluate under the worst-case measure,

$$\mathbb{E}_{-a_t}\left[exp\left(\theta log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{c,t+1} + r_{i,t+1}\right)\right] \quad = \quad 1 \tag{10}$$

,

where $r_{c,t+1}$ is the log return on the consumption claim and $r_{i,t+1}$ is the log return on any asset.

First, conjecture that the log price-consumption ratio, z_t, and the log price-dividend ratio for a dividend claim, $z_{t,m}$,follow,

$$z_t = A_0 + A_1 a_t, \tag{11}$$

$$z_{t,m} = A_{0,m} + A_{1,m} a_t. \tag{12}$$

The log return on consumption claim and log return on dividend claim are given by the Campbell and Shiller (1988b) approximation,

$$r_{c,t+1} = k_0 + k_1 z_{t+1} + \Delta c_{t+1} - z_t. \tag{13}$$

$$r_{m,t+1} \quad = \quad k_{0,m} + k_{1,m} z_{t+1,m} + \Delta d_{t+1} - z_{t,m}. \tag{14}$$

Return on Consumption Claim

In order to solve A_0, A_1, I substitute (11), (13), (8), and (9) into Euler equation (10). z_t can be found by the method of undetermined coefficients, using the fact that the Euler equation must hold for all values of state variable a_t. Collecting all terms involving a_t, it follows that,

$$A_1 = \frac{1-\frac{1}{\psi}}{k_1(\lambda-\kappa)-1},$$

and collect all terms involving constant implies that

$$A_0 = \frac{log\beta+(1-\frac{1}{\psi})\mu_c+k_0+k_1A_1(1-\lambda)a+0.5\theta(1-\frac{1}{\psi}+k_1A_1\kappa)^2\sigma^2+0.5\theta(k_1A_1\sigma_e)^2}{1-k_1}.$$

Pricing Kernel/IMRS

The log pricing kernel is,

$$m_{t,t+1} = logM_{t,t+1} = \theta log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{c,t+1}.$$

And given the solution for return on consumption claim, $r_{c,t+1}$, the innovation to pricing kernel can be written as ,

$$m_{t,t+1} - \mathbb{E}_{-a_t}(m_{t,t+1}) = ((\theta-1)k_1A_1\kappa-\gamma)\sigma\eta_{t+1} + (\theta-1)k_1A_1\sigma_e e_{t+1},$$

Risk-Free Rate

Given $r_{c,t+1}$ solved above, the log pricing kernel, can be used to solve the risk-free rate,

$$r_{f,t} = log(\frac{1}{\mathbb{E}_{-a_t}(M_{t,t+1})}).$$

The solution for risk-free rate is $r_{f,t} = A_{0,f} + A_{1,f}a_t$, with

$$A_{0,f} = \begin{array}{c} -\theta log\beta + \gamma\mu_c - (\theta-1)k_0 - (\theta-1)k_1A_0 - (\theta-1)k_1A_1(1-\lambda)a- \\ 0.5((\theta-1)k_1A_1\sigma_e)^2 - 0.5((\theta-1)k_1A_1\kappa-\gamma)^2\sigma^2 + (\theta-1)A_0 \end{array},$$

$$A_{1,f} = -\frac{1}{\psi}.$$

Return on Dividend Claim

Given the solution for log return on consumption claim, substitute $r_{c,t+1}$, (12), (14), (8), and (9) into Euler equation (10) and use the method of undetermined coefficients, $A_{0,m}, A_{1,m}$ can by found in a similar way,

$$
\begin{aligned}
A_{1,m} &= \frac{\zeta - \frac{1}{\psi}}{k_{1,m}(\lambda - \kappa) - 1} \\
A_{0,m} &= \frac{\begin{array}{c} \theta log\beta + (\zeta - \gamma)\mu_c + \mu_d + (\theta - 1)(k_0 + A_0(k_1 - 1)) + k_{0,m} \\ +0.5[((\theta - 1)k_1 A_1 + k_{1,m}A_{1,m})^2\sigma_e^2 + \sigma_d^2 + ((\theta - 1)k_1 A_1 \kappa + k_{1,m}A_{1,m}\kappa - \gamma + \zeta)^2\sigma^2] \\ +(\theta - 1)k_1 A_1(1 - \lambda)a + k_{1,m}A_{1,m}(1 - \lambda)a \end{array}}{1 - k_{1,m}}
\end{aligned}
$$

The expected equity return are calculated under the reference measure that fit the data best, given $A_{0,m}, A_{1,m}$, it is straight forward to find the expected market return under reference measure, $\mathbb{E}_t(r_{m,t+1}) = A_{0,E} + A_{1,E}a_t$, with

$$
\begin{aligned}
A_{1,E} &= A_{1,m}(k_{1,m}\lambda - 1) \\
A_{0,E} &= k_{0,m} + (k_{1,m} - 1)A_{0,m} + k_{1,m}A_{1,m}(1 - \lambda)a + \zeta\mu_c + \mu_d
\end{aligned}
$$

The innovation in market return $r_{m,t+1} - \mathbb{E}_t(r_{m,t+1}) =$

$$(k_{1,m}A_{1,m}\kappa + \zeta)\sigma\eta_{t+1} + k_{1,m}A_{1,m}\sigma_e e_{t+1} + \sigma_d u_{t+1}.$$

The conditional variance of market return is

$$(k_{1,m}A_{1,m}\kappa + \zeta)^2\sigma^2 + \sigma_d^2 + (k_{1,m}A_{1,m}\sigma_e)^2.$$

And the equity premium is given by $\mathbb{E}_t(r_{m,t+1} - r_{f,t}) + 0.5Var_t(r_{m,t+1})$.