Optimal Policies with an Informal Sector[1]

Katherine Cuff
McMaster University

Nicolas Marceau
Université du Québec à Montreal

Steeve Mongrain
Simon Fraser University

Joanne Roberts
University of Calgary

12 October 2010

[1]We have benefited from comments by Patrick Francois, Wolfram Richter, participants of the 2009 Public Economics Conference in Honour of Robin Boadway, two anonymous referees and the co-editor Wojciech Kopczuk. All authors are grateful to the Social Sciences and Humanities Research Council of Canada for financial support. Cuff and Roberts also thank the Canada Research Chair program for its support, and Mongrain also thanks Metropolis BC, Center of Excellence for Research on Immigration and Diversity for its support. The first author thanks John Weymark and the Department of Economics at Vanderbilt University for hosting her research leave while this paper was written. The usual disclaimer applies.

Abstract

This paper characterizes optimal policies in the presence of tax evasion and undocumented workers. In equilibrium, domestic workers may work exclusively in the formal sector or also in the informal sector. Surprisingly, in equilibrium, wages are always equalized between domestic and undocumented workers, even if they do not work in the same sectors of the economy. This is driven by the interaction of firm level decisions with optimal government policy. We also find that enforcement may not always be decreasing in its cost, and that governments will optimally enforce labour market segmentation if enforcement costs are not too high.

Key Words: Informal Labour Market; Enforcement; Undocumented Workers; Public Good Provision

JEL: H32, H26, K42

1 Introduction

The informal economy affects not only the size and scale of productive output, but also optimal government policy. This sector arises for a variety of reasons: perhaps primarily, as a source of employment for undocumented workers and as a method of evading taxes for employers. These two motivations have been studied independently; in this paper, we look at them jointly and find that illegal immigration has a large impact on the nature of optimal tax and enforcement policy, and interacts with standard tax evasion incentives, playing an important role not only in the determination of equilibrium wages, but also in the organization of production across the formal and informal sectors.

In developed countries, illegal immigration, tax evasion, and the informal economy are of sufficient importance to impact on the performance of the economy. Even conservative estimates suggest these phenomena are large and economically significant. According to a report of the Pew Hispanic Center, the number of illegal immigrants living in the United States was 11.9 million in March 2008, of which 8.3 million participated in the U.S. labor force (Passel and Cohn, 2009). These numbers imply that unauthorized immigrants are close to 4% of the U.S. population and no less than 5.4% of its workforce. Estimates of the number of illegal immigrants in Canada by police and immigration personnel range between 50,000 and 200,000 according to the Canadian Encyclopedia.[1] Given estimates of this size, it is not surprising that immigration policy is the focus of much public debate. Tax evasion by individuals is also an important phenomenon.[2] For example, Slemrod and Yitzhaki (2002) report that in the United States, according to the Internal Revenue Service, 17% of personal income tax liabilities were simply not paid in 1992. Finally, while measuring the size of the informal sector is notoriously difficult, Schneider and Enste (2000) provide estimates for a large number of countries. According to their estimates for the early nineties, the smallest informal sectors (8-10% of the economy) were in Austria, Switzerland, and the United States. At the other extreme were some developing countries where the informal sector represented 68-76% of the economy (e.g. Egypt, Nigeria, Thailand, Tunisia). As for Canada, its informal sector ranges between 10-13.5% of its economy.

The theoretical literature on each of the above phenomena is large but somewhat segmented in that it tends to address each of them separately. For example, the tax

[1] See the article on *Immigration Policy* at www.thecanadianencyclopedia.com.

[2] The evidence of tax evasion by firms is very limited.

evasion literature is not really concerned with the informal sector or illegal immigration. Initiated by Allingham and Sandmo (1972) and surveyed by Slemrod and Yitzhaki (2002), the tax evasion literature mainly focuses on the decision by individuals, otherwise perfectly honest, to conceal a portion of their income from the tax authorities. Following Reinganum and Wilde (1985), an important secondary strand of that literature characterizes the optimal auditing policies of a tax authority facing individuals behaving à la Allingham and Sandmo (1972). Ethier (1986) initiated the literature on illegal immigration by studying its impact on the host country, while Bond and Chen (1987) enriched Ethier's model by adding a second country and capital mobility to examine the welfare effects of firm level enforcement. It is probably fair to say that a significant portion of the literature that followed these papers focuses on the impact of illegal immigrants on the well-being of domestic workers,[3] and that tax evasion was not a primary issue of concern for those working in this area. Finally, there is a theoretical literature on the informal sector. For example, Rauch (1991), Fortin *et al.* (1997), Fugazza and Jacques (2003), and de Paula and Scheinkman (2007), all model the choice of entrepreneurs to operate in the formal or the informal sector, based on factors like scale economies, wage regulations and taxes. However, this literature is not concerned with the presence of illegal immigrants despite the fact that their very presence may affect this choice.

Few models have integrated the above three phenomena despite the fact that there are obvious connections between them, and no paper that we are aware of has looked at optimal policy in this context. The presence of undocumented workers reduces the cost to firms of entering the informal sector, relative wages affect the incentives of documented or domestic workers to work in either sector, and the willingness of firms to move into the informal sector reduces the capacity of the state to raise tax revenue and fund public goods. In this paper, we allow for all of these channels.

The starkest result that we find is that wages are always equalized across the formal and informal sectors (except of course in the presence of a binding minimum wage in the formal sector). Clearly, when the labour market is what we call non-segmented, where both undocumented and some domestic workers work in the informal sector, wages are equalized across sectors. However, this is the case even when the labour market equilibrium is characterized as being segmented, where domestic workers only

[3]To date, there is no consensus on the empirical impact of immigration (legal/illegal) on the native population (employment, wages). See Borjas (1999) and Borjas, Grogger and Hanson (2008).

work in the formal sector and undocumented workers work in the informal sector.[4] This is due to the fact that in such a segmented equilibrium wages are determined by a combination of firm decisions and optimal policy.

Enforcement and taxes interact in somewhat subtle ways. For example, in contrast to standard findings in the literatures on crime and tax evasion,[5] optimal enforcement may not always be decreasing in its cost. In a segmented equilibrium, optimal enforcement and taxes are complementary policies. Since enforcement is costly in terms of the public good, when the marginal value of the public good is high, increasing the cost of the enforcement may actually lead the government to want to increase enforcement to maintain public good provision. When the marginal benefit is small, increasing the cost of enforcement leads to a reduction in optimal enforcement. We also find that increasing the number of undocumented workers increases the cost of public good provision. This is because more undocumented workers reduce informal wages, making it more difficult to raise tax revenue in the formal sector. As a consequence, if society places insufficient weight on the consumption of the public good by these undocumented workers, optimal public good provision will fall as the total number of undocumented workers increases.

Although, market segmentation maximizes the size of the tax base, if the cost of enforcement is too high, it will not be socially optimal to ensure all domestic workers stay in the formal sector. For these cost ranges, optimal policies will enforce a non-segmented equilibrium and domestic workers will choose to work in both the formal and informal sectors. In a non-segmented equilibrium, taxes and enforcement are no longer tied together and the size of the formal sector will be decreasing in the cost of enforcement and the number of undocumented workers. We also find that the optimal public good provision behaves in the same way as in the segmented equilibrium, but for slightly different reasons.

We extend our base model in several important ways. We also consider amnesties for undocumented workers, and find that at the margin, they are socially beneficial. We also consider what happens when firms in the formal sector are obligated to

[4] Djajic (1997) has a related model and considers a segmented equilibrium, but in that model, wages of the two sectors are disconnected in a segmented equilibrium. Although he identifies the possibility of these two types of equilibria, there is no discussion in his paper about the optimality of either type of equilibrium nor conditions under which one type of equilibrium is socially preferred to the other.

[5] See, for example, Kaplow (1990) and Becker (1968).

pay a minimum wage to workers. A minimum wage breaks the arbitrage condition linking formal and informal sector wages. It also increases the cost of operating in the formal sector, and so it can reduce the ability of the government to collect taxes. Consequently, the presence of a minimum wage strengthens the need for enforcement. Finally, we consider what happens when the number of undocumented workers or the level of illegal immigration is endogenous. We obtain similar results, but also find an additional interesting feature: the government can use the level of public provision directly as a way of altering the number of illegal migrants and as a lever on the informal wage. This is due to the fact that the migration decision results from comparing source country utility with the destination country utility. Increasing the public good provided makes a destination country more attractive, which encourages migration. The arbitrage condition then implies that informal sector wages must fall. We are unaware of other papers that formally model this mechanism – that reducing the public good can substitute for increased enforcement.

To our knowledge, there are only two papers similar to ours in the literature. In Djajic (1997), the incentive of firms to hire undocumented immigrants arises because of a wage differential between the formal and the informal sectors, but not from the obligation to pay taxes when hiring domestic workers in the formal sector. In our model, firms may be tempted by the informal sector because of a wage differential but also because they want to evade taxes. Djajic also provides a positive analysis of the impact of government policies (enforcement, increase in the stock of illegal workers) but he does not characterize optimal policies. Epstein and Heizler (2007) construct a partial equilibrium model in which a representative firm may hire domestic workers and/or undocumented immigrants. This is in contrast with our general equilibrium model in which firms cannot simultaneously hire both types of workers. Epstein and Heizler also do not tax firms to provide a public good; while in our model, tax evasion incentives play a central role. Like us, they perform an analysis of optimal enforcement policies.

The paper is organized as follows: Section 2 presents the basic version of the model, Section 3 characterizes optimal policies, and Section 4 examines the extensions by incorporating amnesties, a minimum wage, and endogenous migration. Section 5 concludes.

2 The Model

We model a simple economy in which firms may choose to operate in either an informal sector to evade taxes, or to operate in a formal and regulated sector. There are M domestic workers who can work either in the formal sector (M_F) or the informal sector (M_I) where $M_F + M_I = M$. There are also U undocumented workers who can only work in the informal sector where $M > U$. Each domestic and undocumented worker, if hired, supplies one unit of labour inelastically. Domestic workers choose to work in the sector offering the highest wage.

The economy has N entrepreneurs with varying productivity θ. For simplicity, we assume that productivity is uniformly distributed on $[0, 1]$. Each entrepreneur is endowed with an income of k that can be consumed or invested to start-up a firm. The number of firms in the economy is endogenous and depends on government policies. Entrepreneurs can choose to not operate a firm (N_0 make this choice), to start-up a firm in the formal sector (N_F make this choice) or to start-up a firm in the informal sector (N_I make this choice) so $N_0 + N_F + N_I = N$. Both informal and formal sector firms produce the same good X, which is sold at an exogenous price normalized to one. To produce output, entrepreneurs need to hire one worker and, to guarantee full employment, we assume that $N > M + U$.

All $M + N + U$ individuals have an identical utility function $x + v(G)$, with $v' > 0 > v''$ and $v'(0) \to \infty$, where x is consumption of a private good and G is the amount of a public good provided by the government. An entrepreneur who does not start a firm can consume his endowment and obtain $\Pi_0(\theta) = k + v(G)$. Operating in the formal sector requires k to be invested.[6] A formal firm produces θ units of the private good X, and pays the formal wage w_F to its worker and the tax t to the government.[7] This yields utility in the formal sector of

$$\Pi_F(\theta) = \theta - w_F - t + v(G). \tag{1}$$

[6]The start-up investment costs could also be some positive fraction of k — with the remainder of k being consumed by the entrepreneur operating the firm — without changing our results.

[7]We assume that θ is unobservable. If it were observable, the government would use this information and a proportional tax to directly target lower taxes to firms on the formal-informal sector margin and to remove all incentive to enter into the informal sector. Making θ unobservable is the simplest way to highlight the formal-informal sector decisions that we are interested in.

An informal firm has the same investment costs and output[8] as a formal firm, but pays the informal market wage w_I instead of the formal wage. Firms in the informal sector are exposed to penalties, but they also have the ability to conceal their identity. We assume a very simple concealment technology, where a firm in the informal sector can perfectly avoid detection at a cost of $e\theta$.[9] This cost is increasing with the amount of public enforcement e, like in Slemrod (1994), and since larger firms are more costly to conceal, this cost is also increasing in θ, which is similar to Kopczuk (2001).[10] As long as the cost of concealment is low relative to the expected sanction, all firms in the informal sector will choose to invest in concealment.

This yields utility in the informal sector of

$$\Pi_I(\theta) = \theta - w_I - e\theta + v(G). \tag{2}$$

The government levies taxes on formal sector firms to finance the provision of a pure public good G that is available to all residents and it may also invest in costly enforcement to reduce tax evasion. The cost of a unit of the public good is unity. Similarly to Slemrod (1994), we model enforcement as any resources devoted to increase compliance, including audits, and so cost of enforcement is simply assumed to be given by $c \cdot e$. Therefore, the government budget constraint is

$$G + ce = N_F t. \tag{3}$$

where N_F is the number of firms (entrepreneurs) operating in the formal sector and paying taxes (endogenized below).

2.1 Entrepreneurs' Decision

Now, consider the optimal decisions of entrepreneurs. Given wages and government policies, entrepreneurs will decide whether or not to start a firm, and if they start a firm, which type. We restrict attention to the case where at least some entrepreneurs want to start formal firms.[11] Note that the slope of the utility function (with respect

[8] Informal firms could also be less efficient producers as in Fortin *et al.* (1997). This would affect the marginal decision between the sectors, as expected, but our main results would not be affected.

[9] For similar frameworks, see Kopczuk (2001) and Slemrod (1994, 2001).

[10] Empirical support for the relationship between size of firm and detection can be found in Kleven, Kreiner and Saez (2009) and Gordon and Li (2009).

[11] Given our restrictions on $v(G)$, it will never be optimal for government policy to foreclose this sector completely.

to entrepreneur's productive ability) is 1 in the formal sector, and $1-e$ in the informal sector. The ability level $\hat{\theta}$ that makes an entrepreneur indifferent between starting a firm in the formal sector and starting a firm in the informal sector is determined from the intersection of the two utility functions (see Figure 1). Because the relative cost of operating in the informal sector is increasing in θ, all entrepreneurs with $\theta > \hat{\theta}$ prefer to operate in the formal sector, while all entrepreneurs with $\theta < \hat{\theta}$ prefer operating in the informal sector where

$$\hat{\theta} = \frac{w_F - w_I + t}{e}. \tag{4}$$

Analogously, we define $\bar{\theta}$ as the ability that makes an entrepreneur indifferent between starting a firm in the informal sector and not starting a firm at all. Since utilities are increasing in productivity, all entrepreneurs with $\theta > \bar{\theta}$ prefer starting a firm, while entrepreneurs with $\theta < \bar{\theta}$ prefer to not start a firm where

$$\bar{\theta} = \frac{w_I + k}{1 - e}. \tag{5}$$

Since $k > 0$, the least productive entrepreneur ($\theta = 0$) never starts a firm. Consequently, $\bar{\theta} > 0$.

With $\hat{\theta} > \bar{\theta}$, as in Figure 1, an informal sector will exist. Entrepreneurs below $\bar{\theta}$ do not start-up a firm, those between $\bar{\theta}$ and $\hat{\theta}$ start a firm in the informal sector, and those above $\hat{\theta}$ start a firm in the formal sector. In this situation, formal sector labour demand will be given by $N_F = N(1 - \hat{\theta})$ and informal sector labour demand will be given by $N_I = N(\hat{\theta} - \bar{\theta})$.

2.2 Equilibrium Wages

To close the model, we need: (1) labour demand to equal labour supply in both sectors, (2) a wage arbitrage condition equating wages across the sectors to hold when some domestic workers work in the informal sector since domestic workers are perfect substitutes across sectors[12] and (3) the number of firms in the formal sector to not exceed the number of domestic workers. Note that when only undocumented workers work informally condition (2) does not need to be satisfied, as undocumented workers cannot work in the formal sector. The market will clear in this case as long as the formal wage is at least as large as the informal wage so that all domestic workers

[12] Of course, if informal sector workers could be partially excluded from the public good that would affect domestic workers' willingness to accept employment in the formal sector.

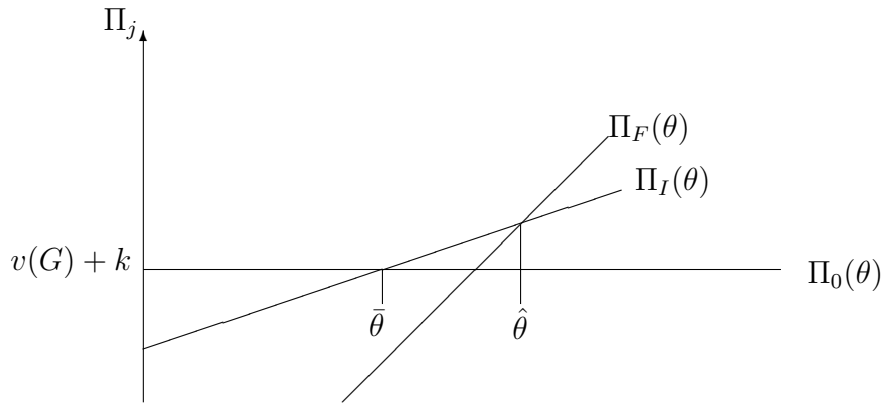

Figure 1: Utility in the Formal and Informal Sectors ($\hat{\theta} > \bar{\theta}$).

prefer to work in the formal sector. Consequently, we distinguish between two types of equilibria. In the first type of equilibrium, labour markets are **segmented** and no domestic workers are employed in the informal sector so $M_F = M$ and $M_I = 0$. In the second type of equilibrium, labour markets are **non-segmented** and some domestic workers choose to work in the informal sector so $M_I > 0$. The type of equilibrium obtained depends on government policy and so will be endogenous.

For condition (1) to be met, the supply of workers in the informal sector must equal the demand for workers by informal firms:

$$M_I + U = N(\hat{\theta} - \bar{\theta}); \tag{6}$$

and the supply of domestic workers must equal the demand for workers by formal firms:

$$M_F = N(1 - \hat{\theta}). \tag{7}$$

From (6) and (7), and using $M = M_I + M_F$ we find that $\bar{\theta}^* = 1 - m - u$, where $m = M/N$ and $u = U/N$. Therefore, in any equilibrium there is full employment of all undocumented and domestic workers.

Using this full employment condition, together with the definition of $\bar{\theta}$ given by (5) we can solve for the wage in the informal sector as a function of government policies.

In any equilibrium, the wage in the informal sector, w_I, is given by[13]

$$w(e) = (1-e)(1-m-u) - k. \tag{8}$$

The equilibrium informal wage is decreasing in the amount of enforcement because enforcement reduces labour demand in that sector.

All entrepreneurs with productivity θ at least as large as $\bar{\theta}^* = 1-m-u$ will start up a firm in either the formal or informal sector, and produce θ. Therefore, total output in the economy is given by

$$N \int_{1-m-u}^{1} \theta d\theta, \tag{9}$$

and is independent of taxes and enforcement. In other words, the size of the economy is fixed for given populations of domestic and undocumented workers. Therefore, the government's problem is a distributional one. Its policy choices will only affect the distribution of total output in the economy between types of goods (public and private), across different individuals in the economy (entrepreneurs, domestic workers and undocumented workers), and across sectors (formal and informal). All of these margins are obviously related to one another.

We can now solve for the equilibrium wage in the formal sector. In any non-segmented equilibrium, wages must be equalized across the sectors. Therefore, $w_F = w(e)$ and wages will be independent of the tax rate, and decreasing in enforcement. In a segmented equilibrium however, the formal wage is given by $w_F = \max\{W(t,e), w(e)\}$, where

$$W(t,e) = w(e) + (1-m)e - t. \tag{10}$$

The wage $W(t,e)$ is the labour market clearing wage when all domestic workers are employed in the formal sector and is obtained from using the formal sector labour market clearing condition (7) with $M_F = M$ and the definition of $\hat{\theta}$ (4) given $w_I = w(e)$. This equilibrium formal wage structure has important consequences for the effect of taxes and enforcement on the formal wage, in particular,

$$\frac{\partial w_F}{\partial t} = \begin{cases} -1: & \text{if } w_F = W(e,t); \\ 0: & \text{if } w_F = w(e). \end{cases} \tag{11}$$

[13] We assume that enforcement is always such that $e \leq 1 - \frac{k}{1-m-u}$, so that all wages are non-negative.

$$\frac{\partial w_F}{\partial e} = \begin{cases} u: & \text{if } w_F = W(e,t); \\ -(1-m-u): & \text{if } w_F = w(e). \end{cases} \tag{12}$$

If $w_F = W(e,t)$ then the formal wage is increasing in public enforcement since enforcement decreases labour demand in the informal sector and thereby increases labour demand in the formal sector. Alternatively, the formal wage will be decreasing in enforcement if $w_F = w(e)$.

Finally, the last requirement or feasibility constraint is that the number of formal firms cannot exceed the number of domestic workers, since undocumented workers are excluded from that sector. We first define $m_F = M_F/N$ as the proportion of entrepreneurs operating a formal firm. It follows from the formal labour market clearing condition (7) and the definition of $\hat{\theta}$ given by (4) that

$$m_F = 1 - \frac{w_F - w(e) + t}{e}. \tag{13}$$

Thus, the feasibility constraint can be written as, $m_F \leq m$ or using (13)

$$w_F - w(e) + t - e(1-m) \geq 0. \tag{14}$$

When labour markets are segmented and no domestic workers are employed in the informal sector, $m_F = m$ and $m_I = 0$. Under this labour market configuration condition (14) must be binding. When labour markets are not segmented and some domestic workers choose to work in the informal sector, $m_F < m$ and $m_I > 0$. Arbitrage between the two sectors requires that $w_F = w(e)$, but condition (14) is not binding. Consequently, for any labour market configuration, at least one of condition (14) and $w_F \geq w(e)$ must be binding. Moreover, only in a segmented labour market is it possible for both conditions to be binding, implying that even if a labour market is segmented and domestic and undocumented workers do not work in the same sectors, it is possible for wages to be equalized across sectors. We later show that wages will be equalized across sectors, even in a segmented equilibrium, under optimal government policies.

How policies affect the proportion of entrepreneurs operating formally or the size of the formal sector depends on whether the feasibility constraint is binding, that is,

$$\frac{\partial m_F}{\partial t} = \begin{cases} 0: & \text{if } m_F = m; \\ -1/e: & \text{if } m_F < m, \end{cases} \tag{15}$$

$$\frac{\partial m_F}{\partial e} = \begin{cases} 0: & \text{if } m_F = m; \\ t/e^2: & \text{if } m_F < m. \end{cases} \tag{16}$$

As one would expect: higher taxes cannot lead to more compliance, and higher enforcement cannot lead to less compliance.

2.3 Welfare

The government has a utilitarian objective and cares about all individuals in the economy, including, to a perhaps lesser degree, undocumented workers assigning them a welfare weight $\alpha \in [0, 1]$. One interpretation for $\alpha < 1$ is political economy. Since undocumented immigrants do not vote, a government may cater less to the needs of this particular population. For the time being, we will restrict government policy to be of only three dimensions: a tax on formal firms, a level of public good, and a level of enforcement. One could also imagine that the government may have some choice over U, perhaps through a choice of border policy. Later, in the extension section, we show simply how optimal policy changes when U is endogenous and briefly discuss the effects of other government instruments such as a minimum wage. We begin by defining the government's objective function.

Definition 1 *Total weighted welfare is given by:*

$$N \int_0^{\bar{\theta}} \Pi_0(\theta) d\theta + N \int_{\bar{\theta}}^{\hat{\theta}} \Pi_I(\theta) d\theta + N \int_{\hat{\theta}}^{1} \Pi_F(\theta) d\theta$$
$$+ M_F \left[w_F + v(G) \right] + \left(M_I + \alpha U \right) \left[w_I + v(G) \right].$$

The first three terms are the sum of utilities of the entrepreneurs who do not start-up firms, start up firms in the informal sector and start-up firms in the formal sector, respectively. The fourth term is total utility for domestic workers employed in the formal sector and the last term is total utility for workers (with weight $\alpha \in [0, 1]$ on undocumented workers) employed in the informal sector.

Using the expressions for entrepreneurs' utilities, the informal sector wage, and the labour market clearing conditions, total weighted welfare as a function of government

policies can be written as

$$\Omega(t,e,G;\alpha) = N\int_{1-m-u}^{1}\theta d\theta + [N-M-U]k - N\int_{1-m-u}^{1-m_F} e\theta d\theta - N\int_{1-m_F}^{1} td\theta$$
$$-(1-\alpha)U\left[(1-e)(1-m-u)-k\right] + (M+N+\alpha U)v(G),$$

where m_F is determined endogenously.

Any wage paid by entrepreneurs is received by workers. Consequently, terms involving wages have no net effect on total welfare if all workers are counted equally. If the welfare of undocumented workers is discounted ($\alpha < 1$), then the welfare loss to entrepreneurs who pay the informal wage is greater than the welfare gain to undocumented workers who receive the informal wage. Consequently, there will be a welfare effect of enforcement policies through changes in the informal wage if $\alpha \neq 1$.

3 Optimal Policies

The government maximizes its weighted utilitarian welfare function $\Omega(t,e,G;\alpha)$ subject to its budget constraint (3) given the formal labour market clearing condition $N_F = Nm_F$, and the feasibility constraint (14). Let λ be the Lagrange multiplier on the feasibility constraint. We first solve for the first order conditions of the government's maximization problem and then characterize optimal policies in the segmented equilibrium and non-segmented equilibrium separately. All proofs are gathered in the Appendix.

Lemma 1 *The first order conditions on t and e are given by:*

$$[M+N+\alpha U]v'(G)\left[1+\frac{t}{m_F}\frac{\partial m_F}{\partial t}\right] = 1-\frac{\lambda}{Nm_F}\left(1+\frac{\partial w_F}{\partial t}\right); \tag{17}$$

$$[N+M+\alpha U]v'(G)\left[t\frac{\partial m_F}{\partial e}-\frac{c}{N}\right] = \int_{1-m-u}^{1-m_F}\theta d\theta-(1-\alpha)u(1-m-u)-\frac{\lambda}{N}\left[\frac{\partial w_F}{\partial e}-u\right]. \tag{18}$$

Both conditions state that the social marginal benefit of the policy (taxation or enforcement) in terms of additional revenue for public good provision must be equal to the social marginal cost of the policy which depends both on the elasticity of the tax base and the multiplier of the feasibility constraint. Taxation and enforcement

both have a direct social cost as firms must either pay the tax or incur concealment costs. When $\alpha < 1$ there is also an indirect social benefit of enforcement. Increasing enforcement reduces the informal wage, which increases profits at the expense of the welfare of undocumented workers. Given the different welfare weights on U and N, this redistribution is socially desirable. However, the social cost associated with concealment activities is always greater than this social benefit through changes in the informal wage and we can state the following Lemma.

Lemma 2 *For any $\alpha \in [0,1]$ and $m_F \leq m$:*

$$\int_{1-m-u}^{1-m_F} \theta d\theta - (1-\alpha)u(1-m-u) > 0. \tag{19}$$

Later we define more precisely the first-order conditions when we consider the segmented and the non-segmented equilibria individually, but for the moment we determine a condition under which the solution to the above conditions will be a global maximum of the government's problem.

Lemma 3 *The sufficient conditions for the government's constrained maximization problem are satisfied if $[M + N + \alpha U]v'(G) > 1/2$ for all possible value of G.*

Now that we know that the government's maximization problem delivers a global maximum, we can assess the condition under which it is optimal to have a segmented equilibrium or, equivalently, ensure that the feasibility constraint (14) is binding.

Proposition 1 *A necessary condition for the government to optimally enforce a segmented equilibrium is $c < N(1-m)^2$, and a sufficient condition for the government to optimally enforce a non-segmented equilibrium is $c \geq N(1-m)^2$.*

The government will optimally enforce segmented labour markets only if enforcement costs are not too high. Not surprisingly, if enforcement is too costly, it is preferable to leave some domestic workers in the informal sector. We now examine the properties of each type of equilibria in more detail.

3.1 Segmented Equilibrium

In a segmented equilibrium, all domestic workers choose to work in the formal sector, so the constraint (14) is binding and $m_F = m$. Since all domestic workers are

employed in the formal sector, public good provision is given by $G = Mt - ce$. From Lemma 1, the first order conditions on t and e become:

$$[M + N + \alpha U]v'(G) = \left[1 - \frac{\lambda}{M}\left(1 + \frac{\partial w_F}{\partial t}\right)\right]; \tag{20}$$

$$[N + M + \alpha U]v'(G)c + \int_{1-m-u}^{1-m} \theta d\theta - (1-\alpha)u(1-m-u) = \lambda\left[\frac{\partial w_F}{\partial e} - u\right]. \tag{21}$$

Equation (20) is a modified Samuelson rule where the marginal cost of public funds depends on λ. What is particularly interesting in a segmented equilibrium is that the tax base does not depend on government policies since $m_F = m$. However, as the government increases taxes, it must also increase enforcement to maintain labour market segmentation, explaining why the Samuelson rule depends on λ.

Public enforcement has no direct benefit, since the number of firms in the formal sector is constant. However, public enforcement must be sufficiently high to guarantee that $m_F = m$ binds. This requirement is represented by the term on the right hand side of (21). On the left hand side of this same equation, we can see that public enforcement is socially costly for two reasons. First, monitoring uses up government resources and reduces the amount of public good that can be provided. Second, firms operating in the informal sector devote resources to avoiding detection which is socially costly and, by Lemma 2, this direct social cost outweighs any potential indirect social benefit via changes in the informal wage.

It is worth noting that if $W(e,t) > w(e)$, then changes in policies have no effect on the constraint, (i.e., from (11) and (12) the terms in the square brackets multiplying λ are both zero). Such a wage guarantees full employment of domestic workers in the formal sector, and any change in policies, e or t, will change the formal wage and ensure the constraint continues to bind. Only if $w(e) \geq W(e,t)$, implying that wages are equalized across sectors, do policy changes affect the constraint. In which case, any increase in the tax must be accompanied by an increase in enforcement to keep the constraint binding. This observation leads to the following proposition.

Proposition 2 *In a segmented equilibrium, the wages in both the formal and informal sectors will be the same and given by* $w(e^*) = (1-e^*)(1-m-u) - k$ *where* e^* *is the optimal level of public enforcement.*

The fact that policies are set such that wages are equalized is somewhat surprising. The first thing to note is that when the formal wage exceeds the informal wage, a \$1

increase in taxes leads to a \$1 decrease in formal wage. Such an increase in the tax has no impact on firms' sectoral choices, and so the government does not need to adjust enforcement to guarantee segmentation. Since the tax base is fixed, taxing firms is equivalent to a lump sum tax. Obviously, the government will want to take advantage of this "cheap" form of taxation, but by increasing taxes it pushes the formal wage down closer to the informal wage. Similarly, a reduction in enforcement is matched by a reduction in the formal wage and an increase in the informal wage. Enforcement also has no impact on firms' sectorial choices. Consequently, the government wants to reduce costly enforcement as much as possible while maintaining labour market segmentation. Both of these forces, increasing taxes and reducing enforcement, push wages to equalization.

If public enforcement was exogenous, as in Djajic (1997), then there may be a strictly positive net wage differential between the two sectors, and firms may be attracted to the informal sector hoping to reduce their wage bill. With endogenous enforcement, this is no longer the case. In our framework, this wage equalization is largely due to the assumption that labour markets are perfectly competitive. Since workers bear taxes, taxes do not affect firms' sectoral allocation decisions.

As a direct consequence of having equal wages across sectors in a segmented equilibrium, the interaction between enforcement and taxes is clear. Any increase in taxes must be coupled by an increase in enforcement, as stated in the corollary below.

Corollary 1 *In a segmented equilibrium, a unit increase in the tax, must be accompanied by an increase in enforcement of* $\frac{1}{1-m}$ *units.*

We now turn to the characterization of optimal policies. Using the two first-order conditions (20) and (21), the optimal provision of public good is determined by the following modified Samuelson condition:

$$[N+M+\alpha U]v'(G) = \left[\frac{1+\frac{1}{M(1-m)}\left(N\int_{1-m-u}^{1-m}\theta d\theta-(1-\alpha)U(1-m-u)\right)}{1-\frac{c}{M(1-m)}}\right]. \quad (22)$$

Using this condition, we can state the following Proposition:

Proposition 3 *In a segmented equilibrium, public good provision is a) under-provided, b) decreasing in enforcement cost, and c) unambiguously decreasing in the number of undocumented workers when* $\alpha = 0$.

Efficiency requires that the sum of the marginal benefits from consumption of the public good equals the marginal cost of provision. If firms were unable to evade taxes, then we would be in the first-best and the sum of the marginal benefits of the public good would be equal to 1. Given firms' ability to evade taxes in the informal sector, however, the public good will be inefficiently provided even with costless enforcement ($c = 0$) as the numerator on the right-hand side of the Samuelson condition is greater than 1. With costly public enforcement, $c > 0$, the denominator of (22) is also less than one and therefore the public good will be further under-provided relative to the first-best. Intuitively, there are two reasons for under-provision despite the fact that the tax base is inelastic. First, higher taxes must be matched with higher enforcement which is costly. Second, higher enforcement stimulates concealment activity which is also socially costly and is always greater than any potential social benefit from a lower informal wage.

An increase in the marginal enforcement costs c increases the social marginal cost of providing the public good. Therefore, the optimal amount of G must go down. In a segmented equilibrium, an increase in the number of undocumented workers directly implies that more entrepreneurs operate in the informal sector. As a consequence, more firms devote resources to concealment activities. Therefore, the social cost of enforcement goes up. When $\alpha = 0$, the increase in the number of undocumented workers has no effect on the sum of the marginal benefit of the public good. Consequently, the amount of public good being provided must go down as U increases. When $\alpha > 0$, the sum of benefits also increases, which if α is large enough could override the above effect and increase the optimal level of public good provision.

Given optimal policies, the government's budget constraint can be rewritten as $G = tM - ce = [M(1 - m) - c]e$. It follows from Proposition 3 that when $\alpha = 0$, an increase in the number of undocumented workers also *reduces* optimal enforcement. Proposition 3, however, does not necessarily imply that optimal enforcement decreases with enforcement cost c. Intuitively, the government could compensate for the diminution of available resources by increasing taxes (and enforcement as stated by Corollary 1). The following Proposition clarifies these issues.

Proposition 4 *In a segmented equilibrium, optimal enforcement is a) increasing (decreasing) in enforcement cost c when the elasticity of the marginal benefit of the public good $\epsilon_G = -v''(G)G/v'(G)$ is greater than (less than) unity, and b) decreasing in the number of undocumented workers when $\alpha = 0$.*

With an increase in enforcement cost, the government has a direct incentive to reduce enforcement. This is similar to a substitution effect. At the same time, the increase in enforcement cost uses up resources, which reduces the amount of public good the government can provide. The marginal benefit for the public good is then higher, and so the government may want to increase taxes to increase provision. Since taxes and enforcement are linked together, enforcement must also increase. This is similar to an income effect. If ϵ_G is large, then a small decrease in G (via an increase in c) results in a large increase in the marginal benefit of the public good. Consequently, the government implements a large increase in taxes, which must be combined with a large increase in enforcement – the income effect dominates the substitution effect. The converse may also hold.[14]

To our knowledge, this result that optimal public enforcement may be increasing in the marginal cost of enforcement is original. Other papers have considered a framework with tax evasion and public good provision (e.g. Kaplow 1990, Cremer and Gahvari 2000), but their focus was different and the phenomenon we have identified was not uncovered.[15]

3.2 Non-Segmented Equilibrium

In a non-segmented labour market, some domestic workers work in the informal sector alongside undocumented workers. Formal and informal wages must be equal in equilibrium,$w_F = w(e)$, since domestic workers are perfect substitutes across sectors, and it follows from (13) that in equilibrium $m_F = 1 - t/e$. Further, the feasibility constraint is simply not binding, $m_F < m$, and $\lambda = 0$. From Lemma 1, the first order conditions on t and e become:

$$[M + N + \alpha U]v'(G) = \frac{1}{1 + \frac{t}{m_F}\frac{\partial m_F}{\partial t}}; \tag{23}$$

[14] A simple example where optimal enforcement is increasing in c is when preferences for the public good are given by $v(G) = \frac{G^{(1-\rho)}}{(1-\rho)}$ with $\rho > 1$. Under this example, $e^* = \left[\frac{M+N+\alpha U}{M(1-m)+N\int_{1-m-u}^{1-m}\theta d\theta-(1-\alpha)u(1-m-u)}\right]^{1/\rho}[M(1-m)-c]^{\frac{1-\rho}{\rho}}$, which is increasing in c given $\rho > 1$.

[15] Kaplow (1990) examines the optimal tax formulae (à la Ramsey) in the presence of tax evasion and enforcement, so his focus is on the relative size of various consumption taxes rather than on the tradeoff between public enforcement and the overall level of taxes. As for Cremer and Gahvari (2000), they examine tax competition and fiscal harmonization in the presence of tax evasion and do not focus on the characterization of optimal enforcement in a one country/government model.

$$[M + N + \alpha U]v'(G)\left[Nt\frac{\partial m_F}{\partial e} - c\right] = N\int_{1-m-u}^{1-m_F}\theta d\theta - (1-\alpha)U(1-m-u). \quad (24)$$

The first equation is a Samuelson rule with an elastic tax base. The second equation shows the trade off between a marginal increase in tax revenue due to higher enforcement and the associated marginal costs (net of the effect on the informal wage when $\alpha < 1$). As in Section 3.1, we will look at the properties of public good provision, but now, we must pay attention to the equilibrium size of the informal sector. In a segmented equilibrium, the size of the informal sector was determined by the number of undocumented workers. In a non-segmented equilibrium, however, the size of the informal sector depends on government policies.

Proposition 5 *In a non-segmented equilibrium, the size of the formal sector (m_F) is a) decreasing in enforcement cost, b) decreasing in the welfare weight put on undocumented workers, and c) decreasing in the number of undocumented workers.*

Higher enforcement costs increase the resources needed to keep firms in the formal sector, so the government optimally allows more firms to operate in the informal sector. With a higher welfare weight on undocumented workers, it becomes less attractive to use enforcement as a tool to lower the informal wage and increase firms' profits. More undocumented workers imply more firms in the informal sector and more resources devoted to avoid detection. This gives the government an incentive to reduce public enforcement (relative to taxes) so the size of the formal sector shrinks. Unlike in a segmented equilibrium, the ratio of taxes to public enforcement is not pinned down. Consequently, optimal public enforcement and taxes are no longer complementary policies, as stated in Corollary 1. We now look at public good provision.

Proposition 6 *In a non-segmented equilibrium, public good provision is a) under-provided, b) decreasing in enforcement cost, and c) decreasing in the number of undocumented workers when $\alpha = 0$.*

In a non-segmented equilibrium, public good provision behaves the same way as in the segmented equilibrium, but for slightly different reasons. Public goods are under-provided simply because the government faces an elastic tax base, not because enforcement and taxes are tied together. An increase in enforcement cost reallocates resources to enforcement away from public good production. Finally, more undocumented workers reduce the desirability of enforcement due to the additional

burden imposed by concealment activities. This reduction in enforcement reduces the ability of the government to collect taxes because of the reduction in compliance. Again, this is only guaranteed if $\alpha = 0$; if not, the government may want to increase the public good because it cares more about undocumented worker utility.

4 Extensions

4.1 Amnesties

In this section, we consider the consequences of legalizing the status of some undocumented workers. To do this, we conduct what we call a marginal amnesty: we marginally increase M while reducing U by the same amount, i.e., $dU = -dM < 0$. As we consider only marginal population changes, it is natural to assume that the economy remains in the same type of equilibria.

Focusing on a non-segmented equilibrium and noting that wages are equalized across sectors, the difference between the effects of a marginal change in U and M on total welfare (using the Envelope Theorem and given $dU = -dM < 0$) can be written as

$$\frac{d\Omega}{dM} - \frac{d\Omega}{dU} = (1-\alpha)\left[w(e^*) + v(G^*)\right]. \tag{25}$$

Recall, only the informal wage appears in the government's objective function, reflecting redistribution between informal firms and undocumented workers when the two are weighted differently. The marginal decrease in U will affect this term by increasing the informal wage and therefore reduce total welfare but at the same time the marginal increase in M will have the exact offsetting effect on the informal wage and total welfare. Consequently, the effect of the marginal amnesty on total welfare is given by the weighted difference in the utility level of a domestic worker and an undocumented worker. Since in a non-segmented equilibrium, domestic and undocumented workers receive the same level of utility this difference will be positive if undocumented workers are weighted less than domestic workers, i.e., $\alpha < 1$.

The above expression will also hold in the case of a segmented equilibrium but because in a segmented equilibrium replacing an undocumented worker by a documented worker necessarily implies that one more firm will pay taxes there will be an additional term $[N + M + \alpha U]v'(G^*)t^*$ in the expression. Thus, legalizing the status

of an undocumented worker will be welfare-improving in a segmented equilibrium for all values of α.

In the positive analysis of Djajic (1997), an amnesty had no impact in a non-segmented equilibrium and decreased the wage of documented unskilled workers in a segmented equilibrium. We obtain different results for two reasons. First, because we allow for undocumented workers to be valued less, an amnesty may be beneficial even in a non-segmented equilibrium. As in Djajic (1997), granting an amnesty to undocumented workers in a non-segmented equilibrium does not change the number of formal/informal workers. The newly documented worker will simply receive the full weight (instead of a weight of $\alpha < 1$), and welfare may increase. We admit that the difference is due to the way we constructed our welfare function, and consequently it is not very surprising. The second difference is more fundamental and somewhat surprising. In Djajic (1997) where enforcement policy is exogenous, wages in the formal and the informal sector may differ, and in particular an amnesty can affect the ratio of formal to informal sector wages. In this paper, we take into account how optimal polices change with the number of both domestic and undocumented workers, and guarantee that wages are equalized. Our marginal amnesty has no effect on equilibrium wages.

This exercise, of course, is only valid for marginal changes. Legalizing the status of a large number of undocumented workers would necessarily change the optimal policies and possibly the type of equilibria in which the economy rests.

4.2 Minimum Wage

Firms, in addition to evading taxes, may also avoid other (enforced) labour market regulations such as minimum wage legislation by operating in the informal sector. To investigate the impact of this additional potential incentive to operate outside of the formal sector, we now introduce an exogenous minimum wage denoted by $\bar{w}$, that must be paid to all individuals working in the formal sector. Further, for simplicity, we assume that the cost of concealing the evasion of the minimum wage and/or taxes are equivalent.[16]

In our framework, we assume that any documented workers who cannot find a job

[16] Consequently, no firm would ever choose to conceal only one of these regulations. All firms in the formal sector will respect both, and all firms in the informal sector will not.

in the formal sector can (and will) work in the informal sector instead.[17] Consequently, there will be full employment of all workers and the equilibrium informal wage will be given by (8). With a minimum wage, the wage in the formal sector is no longer flexible and is equal to the minimum wage, $w_F = \bar{w}$. The size of the formal sector is now given by:

$$m_F = 1 - \frac{\bar{w} - w(e) + t}{e}. \tag{26}$$

Taxes and the minimum wage affect the size of the formal sector in the same manner. As taxes (or the minimum wage) increase, more entrepreneurs choose the informal sector. An increase in enforcement has the opposite effect on the size of the formal sector.

As before, the government maximizes its objective (given $w_F = \bar{w}$) subject to its budget constraint (3) and the constraint that $m_F \leq m$. The first order conditions on t and e are similar to the ones obtained without a minimum wage. However, an important difference is worth highlighting: each condition contains an additional term $(\bar{w} - w(e))$ multiplied by the derivative of m_F with respect to the relevant policy (t or e). When taxation is increased or enforcement reduced, more firms will operate in the informal sector. Consequently there will be shift of domestic workers from the formal sector where they received the minimum wage to the informal sector where they now receive a lower wage. This shift is socially costly.

We now discuss the optimal policies given the minimum wage in the two types of equilibria, starting with the segmented equilibrium. In general, the introduction of a minimum wage creates excess formal labour supply, but in a segmented equilibrium, all domestic workers must be employed in the formal sector. How is this possible? The answer resides in the government having additional policy instruments that can affect demand for labour in the formal sector. The minimum wage discourages firms from operating formally, but at the same time enforcement discourages firms from operating informally. If e is sufficiently large relative to $\bar{w}$ and t, excess supply can be eliminated so that $m_F = m$. The modified Samuelson condition describing the optimal level of the public good without a minimum wage given by (22) also applies when a minimum wage is present, implying that the level of public good in a segmented equilibrium will be the same with and without a minimum wage. Interestingly however, providing

[17] We have worked out the equilibria and optimal policies for the case in which workers who would like to work in the formal sector and are unlucky cannot switch *ex post* to an informal sector job. This is the case in which an excess supply in the formal sector corresponds to involuntary unemployment. It turns out that the algebra for that case is more involved but that the results are fairly similar.

this same amount of public good given the minimum wage will require higher taxes and more enforcement as the binding condition $m_F = m$ is more restrictive in the presence of a minimum wage. It follows that the informal wage will be lower with a minimum wage than without.

In a non-segmented equilibrium with a minimum wage in place, the formal labour market no longer clears; some workers who want minimum wage jobs may not be able to find them. However, relative to a situation with no informal sector, this excess supply will not translate into unemployment – instead some domestic workers will be forced to work in the informal sector and $m_F < m$. Without a minimum wage, the decision to operate legally is based solely on the difference between taxes and enforcement, implying that for the marginal entrepreneur, the gain from evading taxes must equal the cost of concealment activities. The presence of a minimum wage gives entrepreneurs an additional reason to participate in the informal sector. Consequently, for the marginal entrepreneur the cost of concealment activities is strictly larger than the gain from evading taxes.

A minimum wage affects the provision of the public good because it affects the cost of using both instruments. On the one hand, a higher level of public good can be achieved by increasing taxes. An increase in taxes pushes more firms to operate informally, and this is costly because more firms devote resources to concealment activities instead of paying taxes; moreover, with a minimum wage, the cost of concealment activities is strictly larger than taxes paid. On the other hand, a higher level of the public good can be achieved by increasing enforcement instead. With higher enforcement, fewer firms operate informally. Consequently, we cannot say if the presence of a minimum wage implies higher or lower levels of the public good in a non-segmented equilibrium. However, we can argue that the presence of a minimum wage favours the use of enforcement versus taxes in a non-segmented equilibrium. Intuitively, a minimum wage makes the informal sector more attractive, so the government reacts by monitoring more and taxing less.

4.3 Endogenous Undocumented Immigrants

So far, we have considered the number of undocumented immigrants as an exogenous variable, however it is perhaps more natural to assume that the level of undocumented migration is a function of the opportunities available in the host country. We now assume that undocumented workers enter the country as long as jobs paying some

fixed reservation utility w_R exist. This reservation utility can be thought of as simply the wage in the source country, or as that wage augmented by any moving or migration costs, plus any possible utility from public good provision in the source country.[18] In this way, we could imagine that captured within these costs is some border control policy. Varying this reservation utility can be thought of as a very reduced form way of capturing the impact of border policy.

Without affecting the general structure, the addition of endogenous immigration choices introduces new trade offs in the design of optimal policies. On the one hand, most of the important results still apply. On the other hand, a government who maximizes total welfare may find it advantageous to expand public spending in order to attract cheap labour from abroad, and therefore border enforcement may be more desirable than firm-level enforcement.

We assume that any informal wage satisfying $w_I + v(G) > w_R$ will induce undocumented immigrants to migrate until $w_I = w_R - v(G)$.[19] We can use this expression to determine the number of undocumented workers as a function of government policies by first solving for

$$\bar{\theta} = \frac{w_R - v(G) + k}{1 - e}, \tag{27}$$

and noting that there will still be full employment of all domestic and undocumented workers since both labour markets clear. Together, this implies that the ratio of undocumented workers to entrepreneurs is given by

$$\tilde{u} = \tilde{U}/N = (1 - m) - \frac{w_R - v\,(G) + k}{1 - e}. \tag{28}$$

This ratio is decreasing in the reservation utility and in the number of domestic workers, and increasing in the level of public good provision.

[18] Our model is related to that of Harris and Todaro (1970) on rural/urban migration. In Harris-Todaro model, agents migrate until expected income between locations is equalized. In ours, agents migrate until utility, including that from the public good, is equated across locations. In this way, it is closer to Wildasin (1986) and Wellisch (2000) where migration is modeled by equating utilities.

[19] For a very low level of c, the equilibrium may involve $w_I = 0$. This will occur if G is sufficiently high to guarantee that $w_R = v(G)$. High G is often not optimal, since $v(G)$ has diminishing returns, and the enforcement which is necessary to sustain the formal sector tax base is costly. In this corner solution, it is immediately clear how G plays an important role in agents' migration decisions.

Defining the government objective analogously to Definition 1,[20] we can see that possible equilibria still take one of two forms: segmented, potentially allowing for a formal wage higher than the informal wage, or non-segmented in which case the formal wage will be driven down to the informal wage (reservation utility w_R less utility of public good provision). We consider these two cases in turn.

In a non-segmented equilibrium, the labour market clearing condition requires that $w_F = w_R - v(G)$. Therefore, from the definition of m_F given by (13) we have $m_F = 1 - t/e$. From (28), it follows that increasing enforcement shrinks the informal sector and reduces the number of undocumented immigrants. If the reservation utility is loosely taken to proxy for some border enforcement policy, then increasing border enforcement (increasing w_R) discourages entry and reduces the size of the informal sector given other government policies. Public good provision on the other hand, increases the size of the informal sector by encouraging migration of undocumented immigrants. Naturally, it requires that this public good is perfectly non-excludable. Note that in this environment, entrepreneurs will have a direct benefit from the public good as well as an indirect benefit from reduced wages. Workers will have the same direct effect, but the opposite indirect effect.

The government chooses taxes using exactly the same tradeoffs as before, except now there is an additional benefit of higher taxes. By raising taxes and increasing public good provision, the government lowers the informal wage and expands the size of the informal sector. Expanding the informal sector has both positive and negative consequences. Because more firms operate in the informal sector, more wasteful concealment costs are incurred. At the same time, total production increases. Lower wages also redistributes income from undocumented workers to entrepreneurs which is socially beneficial if the welfare weight on undocumented workers is less than one. Notice however, that it also redistributes income away from domestic workers since their wages are tied to those in the informal sector. Enforcement has the complete reverse effect. When e increases, it causes the size of the informal sector to shrink and total production to fall as $\bar{\theta}^*$ increases.

[20] Total weighted welfare $\Omega(t, e, G; \alpha, w_R)$ is given by:

$$\begin{aligned}\Omega(t, e, G; \alpha, w_R) \quad &= N \int_{1-m-\tilde{u}}^{1} \theta d\theta + [N - M - \tilde{U}]k - N \int_{1-m-\tilde{u}}^{1-m_F} e\theta d\theta - N \int_{1-m_F}^{1} t d\theta \\ &\quad -(1-\alpha)\tilde{U}[w_R - v(G)] + [N + M + \alpha\tilde{U}]v(G)\,.\end{aligned}$$

where $G = Nm_F t - ce$, and both m_F and $\tilde{U}$ are endogenous.

It is also interesting to see what happen when w_R falls, as if border enforcement were to be relaxed. Lower reservation wage w_R expands the size of the informal sector, but only along one margin. It increases the number of firms operating in the informal sector, without changing the number of firms operating in the formal sector. Consequently, relaxing border enforcement is more attractive relative to relaxing enforcement on firms. Obviously, this is only true if domestic workers and firms are weighted equally; if workers are valued more, increasing border enforcement may be more desirable since it increases wages.

In a segmented equilibrium, the formal labour market clearing condition is $M = Nm_F$ which yields the equilibrium formal wage: $W(t, e, w_R) = (1 - m)e + w_R - t$ provided $W(t, e, w_R) \geq w_R - v(G)$ otherwise the formal wage is equal to the informal wage. When considering the optimal policy in the segmented equilibrium, much of the same intuition will carry through as discussed for the case of the non-segmented equilibrium. The government can expand production by reducing the cost of setting up a firm in the informal sector by decreasing e or increasing G. It can strategically manipulate the amount of the public good to reduce informal sector wages and redistribute the surplus toward entrepreneurs operating in the informal sector.[21] It can also reduce border enforcement to expand output. Further, even with endogenous illegal immigration, Proposition 2 applies. Formal and informal wages will be equalized. Obviously, if domestic workers were to be more valued than entrepreneurs the situation could be different as illegal immigration lowers wages.

5 Conclusion

In this paper, we construct a simple model of tax evasion with an informal sector, and consider the role of undocumented workers on optimal tax and enforcement policy. We find that optimal policies play a crucial role in the wage determination process and lead wages to be equalized even when domestic and undocumented workers are not competing against each other in the same sector.[22] This result does not arise in previous papers that have considered illegal immigration.

We also find that enforcement may not always be decreasing in its cost, and that

[21] Of course, if the public good were partially excludable, this effect would be mitigated.

[22] See the working paper version of this paper (Cuff et al., 2009) for a discussion of how these results are robust to endogenizing the output price and the taxing scheme.

governments will optimally enforce market segmentation if enforcement costs are not too high. We consider several extensions. First, we consider the consequences of a marginal amnesty. Second, we introduce a minimum wage as a means of breaking the wage arbitrage condition and altering agents responsiveness to enforcement policy. Finally, we make the amount of illegal migration endogenous, and find a novel role for the public good. The public good can be used to depress wages in the informal sector, and thereby redistribute income from undocumented workers to entrepreneurs. In this environment as well, we see that firm level enforcement is socially more desirable than border controls.

It seems natural to mention other possible extensions. In an earlier version, we considered worker taxation, partial excludablity of the public good, and different productivity levels across sectors. None of these changed our results in a significant way. Perhaps the most interesting extension is to consider a political economy approach to policy selection. This is beyond the scope of this paper but it is one of the factors that motivated our modeling of the welfare weight on undocumented workers being less than unity. We leave a careful consideration of this issue for future work.

6 Appendix

Proof of Lemma 1: The derivatives of the government's Lagrangian function with respect to t and e are, respectively:

$$[M+N+\alpha U]v'(G)Nm_F\left[1+\frac{t}{m_F}\frac{\partial m_F}{\partial t}\right]-Nm_F$$
$$+N\left[e(1-m_F)-t\right]\frac{\partial m_F}{\partial t}+\lambda\left[\frac{\partial w_F}{\partial t}+1\right]=0.$$

$$[N+M+\alpha U]v'(G)N\left[t\frac{\partial m_F}{\partial e}-\frac{c}{N}\right]-N\int_{1-m-u}^{1-m_F}\theta d\theta+(1-\alpha)U(1-m-u)$$
$$+N\left[e(1-m_F)-t\right]\frac{\partial m_F}{\partial e}+\lambda\left[\frac{\partial w_F}{\partial e}-u\right]=0.$$

Whenever $m_F<m$, it must be the case that $w_F=w(e)$, and from (13), $m_F=1-t/e$. On the other hand, if $m_F=m$ then from (15) and (16), $\frac{\partial m_F}{\partial t}=\frac{\partial m_F}{\partial e}=0$. Consequently, it must be that both $[e(1-m_F)-t]\frac{\partial m_F}{\partial t}=0$ and $[e(1-m_F)-t]\frac{\partial m_F}{\partial e}=0$ always hold and Lemma 1 follows.

Proof of Lemma 2: To see this, note that

$$\int_{1-m-u}^{1-m_F} \theta d\theta - (1-\alpha)u(1-m-u) = \frac{(1-m_F)^2-(1-m)^2}{2} + \frac{u^2}{2} + \alpha u(1-m-u) \quad (29)$$

which is positive for any $\alpha \in [0,1]$ and $m_F \leq m$.

Proof of Lemma 3: The constraint (14) is linear in t and e since wages depend linearly on policies. Consequently, the sufficient condition for the government's maximization problem is satisfied if the government's objective is concave or, equivalently, the Hessian matrix of the objective function is negative semi-definite which requires that $\Omega_{tt} \leq 0$, $\Omega_{ee} \leq 0$ and $\Omega_{tt}\Omega_{ee} - \Omega_{et}\Omega_{te} \geq 0$. We now show these conditions are satisfied provided $[M+N+\alpha U]v'(G) > 1/2$ for all relevant G.

Differentiating (17) with respect to t, we obtain

$$\begin{aligned}\Omega_{tt}(t,e) =& [M+N+\alpha U]v''(G)N^2{m_F}^2\left[1+\frac{t}{m_F}\frac{\partial m_F}{\partial t}\right]^2 \\ & [M+N+\alpha U]v'(G)N\left[2\frac{\partial m_F}{\partial t}+t\frac{\partial^2 m_F}{\partial t^2}\right] - N\frac{\partial m_F}{\partial t}.\end{aligned}$$

Note from (15) that $\frac{\partial^2 m_F}{\partial t^2} = 0$. If $m_F = m$, then $\Omega_{tt}(t,e) < 0$. When $m_F = 1 - t/e$, then a sufficient condition for $\Omega_{tt}(t,e) < 0$ is that $[M+N+\alpha U]v'(G) > 1/2$ for all relevant G.

Next, differentiating, (18) with respect to e, we obtain

$$\Omega_{ee}(t,e) = [M+N+\alpha U]Nv''(G)\left[t\frac{\partial m_F}{\partial e} - \frac{c}{N}\right]^2 + [M+N+\alpha U]Nv'(G)t\frac{\partial^2 m_F}{\partial e^2} + N(1-m_F)\frac{\partial m_F}{\partial e}.$$

If $m_F = m$, then from (15) and (16), $\frac{\partial m_F}{\partial t} = \frac{\partial m_F}{\partial e} = 0$ and $\Omega_{ee}(t,e) < 0$. Otherwise, we note from (16) that $\frac{\partial^2 m_F}{\partial e^2} = -\frac{2}{e}\frac{\partial m_F}{\partial e} < 0$ and we can rewrite $\Omega_{ee}(t,e)$ as

$$\Omega_{ee}(t,e) = [M+N+\alpha U]Nv''(G)\left[t\frac{\partial m_F}{\partial e} - \frac{c}{N}\right]^2 - Ne\Big([M+N+\alpha U]v'(G)2-1\Big)\left[\frac{\partial m_F}{\partial e}\right]^2$$

which is clearly negative when $[M+N+\alpha U]v'(G) > 1/2$.

Finally, differentiating (17) with respect to e, we obtain

$$\begin{aligned}\Omega_{te}(t,e) = \Omega_{et}(t,e) =& [M+N+\alpha U]v''(G)N^2 m_F\left[t\frac{\partial m_F}{\partial e} - \frac{c}{N}\right]\left[1+\frac{t}{m_F}\frac{\partial m_F}{\partial t}\right] \\ & + [M+N+\alpha U]Nv'(G)\left[t\frac{\partial^2 m_F}{\partial e \partial t} + \frac{\partial m_F}{\partial e}\right] - N\frac{\partial m_F}{\partial e}.\end{aligned}$$

If $m_F = m$, then $\Omega_{tt}(t,e)\Omega_{ee}(t,e) - \Omega_{te}(t,e)\Omega_{et}(t,e) = 0$. When $m_F < m$, the determinant of the Hessian can be written as (noting from (16) that $t\frac{\partial^2 m_F}{\partial e \partial t} = \frac{\partial m_F}{\partial e}$):

$$N^2\Big([M+N+\alpha U]v'(G)2-1\Big)\frac{\partial m_F}{\partial t}[M+N+\alpha U]v''(G)\left[t\frac{\partial m_F}{\partial e}-\frac{c}{N}\right]^2$$
$$-N^3{m_F}^2e\Big([M+N+\alpha U]v'(G)2-1\Big)\left[\frac{\partial m_F}{\partial e}\right]^2[M+N+\alpha U]v''(G)\left[1+\frac{t}{m_F}\frac{\partial m_F}{\partial t}\right]^2$$
$$-2N^2m_F[M+N+\alpha U]v''(G)\left[t\frac{\partial m_F}{\partial e}-\frac{c}{N}\right]\left[1+\frac{t}{m_F}\frac{\partial m_F}{\partial t}\right]\Big([M+N+\alpha U]v'(G)2-1\Big)\frac{\partial m_F}{\partial e}$$

And $\Omega_{tt}(t,e)\Omega_{ee}(t,e) - \Omega_{te}(t,e)\Omega_{et}(t,e) > 0$ when $[M+N+\alpha U]v'(G) > 1/2$.

Proof of Proposition 1: Consider the left hand side of first order condition on e, (18), *just* as the constraint is about to become binding, i.e. as $t \to e(1-m)$. Since the constraint is not yet binding $\lambda = 0$, but $m_F \to m$. Therefore, the left hand side of (18) becomes

$$[M+N+\alpha U]v'(G)\left[N(1-m)^2-c\right] - N\int_{1-m-u}^{1-m}\theta d\theta + (1-\alpha)U(1-m-u). \quad (30)$$

If follows from Lemma 2 that if $c \geq N(1-m)^2$, a segmented equilibrium cannot be the solution to the maximization problem since (30) would be negative. Consequently, when $c \geq N(1-m)^2$ it must be the case that the non-segmented equilibrium is optimal. For a segmented equilibrium to be optimal, it must be that $c < N(1-m)^2$ but this condition is not sufficent.

Proof of Proposition 2: We show by contradiction that in a segmented equilibrium the government sets t and e such that wages are equalized across sectors. Imagine that t and e are set such that $w_F = W(e,t) > w(e)$, then $\partial w_F/\partial t = -1$. Consequently, (20) would become:

$$[M+N+\alpha U]v'(G) = 1. \quad (31)$$

The government would want to set t such that the sum of marginal benefits from the public good equals to one. When t increases, wage $W(e,t)$ decreases. For any given e, if the t that satisfies (31) is such that $W(e,t) \leq w(e)$ then the proof is complete. If $W(e,t) > w(e)$, we must look at equation (21). In such a case $\partial w_F/\partial e = u$, so (21) becomes

$$-[M+N+\alpha U]v'(G)c - N\int_{1-m-u}^{1-m}\theta d\theta + (1-\alpha)U(1-m-u) < 0., \quad (32)$$

where the inequality follows from Lemma 2. Since, the first order condition is always negative, the government will want to set e as low as possible, and so $W(e,t) < w(e)$. Consequently, wages w_F and $w(e)$ must be equalized.

Proof of Proposition 3: In a segmented equilibrium, optimal public good provision is determined by (22). From Proposition 1, a segmented equilibrium requires that $c < N(1-m)^2$ which implies that $1 - \frac{c}{M(1-m)} \in [0,1]$. Together with Lemma 2, this proves that statement a) is true. As for statement b), an increase in c increases the right-hand side of (22) and therefore, $v'(G^*)$ must go up. Since $v'' < 0$, this means G^* must go down. Finally, we now look at statement c). When $\alpha = 0$, U only appears on the right-hand side of (22). Differentiating the right-hand side with respect to U with $\alpha = 0$, we obtain $\frac{1}{M(1-m)-c}u > 0$. Therefore, with $\alpha = 0$ an increase in U increases $v'(G^*)$ and since $v'' < 0$, then G^* must go down.

Proof of Proposition 4: Define

$$F(e;c) \equiv [N+M+\alpha U]v'(G^*) - \left[\frac{1 + \frac{1}{M(1-m)}\left(N\int_{1-m-u}^{1-m}\theta d\theta - (1-\alpha)U(1-m-u)\right)}{1 - \frac{c}{M(1-m)}}\right],$$

where $G^* = [M(1-m)-c]e^*$ and $F(e;c) = 0$ yields optimal enforcement as a function of its marginal cost. Totally differentiating $F(e,c)$, we obtain $\frac{de^*}{dc} = -\frac{F_c}{F_e}$ where:

$$F_e = [N+M+\alpha U]v''(G)(M(1-m)-c) < 0,$$

$$F_c = -[N+M+\alpha U]v'(G)\frac{1}{M(1-m)-c}\left[1 + \frac{v''(G)}{v'(G)}G\right]$$

Therefore, optimal enforcement is increasing in c if $v''(G)G/v'(G) < -1$, decreasing in c if $v''(G)G/v'(G) > -1$ and independent of c if $v''(G)G/v'(G) = -1$. We define the elasticity of the marginal benefit of the public good as $\epsilon_G = -v''(G)G/v'(G)$. This proves statement a). From Proposition 3 part c), we know that $dG^*/dU < 0$ when $\alpha = 0$. Since $G^* = [M(1-m)S - c]e^*$, optimal enforcement is also decreasing in U when $\alpha = 0$. This proves statement b).

Proof of Proposition 5: Eliminating $v'(G)$ from the first-order conditions (23) and (24), and substituting in the expressions from (15) and (16) yields

$$\frac{2m_F - 1}{(1-m_F)^2 - c/N} = \frac{2m_F}{((1-m_F)^2 - \bar{\theta}^2) - 2(1-\alpha)u\bar{\theta}}, \tag{33}$$

where $\bar{\theta} = 1 - m - u$. Manipulating (33), allows us to define

$$R(m_F; c, \alpha, u) \equiv (1-m_F)^2 - 2m_F\frac{c}{N} + (2m_F - 1)(1-m-u)\Big[(1-m-u) + 2(1-\alpha)u\Big] \tag{34}$$

where $R(m_F; c, \alpha, u) = 0$ yields the equilibrium value of m_F as a function of the marginal cost of enforcement, the welfare weight on the undocumented workers and the number of undocumented workers. Totally differentiating $R(m_F; c, \alpha, u)$, we obtain

$$\frac{dm_F}{dc} = -\frac{R_c}{R_{m_F}}, \quad \frac{dm_F}{d\alpha} = -\frac{R_\alpha}{R_{m_F}}, \quad \frac{dm_F}{dU} = -\frac{R_U}{R_{m_F}},$$

where:

$$R_c = -2m_F/N < 0; \tag{35}$$

$$R_\alpha = -2(2m_F - 1)u(1-m-u) < 0; \tag{36}$$

$$R_u = -2(2m_F - 1)\Big[(1-\alpha)u + \alpha(1-m-u)\Big] < 0; \tag{37}$$

$$R_{m_F} = -2(1-m_F) - 2\frac{c}{N} + 2(1-m-u)\Big[(1-m-u) + 2(1-\alpha)u\Big]. \tag{38}$$

In equilibrium, $1 - m_F > 1 - m - u$. Therefore, if $(1 - m + u - 2\alpha u) < 1$ then R_{m_F} will be negative for all α. Note that $(1 - m + u - 2\alpha u)$ is monotonically decreasing in α and at $\alpha = 0$ the expression is $(1 - m + u) < 1$ since $m > u$. Therefore, for all α we have $(1 - m + u - 2\alpha u) < 1$ and $R_{m_F} < 0$.

Proof of Proposition 6: In a non-segmented equilibrium, public good provision is determined by (23), which can be written as $[M + N + \alpha U]v'(G) = \frac{m_F}{2m_F - 1} > 1$. This proves that statement a) is true. As for statement b), an increase in c decreases m_F, which increases the right-hand side of (23) and therefore, $v'(G)$ must go up. Since $v'' < 0$, this means G must go down. Finally we will look at statement c). When $\alpha = 0$, U only appears on the right-hand side of (23). An increase in U decreases m_F, which increases the right-hand side of (23) and therefore, $v'(G)$ must go up. Therefore, with $\alpha = 0$ an increase in u increases $v'(G)$ and since $v'' < 0$, G must go down.

References

[1] Allingham, M.G., and Sandmo, A., 1972. Income tax evasion: a theoretical analysis. Journal of Public Economics 1 (3-4), 323–338.

[2] Becker, G.S., 1968. Crime and punishment: an economic approach. Journal of Political Economy 76, 169–217.

[3] Bond, E. and Chen, T.-J., 1987. The welfare effects of illegal immigration. Journal of International Economics 23 (3-4), 315–328.

[4] Borjas, G., Grogger, J., Hanson, G., 2008. Imperfect substitution between immigrants and natives: a reappraisal. Working Paper 13887, National Bureau of Economic Research.

[5] Borjas, G., 1999. The economic analysis of immigration. In: Ashenfelter, O.C., Card, D. (Eds.), Handbook of Labor Economics. North-Holland, Amsterdam, pp. 1697–1760.

[6] Cremer, H., Gahvari, F., 2000. Tax evasion, fiscal competition and economic integration. European Economic Review 44, 1633–1657.

[7] Cortes, P., 2008. The effect of low-skilled immigration on U.S. prices: evidence from CPI data. Journal of Political Economy 116(3), 381–422.

[8] Cuff, K., Marceau, N., Mongrain, S., Roberts, J., 2009. Optimal policies and the informal sector. Working Paper 2009-14, McMaster University, Department of Economics.

[9] Djajic, S., 1997. Illegal immigration and resource allocation. International Economic Review 38 (1), 97–117.

[10] Epstein G., Heizler, O., 2007. Illegal migration, enforcement and minimum wage. Working Paper 08/07, Centre for Research and Analysis of Migration Discussion Paper Series.

[11] Ethier, W., 1986. Illegal immigration: the host-country problem. American Economic Review 76 (1), 56–71.

[12] Fortin, B., Marceau, N., Savard, L., 1997. Taxation, wage control and the informal sector. Journal of Public Economics 66 (2), 293–312.

[13] Fugazza M., Jacques, J.-F., 2003. Labor market institutions, taxation and the underground economy. Journal of Public Economics 88 (1-2), 395–418.

[14] Gordon, R., Li, W., 2009. Tax structures in developing countries: many puzzles and a possible explanation. Journal of Public Economics 93 (7-8), 855–866.

[15] Harris J., Todaro, M., 1970. Migration, unemployment & development: a two-sector analysis. American Economic Review 60 (1), 126–42.

[16] Kaplow, L., 1990. Optimal taxation with costly enforcement and evasion. Journal of Public Economics 43, 221–236.

[17] Kleven, H., Kreiner, C., Saez, E., 2009. Why can modern governments tax so much? An agency model of firms as fiscal intermediaries. Working Paper 15218, National Bureau of Economic Research.

[18] Kopczuk, W., 2001. Redistribution when avoidance behavior is heterogeneous. Journal of Public Economics 81 (1), 51–71.

[19] Passel, J., Cohn, D., 2009. A portrait of unauthorized immigrants in the United States. Pew Hispanic Center Report No. 107. Available online at http://pewhispanic.org/reports/report.php?ReportID=107, accessed July 17, 2009.

[20] de Paula, Á., Scheinkman, J.A., 2007. The informal sector. Working Paper 13486, National Bureau of Economic Research.

[21] Rauch, J.E., 1991. Modeling the informal sector formally. Journal of Development Economics 35 (1), 33–47.

[22] Reinganum, J., Wilde, L., 1985. Income tax compliance in a principal agent framework. Journal of Public Economics 26 (1), 1–18.

[23] Schneider, F., Enste, D.H., 2000. Shadow economies: size, causes, and consequences. Journal of Economic Literature 38 (1), 77–114.

[24] Slemrod, J., 2001. A general model of the behavioral response to taxation. International Tax and Public Finance 8 (2), 119–128.

[25] Slemrod, J., 1994. Fixing the leak in Okuns bucket. Optimal progressivity when avoidance can be controlled. Journal of Public Economics 55 (1), 41–51.

[26] Slemrod, J., Yitzhaki, S., 2002. Tax avoidance, evasion, and administration. In: Auerbach, A.J., Feldstein, M. (Eds.), Handbook of Public Economics. North-Holland, Amsterdam, pp. 1423–1470.

[27] Wellisch, D., 2000. Theory of public finance in a federal state. Cambridge University Press, Cambridge.

[28] Wildasin, D.E., 1986. Urban public finance. Routledge, London.