An Economic Theory of Fidelity in Network Formation*

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Abstract

We study the stability and welfare properties of fidelity networks. These are networks that form in a mating economy with two types of agents (e.g., men and women). Each enjoys having relationships with the opposite type. However, having multiple partners is viewed as infidelity, which is punished if detected. There is female discrimination in that infidelity is punished more severely for women than for men. We obtain a complete characterization of pairwise stable networks, which is sensitive to the size of the market. In most networks, women obtain their optimal number of partners, whereas men obtain at most their optimum. Furthermore, we show that a pairwise stable network is Pareto efficient if and only if it is female-optimal.

Subsequently, we study how a random, unexpected information shock affects women differently from men. In particular, we introduce and completely characterize *female-information-biased economies*, which are economies in which the diffusion of such a shock affects more women than men in all pairwise stable networks. We show that female-information-biased economies are segmented (or stratified), with the size of each segment being determined by a correspondence. This result generalizes to a probabilistic framework.

We extend the analysis to: (1) economies characterized by *female-to-male subjugation*, a generalization to many-to-many matching markets of the normative principle that underlies relationship formation in one-to-one and many-to-one matching markets; and (2) *hierarchical mating economies*, wherein each agent has a distinct social rank and higher-ranked agents are more desired as partners. We find that any economy of female-to-male subjugation is female-information-biased. Any hierarchical mating economy admits a unique pairwise stable network, and is female-information-biased.

The model has testable implications within and beyond the fidelity context. Applied to sexual markets, our analysis uncovers the social mechanism underlying the gender gap in HIV/AIDS prevalence, and demonstrates the distinct and complementary role of female discrimination, market stratification and social inequality in creating an HIV infection bias against women. Applied to academic markets, our findings highlight a factor in the persistence of inequality between academic institutions, and have implications for how new ideas affect students and faculty differently. Applied to the country-citizen market for highly skilled workers, the model predicts a country's brain drain as a function of its development, and derives implications for the gap between countries in the concentration of new technologies.

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1 Introduction

Fidelity, defined as an exclusive commitment to a partner, is desired and valued in most social and economic relationships. We study network formation in a *fidelity mating economy* with two types of agents (e.g., men and women). Each agent enjoys having relationships with the opposite type. However, having multiple partners is viewed as infidelity, which is punished if detected by the cheated partner. Punishment is both *retributive* and *restorative*.[1] The benefit function is assumed to be the same for all agents, but there is female discrimination in that infidelity after detection is punished more severely for women than for men. Women's optimal number of partners is therefore smaller than men's. We call *fidelity networks* those networks that arise in this environment.

A fidelity mating economy operates under *anonymity* or *secrecy*. *A priori*, agents do not know their partners' true motivation or other partners, and thus do not gain anything from these indirect links. In this sense, the anonymity assumption is consistent with the idea that agents promise fidelity to their partners when forming links, and they all believe each other to be faithful, until infidelity is detected and punished. Anonymity thus implies that agents cannot predict the possible configurations of networks that are likely to arise.

The fidelity model may be viewed as a generalization of the marriage market (Gale and Shapley (1962)). In the latter, each agent has at most one partner, captured in our model if the cost of infidelity for men and women is sufficiently high. Also, as we shall see later, the *reduced form* of the model completely abstracts from the fidelity story, and thus can be used beyond the fidelity context. It consists of a two-sided asymmetric matching market wherein the optimal number of partners is smaller for agents on one side than for those on the other side. Fitting into this category would be, for instance, the market between buyers and sellers, borrowers and banks, faculty and their departments, and doctoral students and their advisors. We will therefore develop applications both within and beyond the fidelity context.

Our goal in this paper is fourfold. First, we study networks that are likely to arise in a fidelity mating economy. Using the notion of pairwise stability, we characterize these networks in terms of the number of partners that each agent obtains.[2,3] Second, we examine their welfare properties, considering welfare for the overall market and for each side of the market. Third, we study the asymmetric effects of the diffusion of a random, unexpected information shock. In particular, we introduce and characterize *female-information-biased economies*, which are economies where a random information shock, after spreading, ends up concentrating more among women than men in all pairwise stable networks.[4] This characterization is particularly useful for

[1] Punishment is retributive because it is a function of the harm caused by the cheater, and it is restorative because it repairs this harm. The cheated thus derives satisfaction (or receives increased utility) from the cheater being punished. This property is consistent with recent experimental studies on altruistic punishment of deviant or uncooperative behaviors (e.g., Falk, Fehr and Fischbacher (2005), Fehr and Schmidt (1999), de Quervain et al. (2004), Fehr, Fischbacher and Kosfeld (2005)). The retributive and restorative nature of punishment is also consistent with the traditional practice of providing a victim with some sort of compensation, maybe a monetary transfer, from his victimizer.

[2] See Gale and Shapley (1962) for a first use of pairwise stability in the matching literature. This definition and its variations have been used in subsequent studies (e.g., Feldman (1973), Knuth (1976), Blair (1988); also see the classical study of Roth and Sotomayor (1990) for an appraisal). Within networks, Jackson and Wolinsky (1996) provide the standard definition. Our definition is related to both, as we discuss later.

[3] We extend our analysis to a dynamic setting in Pongou and Serrano (2009), fully characterizing networks that arise and persist in the very long run. The static and the dynamic approaches turn out to be complementary in theory and in applications.

[4] A wide variety of shocks exist in the real world. In a sexual network, a shock may be an instance of becoming infected with the AIDS virus from an exogenous source. In a faculty-student network, a shock may be a new research idea. In a financial network linking entrepreneurs and banks, a shock may be an idiosyncratic event in financial intermediaries that causes the failure of a single entity (e.g., a bank run), which in turn has a cascading effect on other connected entities. In all these cases, the question of when the spread of a shock ends up having a greater effect on agents on the "weak side" of the market than those on the opposite side is of particular interest. In answering this question, we assume that shocks are not taken into account in agents' utility because they are unexpected. We do not study agents' response to shocks either. Under certain conditions, it is natural to assume that agents do not respond to shocks. This is the case for instance when they are not aware of them, such as HIV/AIDS in its early stages. We now know that the AIDS virus has existed since at least 1959, but was only discovered in 1981 (Worobey et al. (2008)).

analyzing female-information-bias in environments where networks cannot be easily observed. In the case of HIV/AIDS for instance, this characterization identifies the *structure* of sexual markets that causes HIV/AIDS prevalence to be higher among women than men as a result of female discrimination. The fact that we do not necessarily need to observe the possible sexual networks that arise in these markets to study the gender gap in HIV concentration is good news, especially in light of the difficulty encountered in collecting reliable data on sexual behavior. The characterization of female-information-biased economies also suggests a very simple way in which a social planner could design a two-sided market in order to induce differential information concentration between the two sides of the market.

Fourth, in a further development of our analysis, we extend the fidelity model to two natural classes of mating economies. The first extension is to economies characterized by *female-to-male subjugation*. These are economies in which within each male-female relationship, the woman is subjugated to the man in the sense that she is always available to him whenever he needs her. As we shall see later, the principle of female-to-male subjugation turns out to be a generalization of the normative principle that underlies the formation of monogamous (or one-to-one) and polygynous (or many-to-one) matchings, which constitute an important class of matchings in several markets. The second extension is to *hierarchical mating economies*, wherein each agent has a distinct social rank, and higher-ranked agents are more desired as partners. Agents therefore have lexicographic preferences over numbers of partners and social ranks. In this latter class of economies, we also conduct a number of comparative statics analyses.

Our findings have testable implications for the functioning and outcomes of several markets in the real world. Applied to sexual markets, our analysis uncovers the social mechanism underlying the gender gap in HIV prevalence.[5] Our analysis also sheds light on the effect of social inequality on HIV prevalence, and on the relationship between an individual's social rank and his likelihood of being infected with HIV, which provides the first theoretical foundation to the growing empirical literature on this topic (see, e.g., Mishra et al. (2007), Lachaud (2007), Fortson (2008)). Our model also applies beyond the fidelity context. Applied to academic markets, our findings highlight a factor in the persistence of initial inequality between academic institutions, and have implications for how the dissemination of new ideas affects students differently from faculty. Applied to the country-citizen market for highly skilled workers, the model predicts a country's brain drain as a function of its development, and derives implications for the gap between countries in the concentration of new technologies.

The next section briefly discusses the relevance of fidelity for social and economic relationships, providing further motivation for our general framework. This is followed by an overview of our main findings and applications.

1.1 Fidelity: Relevance and Reality

Fidelity is relevant to the understanding of many types of relationships. In people's private lives, faithfulness between partners is essential to sustaining a marriage. In the academic sphere, exclusivity is required by editors from scholars submitting their papers for publication in academic journals. Similarly, citizens of certain countries

[5] There is a very broad and still growing economics literature on HIV/AIDS (see, e.g., Canning (2006) and the references therein). Our application to bipartite sexual markets, however, is the first theoretical attempt to explain higher HIV/AIDS prevalence among women compared to men, which also informs the ongoing debate on this topic (WHO (2003), UNAIDS (2008)). This is an important issue, given the documented damages of gender imbalance in mortality in several societies (Sen (1999)). Our analysis offers new explanations for the documented gender gap in the concentration of HIV/AIDS, as the only other explanation based on biological or physiological differences between the sexes has not been validated empirically (see Section 7.1 on this).

are precluded from acquiring another nationality.[6] Also, most labor contracts stipulate a *prima facie* duty of loyalty of employees to their employers. In general, in a competitive economic and political environment, the survival and success of organizations including firms, governments, intelligence services, the military, political parties, research labs, and financial institutions depends crucially on employee or member loyalty. In all of these cases, disclosure of important information (regarding technology, R&D programs, political or military strategies, etc.) to rivals by disloyal employees or members can be disastrous.[7]

Fidelity can also determine the outcome of a relationship. It has been argued that brain drain or human capital flight from less developed countries causes shortages in important sectors like education, health and industry (Kapur and McHale (2005), World Bank and IMF (2007)). Thus, a citizen's fidelity to his country might affect its economic development. In the sphere of intimate relationships, infidelity also has dramatic consequences. Psychiatrist Frank Pittman argues that 90% of first time divorces in the United States find their root in the infidelity of one or both partners (Pitman (1990, 1999)). Studies of paternity tests have found that about 1 to 30% of children are conceived as a result of an affair, with rates of non-paternity varying across countries and by socioeconomic status (e.g., Macintyre and Sooman (1991), Cerda-Flores et al. (1999), Bellis et al. (2005)). Globally, 33 million people are infected with the AIDS virus, and infidelity in sexual relationships is the main driver of this epidemic (UNAIDS (2008)).

Despite playing an essential role in the determination of important social, epidemiological and economic outcomes, the notion of fidelity and how it affects the formation of partnerships among self-interested agents who otherwise have a *prima facie* duty of loyalty to each other have not been studied in the economics literature.[8] The goal of the current study is to begin to fill this gap.

1.2 An Overview of the Fidelity Model and Theoretical Results

Our formal economic environment consists of a finite population of two equal-size exogenously determined sets of individuals, say men and women.[9] Each individual derives utility from the number of direct links with agents of the opposite type, while engaging in multiple links is considered an act of infidelity, and is punished if detected by the cheated partner. The benefit function is increasing and concave in the number of partners. Detection of infidelity occurs with positive probability, and women incur a higher cost for infidelity than men. These considerations result in each agent having a single-peaked utility function, with the peak, which is the optimal number of partners, being greater for men than women. Let N be the set of agents, n its cardinality, and s^*_m and s^*_w the optimal numbers of partners for each man and for each woman, respectively.

[6] For instance, Section 31 (a) of the Cameroon nationality code of 1968 states that "Cameroon nationality is lost by any Cameroon adult national who wilfully acquires or keeps a foreign nationality [...]" Several other countries including Austria, China, Germany, and Japan have similar laws.

[7] The ability to secure employee loyalty has been identified as a key determinant of a firm's growth and prosperity (Reichheld (1996)); leakage of technological information and its various economic consequences also have been documented (see, e.g., Mansfield et al. (1982), Mansfield (1985), Helpman (1993), Aghion et al. (2001)).

[8] There is an important literature on networks. Networks have been used to study a wide variety of topics including job search through contact information (Boorman (1975), Montgomery (1991), Calvó-Armengol (2004)), purchasing behavior (Frenzen and Davis (1990)), interaction and learning (Kandori, Mailath and Rob (1993), Ellison (1993)), technology diffusion and adoption (Coleman (1966)), HIV/AIDS risk perceptions (Kohler, Behrman and Watkins (2007)), friendship (Jackson and Rogers (2007a)), labor market transitions (Bramoullé and Saint-Paul (2010)), and pairwise and community insurance (Fafchamps and Lund (2000), Bramoullé and Kranton (2007), Bloch, Genicot and Ray (2008), Gallegatti, Greenwald, Richiardi and Stiglitz (2008)).

[9] The assumption that the number of men is equal to that of women is useful in two important respects. First, as we shall see later, it allows to isolate the effect of female discrimination on several outcomes including the allocation of links within each side of the market, as well as gender difference in information concentration. Second, it greatly facilitates the exposition and interpretation of the results.

1.2.1 Pairwise Stable Networks: Existence, Characterization and Welfare Properties

We first characterize networks that are likely to arise based on the notion of pairwise stability. This solution concept postulates a matching process wherein a link formation between two agents requires their mutual consent, whereas a link severance is unilateral. More formally, a network is pairwise stable if: (i) no individual has an incentive to sever an existing link he or she is involved in; and (ii) no pair of a man and a woman have an incentive to form a new link while possibly at the same time severing some of the existing links they are involved in.

Pairwise stable networks always exist. Their characterization depends on the size of the economy, and in general, reveals a conflict of interest between men and women that the market always resolves in favor of the latter. In characterizing these networks, we distinguish three cases, corresponding to very small economies, small economies, and large economies, respectively.[10]

In very small economies, there exists a unique pairwise stable network in which all men are matched with all women. This situation corresponds to one where there is a short supply of partners from both sides of the market, which makes it optimal for each agent to match with all agents on the opposite side. In small economies, a network is pairwise stable if and only if each woman obtains her optimal number of partners (s_w^*), while each man obtains between zero and all the women in the economy. In large economies, the characterization result is a bit more complex. We find that a network is pairwise stable if and only if all women, except at most $s_w^* - 2$, obtain their optimal number of partners, and each man is matched with no more than his optimal number of partners.[11]

Women therefore "typically" obtain their optimal number of partners. This result is consistent with the intuition that female discrimination causes women to supply a smaller number of links than the ones demanded by men, and therefore, men end up competing for them. Competition guarantees that each woman obtains at least one partner, but it generally leads to some men being unmatched, especially in small and large economies. In certain markets, however, despite male competition, some women may have fewer than their optimal number of partners in some pairwise stable networks, due to the decentralized nature of the matching process. Interestingly, these findings imply that heterogeneity in outcomes does not necessarily imply that agents are heterogeneous with respect to their preferences.

Next, we analyze the welfare properties of pairwise stable networks. We consider two notions of efficiency, namely *strong efficiency* and *Pareto efficiency*. A network is said to be strongly efficient if its total value (the sum of agents' utilities in that network) is maximal. Along the same line, we also consider welfare for each side of the market.[12] A network is said to be Pareto efficient if one cannot increase the utility of one agent (by adding, deleting and/or redistributing links) without decreasing the utility of another agent.

In very small economies, we find that the unique pairwise stable network that exists is both strongly efficient and Pareto efficient, which implies that this network is also male-optimal and female-optimal. In small and large economies, it is possible that no pairwise stable network be strongly efficient[13]; and no pairwise stable network is male-optimal. However, all pairwise stable networks, except those in which some women have fewer

[10] Formally, these three cases correspond respectively to $(1): n \leq 2s_w^*$; $(2): 2s_w^* < n \leq 2s_m^*$; and $(3): n > 2s_m^*$.

[11] Given that $s_w^* - 2$ is at least equal to zero, this result implies that all women obtain their optimal number of partners when $s_w^* = 1$ or $s_w^* = 2$. It is when s_w^* exceeds 2 that at most $s_w^* - 2$ have fewer than s_w^* partners.

[12] This is defined in terms of male-optimality and female-optimality. More precisely, we say that a network is male-optimal if the sum of men's utilities in that network is maximal. Female-optimality is similarly defined.

[13] Note that this does not mean that in all small and large economies, no pairwise stable matching is strongly efficient. Depending on the utility profile of agents, a pairwise stable matching may well be strongly efficient.

than their optimal number of partners, are Pareto efficient and maximize the aggregate welfare of women.

We note that the analysis of welfare properties reveals two types of tension and two types of agreement in small and large economies. There exists a tension between pairwise stability and strong efficiency. However, we show that this tension is minimized in egalitrarian pairwise stable networks, where all agents have the same number of partners (the optimal number for women). Indeed, among all pairwise stable networks, egalitarian networks maximize aggregate welfare, although they do not Pareto-dominate non-egalitarian networks. There also exists a tension between male-optimality and female-optimality. Male-optimality to be achieved would require that each man obtain his optimal number of partners, which would imply that the number of links coming from the women's side strictly exceed the number of links that these women can optimally supply, something that is impossible in pairwise stable networks. Within our framework, the unique underlying factor in both tensions is female discrimination, which paradoxically turns out to be positive for women, in that it guarantees optimality for them.

About agreements, we find that pairwise stability is compatible both with Pareto efficiency and female-optimality. In addition, we show that a pairwise stable network is Pareto efficient if and only if it is female-optimal, a result that relates the two notions of efficiency considered in our analysis. This result implies that only those networks in which a few women (at most $s_w^* - 2$) have fewer than s_w^* partners are Pareto-dominated, and we know that these networks may arise only in large economies when s_w^* exceeds 2.

1.2.2 Communication Potential and Female-information-biased Economies

We now turn to the characterization of *female-information-biased* economies. First, we introduce a simple model of communication transmission that answers the following questions:

- If a random agent in a network unexpectedly receives from an exogenous source a piece of information that he/she communicates to his/her neighbors who in turn communicate it to their other neighbors and so on, what proportion of the population will end up receiving the information?
- What will the male-female difference in the proportion of informed agents be?

The answers to these two questions define two indexes: the *communication or contagion potential* of a network, and the *gender difference in the communication potential* of a network.[14]

A female-information-biased economy is one in which the diffusion of a random information shock affects women more than men in all pairwise stable networks. We find that an economy is female-information-biased if and only if that economy is segmented such that no segment contains more than $4s_w^* + 2$ men and women whenever s_w^* exceeds 1.[15] A segmented economy corresponds to one in which the supply of and demand for partners are subject to constraints that partition the population into mutually exclusive groups of agents.

[14] These contagion indexes are real-valued functions of networks. They naturally lend themselves to the analysis of the aggregate and asymmetric effects of the mechanical diffusion of a wide variety of information shocks. For instance, they can be applied to study the diffusion of new ideas in academic and professional networks, information transmission in social networks, contagion in financial networks, cascading failure in power grids or computer networks, and the spread of sexually transmitted diseases such as HIV/AIDS in sexual networks.

[15] A segmented mating economy is here defined as a collection of economies which operate separately. Each of these economies is called a *segment*. Note that in a segmented economy, s_w^* need not be the same for all segments. In this respect, our characterization result is equivalent to saying that an economy is female-information-biased if and only if it is segmented such that the size of any segment $\mathcal{E}^t$ is determined by a correspondence $\varphi : \mathbb{N}^* \to \mathbb{N}^*$ such that $\varphi(s_w^{t*}) = \mathbb{E}^*$ if $s_w^{t*} = 1$ and $\varphi(s_w^{t*}) = \{k \in \mathbb{E}^* : k \leq 4s_w^{t*} + 2\}$ if $s_w^{t*} > 1$, where $\mathbb{N}^*$ is the set of strictly positive integers and $\mathbb{E}^*$ the set of strictly positive integers that are even. In other words, a segment $\mathcal{E}^t$ can be of any size if the optimal number of partners for women (s_w^{t*}) in that segment is 1, and should not contain more than $4s_w^{t*} + 2$ agents if $s_w^{t*} > 1$. This result is surprising, especially as it depends only on the optimal number of partners for women, and does not depend on the optimal number of partners for men. We will fully clarify the rationale behind it in Section 5.

Examples of segmented economies abound in the real world.[16] Also, there is a very broad sociological and economics literature on segmentation and its various dimensions including homophily, social exclusion, and specialization (e.g., McPherson, Smith-Lovin and Cook (2001), Lazarsfeld and Merton (1954), Currarini, Pin and Jackson (2009), Arrow (1971), Reich, Gordon and Edwards (1973)). What was not known, however, is the implication of segmentation for the asymmetric concentration of information in two-sided markets. In the case of HIV/AIDS for example, our characterization result implies that sexual market segmentation combined with female discrimination leads to HIV/AIDS being more prevalent among women than men. Interestingly, it also implies that when women are faithful to their partners ($s_w^* = 1$), they suffer a greater share of the AIDS burden than men. Furthermore, this result suggests a simple way in which a social planner could design a two-sided asymmetric matching market to induce differential information concentration between the two sides of the market. It shows, for instance, that the segmentation of the faculty-student market by field of specialization may lead to the dissemination of new ideas affecting both sides differently.

Importantly, our characterization result generalizes to a probabilitic framework. In an economy admitting multiple pairwise stable networks like ours, it is difficult to tell the exact probability with which a given pairwise stable network will arise. However, within our framework, we know that a network forms with strictly positive probability if and only if it is pairwise stable. We then show that in an economy, women, on average, concentrate more information than men given any probability distribution over pairwise stable networks if and only if that economy is segmented such that no segment has more than $4s_w^* + 2$ agents whenever s_w^* exceeds 1.

1.2.3 Two Extensions

We extend the fidelity model to two classes of mating economies. The first extension is to economies characterized by female-to-male subjugation, and the second extension is to hierarchical mating economies. An economy of female-to-male subjugation is an economy in which within each couple, the woman is always available to the man whenever he needs her.[17] Under the assumption that each agent is endowed with one unit of time that he/she splits equally among all his/her partners, female subjugation is formally equivalent to saying that within each couple, the woman invests at least as much time in the relationship as the man. This imposes a constraint on network formation that we take into account in the characterization of pairwise stable networks in this environment. This characterization is similar to the one obtained for economies without constraint, with the only difference that in all pairwise stable networks, no woman has more partners than any of her partners. We also find that all economies characterized by female-to-male subjugation are female-information-biased.

A few remarks regarding the predictions of economies of female-to-male subjugation are in order. First, if the optimal number of partners for each woman is 1, an economy under the constraint of female-to-male subjugation admits the exact same set of pairwise stable matchings as the corresponding economy without constraint. This set consists solely of one-to-one and many-to-one matchings in which each man is linked to no more than his optimal number of partners (that is, $s_w = s_w^* = 1$ and $0 \leq s_m \leq s_m^*$). Second, in a mating economy of

[16] Sexual markets, for instance, are generally regarded as stratified by such factors as religion, anti-miscegenation laws, caste, geographical distance, family background, and biological markers, to name a few factors. The academic job market (that is, the faculty-department matching market) is another example, where faculty and departments are matched based on academic specialization, and resulting in several departments (e.g., Economics, Mathematics, English literature, etc.) within a university. Within a university, the faculty-student matching market is also segmented by field.

A mating economy may also be segmented along a discrete time space in which, in each period, an equal number of men and women are on the market, and they are replaced by a new generation of agents in the following period. Each time-segment will be relatively small (because not everybody is on the market at the same time), although the overall economy will look large.

[17] An implicit assumption here is that if a woman has two partners, both will not need her at the same time.

female-to-male subjugation, women always concentrate more information than men in any network that arises. In a sufficiently large and non-segmented economy without constraint, this is the case if and only if $s_w^* = 1$. These two remarks suggest that the principle of female-to-male subjugation can be viewed as a generalization of the normative principle that governs the formation of relationships in monogamous and polygynous cultures, and is therefore far from being restrictive.

Our second extension is to hierarchical mating economies. In these economies, each agent has a distinct social rank, and higher-ranked agents are more desired as partners.[18] We find that each such economies has a unique pairwise stable network. This network presents a pattern of matching in which lower class women match with men who are of higher ranks. We also find that all hierarchical mating economies are female-information-biased. Comparative statics analyses reveal that an increase in the level of discrimination against women increases the quality of their matches, as well as their share of information. We also find that higher-ranked men and women have more partners. Moreover, we study the asymmetric effects of the diffusion of a random information shock in the unique pairwise stable network that arises in the economy. We find that such effects are greater for higher-ranked men and women, as a result of their position in the network. They are also greater for a woman than a man of the same rank. Further, we find that the contagion potential of the unique pairwise stable network that arises in a hierarchical mating economy is at least as high as the contagion potential of any pairwise stable network of the corresponding economy without hierarchy if and only if $s_w^* = 1$. This means that although social inequality, captured in our model by the rankings of agents, always results in a greater concentration of information among women than men, it increases the concentration of information in the overall population if and only if female discrimination is sufficiently severe.

1.3 Highlights of Applications

We apply our theoretical findings to shed light on the functioning and outcomes of certain real-world markets, including heterosexual markets, academic markets, and the country-citizen market for skilled workers.

With respect to heterosexual markets, our theoretical results have testable implications for: (1) whether the distribution of sexual partners among sexually active individuals reveals the latter's preferences; (2) gender difference in the timing of sexual initiation; (3) the distinct and complementary role of female discrimination, market stratification, and social inequality in gender difference in HIV/AIDS prevalence; (4) the effect of an individual's social rank on his/her sexual behavior and his/her likelihood of being infected with HIV; and (5) the role of social inequality in increasing HIV/AIDS prevalence in a population. In particular, in trying to understand the social mechanism underlying the spread of HIV/AIDS, we rely on the documented fact that the virus that later became the AIDS virus originated in non-humans (McClure and Schulz (1989)), and thus was an *exogenous* and *unexpected* shock to humans, as we assume in our model of information transmission. We therefore assume that agents ignore the presence of the disease, or are just oblivious to it when considering a relationship. In this respect, one can view our model as describing the situation of the pandemic in its early stages, when nobody was aware of it.[19] Also, as we view our model as a first attempt to this issue, for tractability

[18] There are several examples of such economies in the real world. In a sexual market, rankings may be based on agents' socioeconomic status. In the academic job market, departments may be ranked based on their prestige, and job candidates may be ranked based on their teaching and research ability. Similarly, within a department, advisors may be ranked based on how good they are at mentoring and placing their students, and students may be ranked based on their ability.

[19] Worobey et al. (2008) provide direct evidence for the existence of HIV-1 in Congo since 1959. However, the virus was discovered and recognized only in 1981. Also, many people are unaware of their HIV status; twenty years after the discovery of the AIDS virus, less than 1% of the urban populations had received an HIV test, and this proportion is still very low today (Kumaranayake and Watts (2001), WHO (2008)).

reasons, we leave out other complexities such as homosexuality and prostitution, and future research ought to address them. Despite these simplications, our theoretical results capture several features of the data, and show how the spread of HIV/AIDS can be studied in a context where sexual networks are *invisible*. In doing so, we uncover some key social determinants of the gender gap in HIV/AIDS prevalence.

In our second application, we consider the department-faculty market and the faculty-student market. The department-faculty market is assumed to be a many-to-one hierarchical matching market. We show how the outcome of such a market is likely to perpetuate initial inequality between academic institutions. The faculty-student matching market is a many-to-many matching market. Our main result here has implications for how the design of such a market (in terms of specialization by field, class size, faculty size, the number of courses to be taken to complete a program) may cause the diffusion of new ideas to affect students and faculty differently. We also consider a hierarchical mentoring market where both faculty (or advisors) and students (advisees) are ranked, and show how higher-ranked faculty differ from their lower-ranked counterparts in terms of the quality and quantity of students they attract, and in terms of their likelihood of being informed of new ideas.

Our third application is the country-citizen market for highly skilled workers. Here, we study a slightly different version of the hierarchical mating economy model. Countries, which are ranked based on a combination of factors, compete for equally skilled workers from different countries. Each country, however, prefers its citizens to foreigners, but values the latter equally. This economy admits multiple pairwise stable networks. These networks present a pattern in which skilled workers born in poor countries migrate to more developed countries. In general, the model predicts a country's brain drain as a function of its initial economic development, and generates implications for the gap between countries in the concentration of new technologies and discoveries.

1.4 Related Literature

To the best of our knowledge, the current study is the first attempt to examine the effect of fidelity as a norm on network formation. However, we view the fidelity model as contributing to two main literatures: the literature on two-sided matching theory, and the growing literature on the formation of social and economic networks. Gale and Shapley (1962) introduced a notion of pairwise stability, and used it to prove the existence of stable marriages in a one-to-one matching market. This classical paper has inspired a number of studies that have significantly advanced our understanding of matching theory and its various applications.[20] In general, this literature studies the existence and welfare properties of stable matchings. Our scope is larger. Within our framework, we prove the existence of stable fidelity networks (using a more flexible and general definition of pairwise stability), and study their efficiency; in addition, we provide a complete characterization of these networks in terms of the number of partners that each agent obtains. This characterization is subsequently useful in characterizing *female-information-biased economies*, a new concept and contribution to the literature.

While the notion of pairwise stability is useful in predicting equilibrium outcomes in many contexts, it is often objected on the ground that it does not allow for more general coalitions (Roth and Sotomayor (1988)). Indeed, if one defines the core and the setwise stable set (Roth (1984b), Sotomayor (1999), Echnique and Oviedo (2006)) with respect to strict coalitional improvements, these two sets will coincide with our set of pairwise stable networks. Still, we view our characterization of these sets, in terms of the optimal number of partners for each

[20]see, e.g., Roth (1984a, 1984b), Crawford and Knoer (1981), Roth and Sotomayor (1989, 1990), Kojima and Pathak (2008), Sotomayor (1999), Kranton and Minehart (2001)), Ergin (2002), Hatfield and Milgrom (2005), Klaus and Klijn (2005, 2009), Konishi and Ünver (2006), Echnique and Oviedo (2006), etc.

side, as a useful tool of analysis within our model.

Our characterization of pairwise stable networks also highlights a feature often noted in the general matching literature- that is, a conflict/coincidence of interest between the two sides of the market in some equilibrium matchings (e.g., Roth (1985)). Our result implies that within the fidelity model, the conflict/coincidence of interest between men and women depends on the size of the market. There is coincidence of interest if and only if the economy is very small. In this case, there exists a unique pairwise stable network which is both male-optimal and female-optimal. In small and large economies, there exists a tension between men and women that is always resolved in favor of women, as they typically obtain their optimal number of partners. In this, our result differs from most studies which show the coexistence of both male- and female-optimal stable matchings.

We also view our study as contributing to the market design literature. In general, this literature focuses on how to design a matching market in order to achieve the stability and efficiency of outcomes. Here, we answer a completely different question. Our focus is on how to design a market to achieve female-information-bias when this is desirable. Our characterization of female-information-biased economies suggests that a social planner can impose a culture of monogamy on one side of the market ($s_w^* = 1$), or can simply resort to market segmentation to achieve such a goal. Interestingly, this result also suggests that even in decentralized markets such as sexual markets, higher female vulnerability to HIV/AIDS is an *unintended* consequence of partner selection criteria that society or agents impose on themselves, resulting in sexual market stratification.

Our study also contributes to the network theory literature. Jackson and Wolinsky (1996) introduced a general framework for the study of the stability and efficiency of social and economic networks. The basic assumption underlying network formation in this study is that agents form and sever links with each other based on the reward from doing so. They develop a notion of pairwise stability, and use it to predict networks that are likely to form. The benefit to a stable network accruing to each agent depends on the productive value of this network and the allocation rule. A natural and important question answered by the authors is whether there exists an allocation rule that ensures that agents form a strongly efficient network. They find that in general, there exists a tension between stability and strong efficiency.

Following Jackson and Wolinsky (1996), Dutta and Mutuswami (1997) further study the relationship between stability and strong efficiency. Their analysis is based on a strategic form game in which each agent announces a set of other agents with whom he wants to form a link, and a link between two agents is formed if both announce it. They define two stability notions, strong stability and weak stability, which they use to predict networks that are likely to form. As in Jackson and Wolinsky (1996), the study of the relationship between stability and strong efficiency also reveals a tension between the two notions.[21]

Our study is related to these works in that we analyze network formation among agents who trade off the cost and the benefit of forming links. We define a notion of pairwise stability that allows for simultaneous link formation and severance, and which differs from the ones proposed in these studies. Our analysis of the relationship between stability and strong efficiency confirms the tension between the two notions as in Jackson and Wolinsky (1996) and Dutta and Mutuswami (1997). A distinctive contribution of our study, however, is

[21] The basic framework in Dutta and Mutuswami (1997) is similar to the one introduced by Aumann and Myerson (1988). Aumann and Myerson (1988) study a two-stage game in which in the first stage, players form bilateral links, yielding a cooperative structure to which the Myerson value (Myerson (1977)) is applied to determine the payoff to each player in the second stage. Extensions and variants of this game have been considered in Dutta, van den Nouweland and Tijs (1996), Slikker and van den Nouweland (2001a), and Slikker and van den Nouweland (2001b).

These pioneering works have spawn a number of studies on strategic network formation (see, e.g., Bala and Goyal (2000), Watts (2001), Jackson and Watts (2002), Page, Wooders and Kamat (2005), Dutta, Ghosal and Ray (2005), Bloch and Jackson (2007)), with some using a dynamic framework (see, e.g., Bala and Goyal (2000), Watts (2001), Jackson and Watts (2002), Dutta, Ghosal and Ray (2005), etc.).

in taking advantage of the bipartite nature of a fidelity mating economy to investigate welfare for each side of the economy. We note a tension between male-optimality and pairwise stability, but pairwise stability agrees with female-optimality in general. We also examine the relationship between pairwise stability and Pareto efficiency, and provide a full characterization of pairwise stable networks that are Pareto efficient. We find that such networks always exist, showing that the tension between stability and efficiency depends on the notion of efficiency under consideration. Another distinctive feature of our study is our focus on fidelity networks, which are networks in which *a priori*, agents do not know the other partners of their partners, and do not gain anything from these indirect links. These networks therefore have different incentive properties and applications.

The fidelity model has been extended in two ways. Pongou (2009b) generalizes it to multi-ethnic societies, deriving testable predictions for the effects of ethnic heterogeneity on sexual behavior and the spread of HIV/AIDS. Pongou and Serrano (2009) focus on the long-run predictions of the model, based on a dynamic and stochastic approach. They characterize networks that are visited a positive amount of time in the very long run, and find that in these networks, women always concentrate more information than men. In addition to their methodological differences, our paper and Pongou and Serrano (2009) differ in their scope, as issues pertaining to efficiency or its relationship to stability are not covered in the latter study. Another distinctive feature of our study is in extending the fidelity model to two newly defined classes of economies including economies of female-to-male subjugation and hierarchical mating economies. When it comes to understanding the effect of female discrimination on gender differences in HIV/AIDS prevalence, the findings of the two papers turn out to complement each other. To be more precise, while Pongou and Serrano (2009) show that discrimination does not favor women in the very long run, our study fully identifies the characteristics of mating economies that produce the same outcome in the early stage of network formation. In a context of limited resources to address gender inequity in HIV/AIDS prevalence and its detrimental social and economic consequences, such an identification calls attention to markets that should be prioritized in initial interventions. When considered together, findings from both studies establish priorities as to where to invest these scarce resources, and with what timing and intensity.

Further, we introduce a simple model to study the asymmetric effect of a random information shock in a network. This model assumes that information travels the network via word-of-mouth or neighbors' contagion, and so does not spread if received by an isolated agent. Recent studies on the effect of network structure on the diffusion of information or the spread of certain behaviors make a similar assumption (see, e.g., Pastor-Satorras and Vespignani (2000, 2001), Morris (2000), Jackson and Rogers (2007b), Jackson and Yariv (2007), and Lopez-Pintado (2008), Lamberson (2010)). Our study, however, differs from this literature in at least two important respects. First, our modeling of information diffusion is conceptually different. Most existing models assume a distribution of connections in the population, and a payoff function whose arguments include an individual's and her neighbors' choice of a certain behavior. Thus each individual faces the choice of adopting a certain behavior, such as buying a new product or not, and this behavior spreads as it is adopted. Our model differs in that it studies "information transmission" (in a context in which transmission is not costly), not "information adoption." Distinguishing between the two notions is important. Within our framework, an agent who receives information about, say a new product, communicates it to her friends, but we are not interested in whether the latter purchase the product or not. In the same way, an agent who is infected with the AIDS virus infects his/her sexual partners; the latter do not make the choice of becoming infected, and the former may not even be aware of his/her HIV status. This leads to significant differences in the assumptions underlying our different approaches. For instance, we do not make any assumptions on the connectivity distribution of the population, but rely only on the knowledge of the number of components and their size. Another distinctive feature of our model is in studying information transmission in bipartite environments, with a focus on understanding how various market and network structures affect gender difference in information concentration. In this regard, we view our characterization of female-information-biased economies as a contribution. This characterization, in particular, speaks to the role of market segmentation or stratification in generating an asymmetry of information concentration in a two-sided market. As mentioned earlier, there is a very broad sociological and economics literature on social and economic segmentation and its different dimensions, although its focus has not been on analyzing the effect of segmentation on information diffusion in two-sided markets in the way we approach this problem. We therefore view our analysis of this problem as complementing this literature.

Finally, our study distinguishes itself by its applications and empirical implications. In the context of HIV/AIDS, the biggest public health crisis of our time, we theoretically document for the first time in a unified framework the role of female discrimination, market segmentation and social inequality in creating an HIV infection bias against women. We also develop applications to academic markets, and to the international market for highly talented workers. By focusing on *market structure*, and not only on *network structure*, we derive implications that are easily testable, as we shall show in Section 7.

1.5 Plan of the Paper

The rest of this paper unfolds as follows. Section 2 introduces the model that forms the basis for our analysis. We characterize pairwise stable networks in Section 3, and study their welfare properties in Section 4. Section 5 introduces a new approach to analyzing the diffusion of information in a network, which we use to characterize female-information-biased economies. Section 6 studies two extensions of the fidelity model. Applications follow in Section 7. We discuss and conclude our study in Section 8, and collect all the proofs in Section 9.

2 The Fidelity Model

The economic environment consists of a non-empty finite set of individuals $N = \{i_1, \ldots, i_n\}$ of size n, partitioned into a set of men M and a set of women W, each of equal size. Each individual derives utility from direct links with opposite sex agents, but engaging in multiple links is an act of infidelity, and is punished if detected by the cheated partner. Punishment is both retributive and restorative. Detection occurs with positive probability. It is assumed that a woman whose infidelity is detected is more severely punished than a man in a similar situation. Networks that arise from this environment are called *fidelity networks*.

2.1 Utility Functions

Let $\overline{M} = M \cup \{\emptyset\}$ and $\overline{W} = W \cup \{\emptyset\}$ be the expanded set of men and the expanded set of women, respectively. A network g is a subset of $\overline{M} \times \overline{W}$ such that $\forall m \in M, (m, \emptyset) \in g \Longrightarrow \forall w \in W$, $(m, w) \notin g$ and $\forall w \in W, (\emptyset, w) \in g \Longrightarrow \forall m \in M$, $(m, w) \notin g$. Here, $(m, \emptyset) \in g$ means that m is connected to no woman in g, and similarly, $(\emptyset, w)$ means that w is connected to no man in g. If an agent is not connected in a network g, we say that such an agent is isolated in that network.

Let g be a network. Since we are dealing with undirected graphs, if $(i,j) \in g$, we will abuse notation and consider that $(j,i) \in g$ (in fact, (i,j) and (j,i) represent the same relationship). Let $i \in N$ be an individual, and $s_i(g)$ the number of opposite type partners that i has in the network g.

The utility that i derives from g is expressed by the following function:

$$u_i(g) = v(s_i(g)) - c(s_i(g)) - h_i(g) + p_i(g)$$

where $v(s_i(g))$ is the utility derived from direct links with opposite type partners in g, and is concave and strictly increasing in $s_i(g)$; $c(s_i(g))$ the cost of infidelity; $h_i(g)$ the disutility to i caused by the (detected) infidelity of his/her partners in g; and $p_i(g)$ the utility derived by i from his/her detected unfaithful partners being punished.

We shall define $c(s_i(g))$, $h_i(g)$ and $p_i(g)$ more precisely. Let $j, k \in N$ be such that $(i,j) \in g$ and $(i,k) \in g$. Let π be the probability that j detects the liaison (i,k), and c the cost incurred by i if j detects that liaison. Because i has $s_i(g)$ partners, he/she will be detected $s_i(g)(s_i(g)-1)$ times with probability π, incurring an average cost of $s_i(g)(s_i(g)-1)\pi c$. So we define the cost function as:

$$c(s_i(g)) = s_i(g)(s_i(g)-1)\pi c$$

Further, assume that i's partner j has another partner l. i derives a certain amount of disutility d from j's liaison with l only if i becomes aware of that liaison, which happens with probability π. So, the average amount of disutility that i derives from the infidelity of his/her partners in g is:

$$h_i(g) = \sum_{j:(i,j)\in g}(s_j(g)-1)\pi d$$

The *restorative* nature of punishment implies that punitive actions repair the harm caused to the cheated by the wrongdoing of the cheater. So, the utility derived derived by i from his/her partners being punished for infidelity upon detection is:

$$p_i(g) = \sum_{j:(i,j)\in g}(s_j(g)-1)\pi d$$

Thus, i's utility from g is:

$$u_i(g) = v(s_i(g)) - s_i(g)(s_i(g)-1)\pi c - \sum_{j:(i,j)\in g}(s_j(g)-1)\pi d + \sum_{j:(i,j)\in g}(s_j(g)-1)\pi d = v(s_i(g)) - s_i(g)(s_i(g)-1)\pi c$$

The utility that i derives from g therefore is just a function of the number of partners that i has in g. In the rest of the paper, we will abuse notation and write:

$$u_i(g) = u_i(s_i(g)) = v(s_i(g)) - s_i(g)(s_i(g)-1)\pi c$$

Assuming that i is an expected utility maximizer, he/she thus maximizes the function $u_i(s_i(g))$. Denote the extension of u_i to the non-negative reals by $\overline{u_i}(s_i)$. Without loss of generality, let $\overline{u}_i$ be twice continuously differentiable. The following remark is straightforward:

Remark 1 *(1)* $\exists s^* \in [1, +\infty[$ *such that* $\overline{u}'(s^*) = 0$, $\forall s \in [0, s^*[$, $\overline{u}'(s) > 0$, *and* $\forall s \in]s^*, +\infty[$, $\overline{u}'(s) < 0$.

(2) $\frac{\partial s^*}{\partial c} \leq 0$

Remark 1 implies that u_i is single-peaked. This result also holds if we more generally assume a twice continuously differentiable convex cost function $c(s_i)$ such that $c(0) = c(1) = 0$, which allows for a more general interpretation of our model, including in non-fidelity contexts. Also, given that the cost incurred per detection is equal for all individuals of the same type, they have the same optimal number of partners. Further, the optimal number of partners for women is smaller than that for men because the former are more severely punished than the latter if their infidelity is detected. Note that if s^* is not an integer, then the optimal number of partners will be either the largest integer smaller than s^* $\lfloor s^* \rfloor$ or the smallest integer greater than s^* $\lceil s^* \rceil$. We also postulate that for no $s \geq 0$, $u_i(s) = u_i(s+1)$. These considerations motivate the following assumption:

Assumption A1. Denoting by s^*_m and s^*_w the unique optimal integer number of partners for men and women, respectively, we assume that $s^*_m > s^*_w$.

2.2 Mating economies

This section introduces the notion of mating economies. Let i be an individual and $P(i)$ the set of *feasible* partners for i. We assume that $j \in P(i) \Longrightarrow i \in P(j)$.[22]

A *trivial mating economy (* also simply called *a mating economy)*, denoted $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$, is a population $N = M \cup W$ endowed with a utility profile $(u_i)_{i \in N}$ such that for any $i \in M$ (resp. $i \in W$), $P(i) = W$ (resp. $P(i) = M$).

It is straightforward that a mating economy $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$ corresponds to a triplet $(N = M \cup W, s^*_m, s^*_w)$, and conversely, to any triplet $(N = M \cup W, s^*_m, s^*_w)$, one can associate a mating economy $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$.

A *segmented mating economy* $\mathcal{E}$ is a finite collection of pairwise disjoint mating economies. More formally, $\mathcal{E}$ is segmented if $\mathcal{E} = (\mathcal{E}^t)_{t \in I_T}$ where $I_T = \{1, ..., T\}$; $\mathcal{E}^t = (N^t = M^t \cup W^t, (u^t_i)_{i \in N^t})$ is a mating economy; for any $t \neq t' \in I_T$, $N^t \cap N^{t'} = \phi$; and $N = \bigcup\limits_{t \in I_T} N^t$.

In a segmented mating economy $\mathcal{E}$, each economy $\mathcal{E}^t$ is called a *segment*, and can be associated with a triplet $(N^t = M^t \cup W^t, s^{t*}_m, s^{t*}_w)$ where s^{t*}_m and s^{t*}_w are respectively the unique optimal integer number of partners for men and women in that segment. We shall pose $|N^t| = n^t$.

A trivial mating economy is therefore simply a segmented mating economy that has only one segment. It can also be called a *non-segmented mating economy.* A segmented mating economy is a collection of mating economies operating separately.

2.3 Elements of Fidelity Networks

Let g be a fidelity network. The elements of N are called vertices. A path in g connecting two vertices i_1 and i_n is a set of distinct nodes in $\{i_1, i_2, \ldots, i_n\} \subset N$ such that for any k, $1 \leq k \leq n-1$, $(i_k, i_{k+1}) \in g$.

Let i be an individual. We denote by $g(i) = \{j \in N : (i,j) \in g\}$ the set of individuals who have i as a partner in the network g. The cardinality of $g(i)$ is called the degree of i. If a set A is included either in M or W, then the image of A in the network g is $g(A) = \bigcup\limits_{i \in A} g(i)$.

We denote respectively by $M(g) = \{i \in M : \exists j \in W, (i,j) \in g\}$ and by $W(g) = \{i \in W : \exists j \in M, (i,j) \in g\}$ the set of men and women who are matched in the network g. We pose $N(g) = M(g) \cup W(g)$.

[22] Note that if $i \in M$ (resp. $i \in W$), then $P(i)$ is included in W (resp. M), but is not necessarily equal to W (resp. M).

A subgraph $g' \subset g$ is a component of g if for any $i \in N(g')$ and $j \in N(g')$ such that $i \neq j$, there is a path in g' connecting i and j, and for any $i \in N(g')$ and $j \in N(g)$ such that $(i,j) \in g$, $(i,j) \in g'$. A network g can always be partitioned into its components. This means that if $C(g)$ is the set of all components of g, then $g = \bigcup_{g' \in C(g)} g'$, and for any $g' \in C(g)$ and $g'' \in C(g)$, $g' \cap g'' = \emptyset$.

2.4 Pairwise Stability

In a society such as the one we are describing, individuals form new links or sever existing links based on the improvement that the resulting network offers them relative to the current network. We say that a network g is pairwise stable if: (i) no individual has an incentive to sever an existing link she is involved in; and (ii) no pair of a man and a woman have an incentive to form a new link while at the same time severing some of the existing links they are involved in.

More formally, given a profile of utility functions $u = (u_i)_{i \in N}$, a network g is pairwise stable with respect to u if:

(i) $\forall i \in N$, $\forall (i,j) \in g$, $u_i(s_i(g)) \geq u_i(s_i(g \setminus \{(i,j)\}))$

(ii) $\forall (i,j) \in (M \times W) \setminus g$, if network g' is obtained from g by adding the link (i,j) and perhaps severing other links involving i or j, $u_i(s_i(g')) > u_i(s_i(g)) \Longrightarrow u_j(s_j(g')) \leq u_j(s_j(g))$ and $u_j(s_j(g')) > u_j(s_j(g)) \Longrightarrow u_i(s_i(g')) \leq u_i(s_i(g))$.

We denote the set of pairwise stable networks of a mating economy $\mathcal{E}$ by $\mathcal{PS}(\mathcal{E})$.

The following example illustrates the concept of pairwise stability, and will often be referred to in the remainder.

Example 1 *Consider a mating economy with 10 men and 10 women. An agent i's utility function is $u_i(s) = s - s(s-1)\pi c_i$ where $\pi = 0.5$, and $c_i = \frac{2}{7}$ if $i \in M$ and $c_i = \frac{2}{3}$ if $i \in W$. It can be checked that $s_w^* = 2$ and $s_m^* = 4$. Consider the three networks represented by Figures 1-1, 1-2 and 1-3. In the first two networks, each woman is at her peak, and no man has more than his optimal number of partners. Thus, nobody has an incentive to sever a link he/she is involved in, and although some men who have less than their optimum (e.g., m_1) would like to have a relationship with women they are not linked to, this is not possible because all women are at their peak. These two networks are therefore pairwise stable. However, the networks represented by Figures 1-3 and 1-4 are not pairwise stable. In Figure 1-3, woman w_4 has four partners, and thus would be better off dropping two of them. In Figure 1-4, woman w_8 has only one partner and man m_6 has no partner, so both will be better off if they form a relationship.*

Now, assume that this economy is segmented due to an exogenous factor, and two segments, each consisting of 5 men and 5 women, result. The first segment contains men $m_1 - m_5$ and women $w_1 - w_5$, and the second segment contains men $m_6 - m_{10}$ and women $w_6 - w_{10}$. Cross-segment relationships are not possible. The network represented by Figure 1-5 is a pairwise stable network of this economy, whereas the networks represented by Figures 1-1, 1-2, 1-3,and 1-4 are not feasible.

In Example 1, we note that in pairwise stable networks, women obtain their optimal number of partners, but men generally do not. A simple intuitive explanation is that women are on the short side of the market, supplying a smaller number of links than those demanded by men. Men therefore end up competing for them. However, we will see in the next section on the general characterization of pairwise networks that under certain conditions, women may obtain less than their optimal number of partners, despite men competing for them.

3 Existence and Characterization of Pairwise Stable Networks

In this section, we prove the existence and provide a complete characterization of pairwise stable networks. The proof of the existence is constructive, and easily derives from the characterization result. More generally, all the possible configurations of pairwise stable networks can be deduced from this result. Also, for simplicity, we state our results only for non-segmented mating economies. In fact, given that a segmented economy is just a collection of pairwise disjoint non-segmented economies, generalizations are straightforward.[23]

We will see that the main characterization result (Theorem 1) is sensistive to the size of the economy. To facilitate its exposition, we state a few preliminary results as lemmas (Lemmas 1-4). Lemma 1 below says that in a pairwise stable network, no individual has more than his/her optimal number of partners.

Lemma 1 *Let g be a pairwise stable network. Then, $\forall(m, w) \in M \times W$, $0 \leq s_m \leq s_m^*$ and $0 \leq s_w \leq s_w^*$.*

The intuition behind the proof of Lemma 1 is that if an agent is linked to strictly more than his/her optimal number of partners in a network, he/she will be better off dropping some of them to be at his/her optimum, implying that that network is not pairwise stable.

The following lemma characterizes pairwise stable networks in very small economies and small economies, corresponding formally to $|M| \leq s_w^*$ and $s_w^* < |M| \leq s_m^*$, respectively. It says that in very small economies, there exists a unique pairwise stable network in which all men are matched with all women. In a small economy, a network is pairwise stable if and only if each woman is matched exactly with her optimal number of partners, and each man is matched with anywhere from no woman at all to all the women in the economy.

Lemma 2 *Let g be a network. (1) is equivalent to (2) and (3).*

(1) g is pairwise stable

(2) If $|M| \leq s_w^*$, then $\forall(m, w) \in M \times W$, $s_m = s_w = |M|$

(3) If $s_w^* < |M| \leq s_m^*$, then $\forall(m, w) \in M \times W$, $0 \leq s_m \leq |M|$ and $s_w = s_w^*$.

We now turn our attention to large economies, which formally correspond to $|M| > s_m^*$. The characterization of pairwise stable networks will depend on the optimal number of partners for each woman (s_w^*). In particular, we distinguish two cases: $s_w^* \in \{1, 2\}$ and $s_w^* > 2$. The following lemma says that if $s_w^* \in \{1, 2\}$, a network is pairwise stable if and only if each woman is matched exactly with her optimal number of partners, and each man is matched with no more than his optimal number of partners.

Lemma 3 *Assume that $|M| > s_m^*$ and $s_w^* \in \{1, 2\}$, and let g be a network. Then (1) and (2) are equivalent.*

1) g is pairwise stable

2) $\forall(m, w) \in M \times W$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$.

According to Lemma 3, each woman is at her peak in any pairwise stable network whenever s_w^* does not exceed 2 (an illustrative example is the large economy defined in Example 1 where $s_w^* = 2$). The following example demonstrates that this is no more true when s_w^* exceeds 2. In fact, it shows that some women may have less than their optimal number of partners in this case.

[23] Note that only two main results in the paper (Theorem 3 and Corollary 1) will explicitly appeal to non-trivial segmented mating economies.

Example 2 *Consider a mating economy with 8 men and 8 women. An agent i's utility function is* $u_i(s) = s - s(s-1)\pi c_i$ *where* $\pi = 0.5$, *and* $c_i = \frac{2}{9}$ *if* $i \in M$ *and* $c_i = \frac{2}{7}$ *if* $i \in W$. *It can be checked that* $s_w^* = 4$ *and* $s_m^* = 5$. *Consider the two networks represented by Figure 2-1 and Figure 2-2, respectively. In the first network, each woman is at her peak and no man has more than his optimal number of partners. One can easily check that it is a pairwise stable network.*

In the second network, one woman (w_1) has only three partners (less than her optimal number of partners), seven women are at their peak, and no man exceeds his optimum. Note that men who are not matched with w_1 are already at their peak, and thus w_1 cannot form a new link although she desires to. Also, no agent has an incentive to sever an existing link he/she is involved in. So this network is pairwise stable. This shows that the characterization of Lemma 3 does not generally hold when the optimal number of partners for women is 3.

The following result now generalizes some features of Example 2. It shows that in large economies, when the optimal number of partners for women is greater than 2: (1) it is possible that a woman obtain fewer than her optimal number of partners in a pairwise stable network. When such women exist: (2) none of them can be unmatched; (3) they all belong to the same component; and (4) their number cannot exceed $s_w^* - 2$ (so, there can only exist a few such women).

Lemma 4 *Assume that* $|M| > s_m^*$ *and* $s_w^* > 2$, *and let g be a pairwise network. Let* $A = \{w \in W : s_w < s_w^*\}$ *be the set of women who are matched with fewer than their optimal number of partners in g.*

1) A may not be empty.

2) If $A \neq \phi$, *each woman in A has at least one partner.*

3) If $A \neq \phi$, *there exists a unique component h of g such that* $A \subset W(h)$.

4) $0 \leq |A| \leq s_w^* - 2$.

We are now ready to state our main result, which partially derives from Lemmas 1-4, and provides a complete characterization of pairwise stable networks in a mating economy of any size.

Theorem 1 *Let g be a network. (1) is equivalent to (2)-(5).*

1) g is pairwise stable

2) $|M| \leq s_w^* \Longrightarrow \forall (m,w) \in M \times W,\ \ s_m = s_w = |M|$

3) $s_w^* < |M| \leq s_m^* \Longrightarrow \forall (m,w) \in M \times W,\ \ 0 \leq s_m \leq |M|$ *and* $s_w = s_w^*$.

4) $|M| > s_m^*$ *and* $s_w^* = 1, 2 \Longrightarrow \forall (m,w) \in M \times W,\ 0 \leq s_m \leq s_m^*$ *and* $s_w = s_w^*$.

5) $|M| > s_m^*$ *and* $s_w^* > 2 \Longrightarrow \exists A = \{w \in W : s_w < s_w^*\}$ *such that:*

- $A = \phi \Longrightarrow \forall (m,w) \in M \times W,\ 0 \leq s_m \leq s_m^*$ *and* $s_w = s_w^*$
- $A \neq \phi \Longrightarrow \forall (m_1, m_2, w_1, w_2) \in \bigcap_{w \in A} g(w) \times (M \setminus \bigcap_{w \in A} g(w)) \times A \times (W \setminus A)$, $|A| \leq s_{m_1} \leq s_m^*,\ s_{m_2} = s_m^*,\ 1 \leq s_{w_1} \leq s_w^* - 1,$ *and* $s_{w_2} = s_w^*$.

In addition, if $|M| \geq s_w^*$ *and g is pairwise stable, then the number of women who have fewer than their optimal number of partners is at most* $s_w^* - 2$.

Theorem 1 shows that the characterization of pairwise stable networks in an economy depends on its size. The equivalence between assertation (1) and assertations (3)-(4) is just a re-statement of Lemmas 2-3. Assertation (5) is a bit involved and deserves a few explanatory comments. Its main appeal, however, is that it is constructive.

It says that in large economies where the optimal number of partners for each woman exceeds 2, it is possible that a woman be matched with fewer than her optimal number of partners (Lemma 4). Denote by A the set of such women. We illustrate the rest of the assertation by resorting to our previous example 2. A can be empty (as shown in Figure 2-1), or non-empty (as shown in Figure 2-2 where $A = \{w_1\}$). If $A = \phi$ in a network, then that network is pairwise stable if and only if each woman has exactly her optimal number of partners and each man has no more than his optimal number of partners (this corresponds to the first part of assertion (5), and is illustrated by Figure 2-1).

If $A \neq \phi$ in a pairwise network, then: any man has at least $|A|$ women; in particular, there are two types of men: those who are linked to all women in A and those who are not. (a) Those who are linked to all women in A obviously have at least $|A|$ women and at most their optimal number of partners s_m^* (Lemma 1) (in Figure 2-2, these men are m_1, m_2 and m_3) ; (b) those who are not linked to all the women in A have exactly s_m^* partners (in Figure 2-2, these men are $m_4 - m_8$). There are also two types of women: Those in A and those not in A. (c) Those in A have at least 1 partner (Lemma 4) and at most $s_w^* - 1$ partners (in Figure 2-2, $A = \{w_1\}$); (d) those not in A have their optimal number of partners s_w^* (in Figure 2-2, $W \setminus A = \{w_2, w_3, w_4, w_5, w_6, w_7, w_8\}$). Conversely, any network in which $A \neq \phi$ and which satisfies (a)-(d) is pairwise stable.

We conclude our comments on Theorem 1 by noting that the fact that the number of women who have fewer than their optimal number of partners does not exceed $s_w^* - 2$ in a pairwise stable network simply derives from the characterization of such networks in the case where $s_w^* \leq |M| \leq s_m^*$ and the case where $|M| > s_m^*$ and $s_w^* = 1, 2$. In those cases, each woman has exactly her optimal number of partners (Lemmas 2-3). In the case where $|M| > s_m^*$ and $s_w^* > 2$, this result comes from Lemma 4-4.

We end this section by stating the following straightforward result, which says that the set of pairwise stable networks of any mating economy is never empty.

Remark 2 *Let $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$ be a mating economy. Then $\mathcal{PS}(\mathcal{E}) \neq \phi$.*

The proof of this existence result is constructive. In fact, it is easy for instance to construct a network in which each agent has the maximal number of partners that each woman can optimally have. Such a network is always pairwise stable.

4 Efficiency and Stability

In this section, we study the welfare properties of pairwise stable networks. We appeal to two concepts of efficiency, namely *Pareto efficiency* and *strong efficiency*. We also consider welfare for each side of the economy. A network is said to be Pareto efficient if one cannot increase the utility of one agent without decreasing the utility of another agent. A network is said to be strongly efficient if its total value, given by the sum of agents' utilities in the network, is maximal. We say that a network is male-optimal if its total value for men (the sum of utilities accruing to men) is maximal. Female-optimality is similarly defined. Those notions are formalized in the following definition.

Definition 1 *Let $\mathcal{E} = (N = M \cup W, (u_i)_{i \in N})$ be a mating economy and g a network.*

1) g is said to be Pareto-dominated by another network g' if for all agent i, $u_i(s_i(g)) \leq u_i(s_i(g'))$, and $u_j(s_{j(}(g)) < u_j(s_j(g'))$ for some agent j. A network that is not Pareto-dominated is said to be Pareto-optimal or Pareto efficient.

2) g is said to be strongly efficient if $\sum_{i\in N} u_i(s_i(g)) \geq \sum_{i\in N} u_i(s_i(g'))$ for all network g'.

3) g is said to be male-optimal (resp. female-optimal) if $\sum_{i\in M} u_i(s_i(g)) \geq \sum_{i\in M} u_i(s_i(g'))$ (resp. $\sum_{i\in W} u_i(s_i(g)) \geq \sum_{i\in W} u_i(s_i(g'))$) for all network g'.

The following straightforward proposition says that in a mating economy, egalitarian pairwise stable networks have the maximal total value in the set of pairwise stable networks.

Proposition 1 *: Let $\mathcal{E} = (N = M \cup W, (u_i)_{i\in N})$ be a mating economy and g an egalitarian pairwise stable network. Then, $\sum_{i\in N} u_i(s_i(g)) \geq \sum_{i\in N} u_i(s_i(g'))$ for all network $g' \in \mathcal{PS}(\mathcal{E})$.*

Jackson and Wolinsky (1996) and Dutta and Mutuswami (1997) remark that there exists a tension between stability and strong efficiency. The following example illustrates a similar tension in fidelity networks.

Example 3 *The economy has 4 men and 4 women. An agent i's utility function is $u_i(s) = 10s - s(s-1)\pi c_i$ where $\pi = 0.5$, and $c_i = 2$ if $i \in M$ and $c_i = 14$ if $i \in W$. It can be checked that $s^*_w = 1$ and $s^*_m = 3$. We will show that no pairwise stable network is strongly efficient. Given Proposition 1, it suffices to provide an unstable network whose aggregate value is strictly greater than the aggregate value of any egalitarian pairwise stable network. Any egalitarian pairwise stable network g in this economy has the aggregate value $\sum_{i\in N} u_i(s_i(g)) = 80$. Figure 3-1 represents such a network. Now consider the unstable network g' where man m_1 is matched with women w_1 and w_2 , man m_2 with woman w_2, man m_3 with woman w_3, and man m_4 with woman w_4 (Figure 3-2). This network is unstable because w_2 has more than her optimal number of partners which is 1. We have $\sum_{i\in N} u_i(s_i(g')) = 82 > 80 = \sum_{i\in N} u_i(s_i(g))$, which completes our illustration.*

Note however that one can find a utility profile implying no tension between strong efficiency and stability. For instance, consider an economy with 4 men and 4 women. An agent i's utility function is $u_i(s) = s - s(s-1)\pi c_i$ where $\pi = 0.5$, and $c_i = \frac{2}{9}$ if $i \in M$ and $c_i = 2$ if $i \in W$. We have $s^*_w = 1$ and $s^*_m = 5$. It can be easily shown that all egalitarian pairwise stable networks are strongly efficient.

We are now ready to state our main result on the relationship between stability and efficiency.

Theorem 2 *Let $\mathcal{E} = (N = M \cup W, (u_i)_{i\in N})$ be a mating economy.*

1) If $|M| \leq s^*_w$, then the unique pairwise stable network that exists is Pareto efficient, strongly efficient, male-optimal and female-optimal.

2) If $|M| > s^*_w$, a Pareto efficient pairwise stable network always exists, but a strongly efficient pairwise stable network may not exist. In addition:

(i) A pairwise stable network g is Pareto efficient if and only if $\forall (m, w) \in g,\ \ 0 \leq s_m \leq s^*_m$ and $s_w = s^*_w$.

(ii) No pairwise stable network is male-optimal.

(iii) A pairwise stable network is Pareto efficient if and only if it is female-optimal.

A few clarifying comments on this result are in order. When $|M| \leq s^*_w$, we find that the unique pairwise stable network that exists in the economy is male-optimal and female-optimal. This is uniquely attributed to the fact each agent obtains the maximum possible number of partners in this network. When $|M| > s^*_w$, all pairwise stable networks are female-optimal, except those where some woman has less than her optimum. However, no pairwise stable network is male-optimal in this case. In fact, male-optimality to be achieved requires that each man obtain his optimal number of partners, which would imply that the number of links coming from the women's side strictly exceed the number of links they can optimally supply, something that is impossible in a pairwise stable stable. One therefore notes a tension between male-optimality and pairwise stability, and another tension between male-optimality and female-optimality. This latter tension reflects a conflict of interest between men and women that is always resolved in favor of women, since in general, pairwise stability is compatible with female-optimality and incompatible with male-optimality. In general, pairwise stability is also compatible with Pareto efficiency.

5 Communication Potential and Female-Information-Biased Economies

In this section, we study how the effect of the diffusion of a random, unexpected, idiosyncratic information shock to a network differs for men and women. For this purpose, we introduce a simple model of information transmission that answers the following questions:

- If a random agent in a network receives from an exogenous source a piece of information that he/she communicates to his/her neighbors, who in turn communicate it to their other neighbors and so on, what proportion of the population will end up receiving the information?
- What will the male-female difference in the proportion of informed agents be?

The answers to these two questions define respectively what we call the *communication or contagion potential* of a network, and the *gender difference in the communication or contagion potential* of a network. These definitions are stated more explicitly in Section 5.1 below. As remarked in the introduction, this model may be used to study the effect of a random idiosyncratic shock in a variety of contexts. We subsequently use them in Section 5.2 to characterize *female-information-biased* economies.

5.1 Communication Potential of a Network

Let g be a network. Assume that an agent $i \in N$ is drawn at random to receive a piece of information γ that he/she communicates to his/her partners in $g(i)$, who in turn communicate it to their other partners, and so on. Information is thus supposed to travel the network via word-of-mouth or neighbors' contagion. If i is not matched with any agent, the information does not spread. Suppose that with equal probability, $\frac{1}{|N|}$, each agent receives the information.[24] We define the *communication or contagion potential* of g as the expected proportion of agents who will receive the information following its diffusion in the network. We also define the *gender difference in communication or contagion potential* as the difference in the expected proportion of men and women who will receive the information. To formalize these notions, we need a few additional definitions.

Let $i \in N$ be an agent such that $g(i) = \emptyset$. We say that i is isolated in the network g. We abuse language and call $\{i\}$ an isolated component of g, thus consisting only of one agent. We denote by $\mathcal{I}(g)$ and $\mathcal{J}(g)$ respectively the set of isolated and non-isolated components of g. Clearly, the set of components of g $C(g) = \mathcal{I}(g) \cup \mathcal{J}(g)$.

Assume that g is a k-component network, and let $C(g) = \{g_1, \ldots, g_k\}$ be the set of its components. Pose $I_k = \{1, \ldots, k\}$. To simplify notation, we write $N(g_i) = N_i$, $M(g_i) = M_i$, $W(g_i) = W_i$, and $|N_i| = n_i$ for $i \in I_k$. We associate each component g_i with the number n_i and its bipartite component vector $(|M_i|, |W_i|)$,

[24] For the sake of concreteness, consider the information to be an instance of becoming infected with the AIDS virus due to a random event. In Section 7, we will provide other applications.

and g with the vector $[(n_i)]_{i\in I_k}$ and its bipartite vector $[(|M_i|,|W_i|)]_{i\in I_k}$. It follows from this that if g_i is an isolated component, its associated vector is either $(1,0)$ or $(0,1)$.

Denote by $\rho(z,\gamma)$ the status of an agent z with respect to the information γ. We pose $\rho(z,\gamma)=1$ if z has received the information and 0 if he/she has not. For any set $B=N,M,W$, let $\Pr(\gamma|B)=\frac{|\{z\in B:\rho(z,\gamma)=1\}|}{|B|}$ be the proportion of agents who have received the information in the population B. We provide below a formula for the expected value of $\Pr(\gamma|N)$ and $\Pr(\gamma|M)-\Pr(\gamma|W)$, denoted respectively $E[\Pr(\gamma|N)]$ and $E[\Pr(\gamma|M)-\Pr(\gamma|W)]$.

Claim 1 *1)* $E[\Pr(\gamma|N)]=\frac{1}{n^2}\sum_{i\in I_k}n_i^2$.
2) $E[\Pr(\gamma|M)-\Pr(\gamma|W)]=\frac{2}{n^2}\sum_{i\in I_k}(|M_i|^2-|W_i|^2)$.

Note that the proof of item (1) of Claim 1 does not use the fact that the network is a bipartite graph. This proof is therefore valid for any network.

Claim 1 provides the foundation for the following definition.

Definition 2 *Let g be a k-component network with the corresponding component vector $[(n_i)]_{i\in I_k}$.*
1) The communication or contagion potential of g is defined as:

$$\mathcal{P}(g)=\frac{1}{n^2}\sum_{i\in I_k}n_i^2.$$

2) If g is a bipartite graph with the corresponding component vector $[(|M_i|,|W_i|)]_{i\in I_k}$, the gender difference in the communication or contagion potential of g is defined as:

$$\mathcal{F}(g)=\frac{2}{n^2}\sum_{i\in I_k}(|M_i|^2-|W_i|^2).$$

3) g is said to be female-information-biased if $\mathcal{F}(g)\le 0$. g is said to be male-information-biased if $\mathcal{F}(g)\ge 0$. A network that is both female-information-biased and male-information-biased ($\mathcal{F}(g)=0$) is said to be gender-information-neutral.

Intuitively, a female-information-biased is one in which women (on average) belong to larger components than men, so that the diffusion of a random information shock has more effect on the former than the latter. The following example further illustrates Definition 2.

Example 4 *Consider the non-segmented economy of Example 1 with 10 men and 10 women. Call g, h and l the networks represented by Figures 1-1, 1-2 and 1-3, respectively. Their component vectors are $[(13);(6);(1)]$, $[(19);(1)]$ and $[(13);(6);(1)]$, respectively. Their bipartite component vectors are $[(7,6);(2,4);(1,0)]$, $[(9,10);(1,0)]$ and $[(7,6);(2,4);(1,0)]$, respectively. One can easily check that the communication potential of these networks is $\mathcal{P}(g)=\frac{206}{400}$, $\mathcal{P}(h)=\frac{362}{400}$ and $\mathcal{P}(l)=\frac{206}{400}$. Also, the gender difference in the communication potential is $\mathcal{F}(g)=\frac{4}{400}$, $\mathcal{F}(h)=-\frac{36}{400}$ and $\mathcal{F}(l)=\frac{4}{400}$. We remark that despite the fact that g and h have the same degree distribution (the number of partners that each agent has does not vary across the two networks), g and h have different communication potential and different gender difference in the communication potential (g is male-information-biased whereas h is female-information-biased). On the other hand, g and l have different degree distributions , but have the same communication potential and gender difference in the communication potential. This shows that in a sexual market for example, simply knowing how many partners people have may* *not be useful in evaluating how vulnerable they are to the diffusion of a random HIV infection. What is really important is the knowledge of a network structure in terms of the number of components that it has and their size.*

It is also important to note that female-information-bias does not depend on certain men being isolated in a network. In fact, in the network represented by Figure 1-5, if we break the link between m_9 and w_{10} and create a new link between m_{10} and w_{10}, no man will be isolated in the resulting network, yet it is easy to check that this network is female-information-biased. Also, there exist male-information-biased networks in which certain men are isolated, such as g.

We will next characterize female-information-biased economies. In the context of HIV/AIDS for example, these are economies in which a higher cost of infidelity punishment for women than men always leads to HIV prevalence being greater among the former than the latter.

5.2 Female-information-biased Economies

We provide below a formal definition of a female-information-biased economy.

Definition 3 *Let $\mathcal{E}$ be a mating economy that is not necessarily trivial. $\mathcal{E}$ is said to be female-information-biased if every pairwise stable network $g \in \mathcal{PS}(\mathcal{E})$ is female-information-biased. $\mathcal{E}$ is said to be male-information-biased if every pairwise stable network $g \in \mathcal{PS}(\mathcal{E})$ is male-information-biased. $\mathcal{E}$ is said to be gender-information-neutral if $\mathcal{E}$ is both female-information-biased and male-information-biased.*

To motivate our question, let us consider the non-segmented economy of Example 1 and reexamined in Example 4. In the latter example, we have seen that that economy admits a pairwise stable network that is male-information-biased and another one that is female-information-biased, none of them being gender-information-neutral. That economy therefore is neither female-information-biased, nor male-information biased. Given an economy $\mathcal{E}$, we would like to find necessary and sufficient conditions on $\mathcal{E}$ for $\mathcal{E}$ to be female-information-biased. We will need two preliminary results (Lemmas 5-6 below).

Let g be a pairwise stable network, g' a non-isolated component of g, and $A = \{w \in W : s_w < s_w^*\}$ the set of women who are matched to fewer than their optimal number of partners in g. The following lemma gives a formula for the number of women involved in g', and provides a lower bound and an upper bound for the number of men in this component under the assumption that A is empty.

Lemma 5 *Let g be a pairwise stable network such that $A = \phi$, and $g' \in \mathcal{J}(g)$ a non-isolated component of g. Then:*

1) $|W(g')| = \frac{|g'|}{s_w^*}$.

2) $\max(\left\lceil \frac{|g'|}{s_m^*} \right\rceil, s_w^*) \leq |M(g')| \leq |g'| - \frac{|g'|}{s_w^*} + 1$.

Combining items (1) and (2) in Lemma 5 yields a complete characterization of pairs $(X, Y) \subset M \times W$ such that there exists a matching between X and Y which is pairwise stable and which directly or indirectly connects any two elements of $X \cup Y$.

The following lemma will be needed too. It says that if each non-isolated component of a bipartite (not necessarily pairwise stable) network g is such that the number of women weakly exceeds the number of men, then that network is female-information-biased.

Lemma 6 *Let g be a network.*

If $\forall g' \in \mathcal{J}(g)$, $|M(g')| \leq |W(g')|$, then $\mathcal{F}(g) \leq 0$.

It is important to remark that while the assumption is made only on the non-isolated components of a bipartite graph, the implication is derived for the entire graph. A good example of a network in which the number of women weakly exceeds the number of men in each non-isolated component is a polygynous network. In such a network, each matched woman has only one partner and each matched man may have more than one partner.

The following result provides a complete characterization of female-information-biased trivial mating economies. It says that a mating economy is female-information-biased if and only if the optimal number of partners for each woman is 1 or the size of the economy does not exceed $4s_w^* + 2$.

Lemma 7 *Let $\mathcal{E} = (N, s_m^*, s_w^*)$ be a mating economy. Assertions (1) and (2) are equivalent.*

1) $\mathcal{E}$ is female-information-biased.

2) $s_w^* = 1$ or $n \leq 4s_w^* + 2$.

It is easy to see that if $s_w^* = 1$, then the economy is female-information-biased. This is because if $s_w^* = 1$, then any pairwise stable network in the economy is a polygynous network (Lemma 6). If $n \leq 4s_w^* + 2$, we prove that any possible pairwise stable network g is such that $\mathcal{F}(g) \leq 0$. The intuition behind why these two conditions are also necessary for the economy to be female-information-biased might not be easy to pin down. However we show by contradiction that if those two conditions are violated (that is, $s_w^* > 1$ and $n > 4s_w^* + 2$), then it is always possible to construct a pairwise stable network g such that $\mathcal{F}(g) > 0$ (Lemma 5 is particularly useful to this part of the proof).

Note for instance that the non-segmented economy of Example 1 violates these two conditions, which is the reason why this economy is not female-information-biased. However, if we partition this economy into two segments as we did in the same example so that each segment contains 5 men and 5 women, then condition $n \leq 4s_w^* + 2$ of Lemma 7 will be satisfied, and each segment of the economy will become female-information-biased (in particular, the gender difference in the contagion potential of the pairwise stable network represented by Figure 1-5 is $-\frac{16}{400}$). This observation motivates our main result for this section. This result says that a non necessarily trivial mating economy is female-information-biased if and only if that economy is segmented such that in each segment t, there are no more than $4s_w^{t*} + 2$ men and women whenever the optimal number of partners for each woman is greater than 1.

Theorem 3 *Let $\mathcal{E}$ be a mating economy that is not necessarily trivial. Assertions (1) and (2) are equivalent.*

1) $\mathcal{E}$ is female-information-biased.

2) $\mathcal{E}$ is a segmented mating economy $(\mathcal{E}^t = (N^t, s_m^{t*}, s_w^{t*}))_{t \in I_T}$ such that $\forall t \in I_T$, $s_w^{t*} = 1$ or $n^t \leq 4s_w^{t*} + 2$.

Note also that all segments need not have the same characteristics. For instance, if an economy has two segments, the optimal number of partners for each woman could be 1 in one segment and 3 in the other. In this case, there will need to be at most 14 individuals in the second segment, and any number of individuals in the first segment for the economy to be female-information-biased.

The characterization result in Theorem 3 is also equivalent to saying that an economy is female-information-biased if and only if it is segmented such that the size of any segment $\mathcal{E}^t$ is determined by a correspondence

$\varphi : \mathbb{N}^* \to \mathbb{N}^*$ such that $\varphi(s_w^{t*}) = \mathbb{E}^*$ if $s_w^{t*} = 1$ and $\varphi(s_w^{t*}) = \{k \in \mathbb{E}^* : k \leq 4s_w^{t*} + 2\}$ if $s_w^{t*} > 1$, where $\mathbb{N}^*$ is the set of strictly positive integers and $\mathbb{E}^*$ the set of strictly positive integers that are even (note that the size of an economy is by definition an even number). In other words, a segment $\mathcal{E}^t$ can be of any size if the optimal number of partners for women (s_w^{t*}) in that segment is 1, and should not contain more than $4s_w^{t*} + 2$ agents if $s_w^{t*} > 1$. The equivalent characterization of female-information-biased economies in terms of correspondences makes it easy to study some properties of these economies. For instance, one can check that φ is descending in the set $\{1, 2\}$ and ascending in the set $\mathbb{N}^* \setminus \{1\}$, with respect to set inclusion.

In the context of HIV/AIDS, Theorem 3 suggests that higher HIV/AIDS prevalence in women compared to men in certain societies may be an *unintended* consequence of sexual market segmentation or stratification. It also implies that in a sufficiently large and homogeneous population where segmentation is unlikely, women do not necessarily bear a greater share of the HIV/AIDS burden than men, despite being discriminated against in infidelity punishment.

The following example shows the relationship between the level of market segmentation and the maximal market size that guarantees female-information-bias.

Example 5 *Assume that in a mating economy, individuals choose their partners based on a set of e criteria* $\{x_1, ..., x_e\}$. *Each criterium* x_i *is a categorical variable with* y_i *categories.*[25] *So, there are* $z = \prod_{1 \leq i \leq e} y_i$ *segments in the economy. Assume that in each segment* $\mathcal{E}^t$, $t \in I_z$, *the optimal number of partners for each woman is* $s_w^{t*} > 1$. *Following Theorem 3, the maximum size of the economy that guarantees female-information-bias is* $\overline{n} = \sum_{t \in I_z} (4s_w^{t*} + 2)$. *This latter equation shows the relationship between the maximum size of the economy and the level of segmentation.*

To further illustrate this relationship, assume in particular that each of the e criteria for partner selection has $y = 5$ *categories, and that in each segment of the economy, the optimal number of partners for each woman is* $s_w^* = 2$. *It follows that the maximum size of the economy that guarantees that the economy is female-information-biased is* $\overline{n} = y^e(4s_w^* + 2) = 10 \times 5^e$. *Figure 4 shows how* $\overline{n}$ *varies as a function of e. We note in particular that if* $e = 1$, *then* $\overline{n} = 50$, *and if* $e = 10$, *then* $\overline{n} = 97,656,250$.

We now extend Theorem 3 to a probabilistic framework. Let $\mathcal{E}$ be an economy. If $\mathcal{E}$ admits multiple pairwise stable networks, it is difficult to tell the exact probability with which a given network will arise. However, within our framework, we know that a network arises with strictly positive probability if and only it is pairwise. Let $(x(g))$ be a probability distribution over networks that assigns to a network g a strictly positive weight $x(g)$ if and only if g is pairwise stable. Denote by $\mathcal{P}(\mathcal{E})$ the set of such probability distributions. The following result, which is a corollary of Theorem 3, says that women, on average, concentrate more information than men given any probability distribution over pairwise stable networks if and only if $\mathcal{E}$ is segmented such that no segment has more than $4s_w^* + 2$ agents whenever s_w^* exceeds 1.

Corollary 1 *Let* $\mathcal{E}$ *be a (non necessarily trivial) mating economy. Assertions (1) and (2) are equivalent.*

1) $\forall (x(g)) \in \mathcal{P}(\mathcal{E})$, $\sum_g x(g)\mathcal{F}(g) \leq 0$.

2) $\mathcal{E}$ *is a segmented mating economy* $(\mathcal{E}^t = (N^t, s_m^{t*}, s_w^{t*}))_{t \in I_T}$ *such that* $\forall t \in I_T$, $s_w^{t*} = 1$ *or* $n^t \leq 4s_w^{t*} + 2$.

The inequality in (1) is strict if and only if in addition to (2), $n^{t_0} > 2s_w^{t_0*}$ *for some segment* $\mathcal{E}^{t_0}$.

[25] In a sexual market, for instance, criteria would generally include socioeconomic and biological considerations such as income, education, occupation, religion, family background, age, height, etc. Religion, for instance, would have christianity, judaism, islam, induism, buddhism, etc. as categories.

6 Two Extensions of the Fidelity Model

In this section, we extend the fidelity model to two natural classes of economies. The first extension is to mating economies of female-to-male subjugation, and the second is to hierarchical mating economies. A mating economy of female-to-male subjugation is a mating economy in which within each couple, the woman is subjugated to the man in the sense that she is always available to him whenever he needs her. A hierarchical mating economy is a mating economy in which agents within each side of the market are ranked according to their social class, and higher-ranked agents are more desired as partners by agents of the opposite type.

6.1 Economies of Female-to-Male Subjugation

The type of discrimination uncovered so far postulates only gender asymmetry in the punishment of infidelity. In this section, we additionally assume female-to-male subjugation in the sense that within each couple, the woman is always available to the man whenever he needs her. This assumption, that we denote $(\mathcal{S})$, translates into gender differential time investment in a relationship as follows: Assuming that each agent is endowed with one unit of time that he/she splits equally among his/her partners, $(\mathcal{S})$ is equivalent to saying that within each couple, the woman invests at least as much time as the man. More formally, let (m, w) be a pair of a man and a woman on a relationship, and s_m and s_w their respective number of partners. The assumption that w is subjugated to m is expressed as :

$$(\mathcal{S}): \quad \tfrac{1}{s_w} \geq \tfrac{1}{s_m}$$

where $\frac{1}{s_i}$ is the amount of time that agent $i \in \{m, w\}$ invests in each of his/her relationships.

We define a mating economy of female-to-male subjugation as a mating economy subject to the constraint of female-to-male subjugation $(\mathcal{S})$. It is formalized by $\mathcal{E}^{\mathcal{S}} = (N = M \cup W, s_m^*, s_w^*, (\mathcal{S}))$.

The following result provides a complete characterization of pairwise stable networks in a mating economy of female-to-male subjugation. It also says that any such economy is female-information-biased.

Theorem 4 *Let $\mathcal{E}^{\mathcal{S}} = (N = M \cup W, s_m^*, s_w^*, (\mathcal{S}))$ be an economy of female-to-male subjugation and g a network. (1) is equivalent to (2)-(5).*

1) g is pairwise stable

2) $|M| < s_w^* \Longrightarrow \forall (m, w) \in M \times W, \quad s_m = s_w = |M|$

3) $s_w^* \leq |M| \leq s_m^* \Longrightarrow \forall (m, w) \in g, \quad s_w^* \leq s_m \leq |M|$ and $s_w = s_w^*$.

4) $|M| > s_m^*$ and $s_w^* = 1, 2 \Longrightarrow \forall (m, w) \in g, \ s_w^* \leq s_m \leq s_m^*$ and $s_w = s_w^*$.

5) $|M| > s_m^*$ and $s_w^* > 2 \Longrightarrow \exists A = \{w \in W : s_w < s_w^*\}$ such that:

- *$A = \phi \Longrightarrow \forall (m, w) \in g, \ s_w^* \leq s_m \leq s_m^*$ and $s_w = s_w^*$*
- *$A \neq \phi \Longrightarrow \forall (m_1, m_2, w_1, w_2) \in \bigcap\limits_{w \in A} g(w) \times (M \setminus \bigcap\limits_{w \in A} g(w)) \times A \times (W \setminus A)$,*

  $|A| \leq s_{m_1} \leq s_m^*, \ s_{m_2} = s_m^*, \ 1 \leq s_{w_1} \leq |A|$, and $s_{w_2} = s_w^*$.

6) In addition, $\mathcal{E}^{\mathcal{S}}$ is female-information-biased.

A few comments on this result are necessary. First, notice that if the optimal number of partners for each woman is 1, the constraint $(\mathcal{S})$ is always satisfied. This implies that any mating economy of female-to-male subjugation $\mathcal{E}^{\mathcal{S}} = (N = M \cup W, s_m^*, s_w^* = 1, (\mathcal{S}))$ admits the exact same set of pairwise stable networks as the corresponding economy without constraint $\mathcal{E} = (N = M \cup W, s_m^*, s_w^* = 1)$. This set consists solely of monogamous and polygynous networks in which each man is matched to no more than his optimal number of partners. Second, in a mating economy of female-to-male subjugation, women always concentrate more information than men in any network that arises. In a sufficiently large economy without constraint, this is the case if and only if $s_w^* = 1$ (Lemma 7). These facts show that the constraint $(\mathcal{S})$ can be viewed as a generalization of the normative principle that underlies the formation of networks in monogamous and polygynous cultures.

One might be tempted to view the set of pairwise stable networks of a constrained economy $\mathcal{E}^{\mathcal{S}}$ as a refinement of the set of pairwise stable networks of the corresponding economy $\mathcal{E}$. However, this is not true in general. In fact, consider the non-segmented economy $\mathcal{E}$ in Example 1. Consider the two networks represented by Figures 1-1 and 1-4, respectively. We know that the network represented by Figure 1-1 is pairwise stable in that economy. However, this is no longer the case if that economy is subject to the constraint $(\mathcal{S})$ (because woman w_6 has two partners m_6 and m_7, but m_7 has only one partner, which violates the constraint $(\mathcal{S})$). As for the network represented by Figure 1-4, it is not pairwise stable in $\mathcal{E}$, but is pairwise stable in $\mathcal{E}^{\mathcal{S}}$.

6.2 Hierarchical Mating Economies

A hierarchical mating economy is a mating economy in which each agent has a distinct social rank and higher-ranked agents are more desired as partners. Agents therefore have lexicographic preferences over numbers of partners and social ranks in this order. More formally, a hierarchical mating economy is formalized by $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ where:

- s_m^* and s_w^* represent the optimal number of partners for men and women, respectively;
- $s_m^* > s_w^*$;
- $\succ_m$ and $\succ_w$ are linear orderings on M and W, respectively. $\succ_m$ represents the ranking of men, and $\succ_w$ represents that of women. $\succ_m$ also represents women's preferences over men's ranks, and $\succ_w$ represents men's preferences over women's ranks.[26]

Intuitively, a hierarchical mating economy is a two-sided market in which each agent falls into a certain social class, which in turn determines his/her desirability as a partner: the higher in the social hierarchy, the more desired by agents of the opposite type. A real-life example is a sexual market where people's desirability as partners may be determined by their income, social class or education. In Section 7, we will provide applications to certain academic matching markets and to the country-citizen market for skilled workers.

We will next prove the existence of a unique pairwise stable network in any hierarchical mating economy, and show that any such economy is female-information-biased.

6.2.1 Characterization

The following lemma will be needed in the proof of our main result. Like Lemma 6, it provides a sufficient condition for a network to be female-information-biased.

[26] On the second interpretation of $\succ_m$and $\succ_w$, note that $\succ_m$, for instance, is not a ranking of the subsets of the set of men by women as it is often the case in the traditional matching literature; $\succ_m$ is a ranking of individual (or singleton) men by women. For our purpose, we do not need a ranking of the subsets of the set of agents on each side of the market.

Lemma 8 *(Pongou 2008) Let g be a network such that $\forall g' \in \mathcal{J}(g)$, $|M(g')| > |W(g')| \Longrightarrow |M(g')| = s_w^*$, and $|M(g')| \leq |W(g')| \Longrightarrow |M(g')| \geq s_w^*$. Then, $\mathcal{F}(g) < 0$.*

Our main result for this section says that any hierarchical mating economy admits a unique pairwise stable network, and is female-information-biased.

Theorem 5 *Let $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ be a hierarchical mating economy.*

1) There exists a unique pairwise stable network $g \in \mathcal{PS}(\mathcal{E}^{\succ})$.

2) $\mathcal{E}^{\succ}$ is female–information-biased. More precisely,

- If $|M| \leq s_w^*$, then $\mathcal{F}(g) = 0$.

- If $|M| > s_w^*$, then $\mathcal{F}(g) < 0$.

The proof of item (1) is constructive. It characterizes the unique pairwise stable network of a hierarchical mating economy in terms of the number of partners that each agent obtains. This network presents a pattern of matching in which women from lower ranks match with men from higher ranks. In the context of HIV/AIDS, item (2) implies that social inequality, captured in our model by the rankings of agents, combined with female discrimination, results in higher HIV/AIDS prevalence in women than men.

We illustrate this result by the following example, also with the purpose of highlighting the difficulty inherent in the proof of the sign of the function $\mathcal{F}(.)$.

Example 6 *Assume a hierarchical mating economy $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ such that $M = \{m_1, ..., m_{|M|}\}$, $W = \{w_1, ..., w_{|W|}\}$, $m_1 \succ_m m_2 \succ_m ... \succ_m m_{|M|}$ and $w_1 \succ_w w_2 \succ_w ... \succ_w w_{|W|}$.*

1) Assume that $|M| = |W| = 4$, $s_m^* = 2$ and $s_w^* = 1$, the unique pairwise stable network is g, associated with the bipartite component vector $[(1,2);(1,2);(1,0);(1,0)]$, and represented by Figure 5-1. We have $\mathcal{F}(g) = -\frac{8}{64} < 0$.

2) Assume that $|M| = |W| = 4$, $s_m^* = 3$ and $s_w^* = 1$, the unique pairwise stable network is h, associated with the component vector $[(1,3);(1,1);(1,0);(1,0)]$, and represented by Figure 5-2. We have $\mathcal{F}(h) = -\frac{12}{64} < 0$.

3) Assume that $|M| = |W| = 11$, $s_m^* = 7$ and $s_w^* = 5$, the unique pairwise stable network is l, associated with the component vector $[(5,7);(5,4);(1,0)]$, and represented by Figure 5-3. We have $\mathcal{F}(l) = -\frac{28}{484} < 0$.

4) Assume that $|M| = |W| = 11$, $s_m^* = 8$ and $s_w^* = 5$, the unique pairwise stable network is q, associated with the component vector $[(5,8);(5,3);(1,0)]$, and represented by Figure 5-4. We have $\mathcal{F}(q) = -\frac{44}{484} < 0$.

We also note that on both sides of the market, the number of partners that each agent has weakly increases with his/her social rank, more so for men than for women. While Figures 5-1, 5-2, and 5-3 show that all women have the same number of partners, Figure 5-4 illustrates that the woman on the bottom has fewer partners than those who are ranked higher. In all four figures, lower ranked men have fewer partners.

Figures 5-1, 5-2, 5-3 and 5-4 also allow us to appreciate some of the difficulties inherent in the general proof of the sign of $\mathcal{F}(.)$. In Figures 5-3 and 5-4, there are more women than men in each non-isolated component. The sign of $\mathcal{F}(g)$ and $\mathcal{F}(h)$ is therefore derived by appealing to Lemma 6. However, in Figures 5-3 and 5-4, there are more women than men in the first component, but more men than women in the second component. It is not therefore possible to appeal to Lemma 6 to figure out the sign of $\mathcal{F}(l)$ or $\mathcal{F}(q)$. We however note that the assumptions of Lemma 8 are satisfied in the case of l, which allows to derives its sign. q does not satisfy the conditions of Lemma 8, but we prove that the sign of $\mathcal{F}(.)$ for networks that have the characteristics of q is negative.

6.2.2 Comparative Statics

In this section, we are going to conduct two comparative statics exercises. The first one studies the effect of an increase in female discrimination on the outcomes of women. The second studies the effect of economic inequality on the prevalence of information in the overall population. In particular, we find that increasing female discrimination (which is equivalent to increasing the optimal number of partners for men while holding constant that of women) increases the quality of women's matches as well as the concentration of information among them. Further, economic inequality leads to higher concentration of information in the population if and only if female discrimination is sufficiently severe. We need a few preliminary definitions.

Let $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ be a hierarchical mating economy. Let $2^M(s_w^*)$ be the set of subsets of M with cardinality not exceeding s_w^*. Define over $2^M(s_w^*)$ the following binary relation denoted R_M: Let $X, Y \in 2^M(s_w^*)$ be two elements of $2^M(s_w^*)$. We say that X is better than Y, denoted XR_MY, if:

- the cardinality of X is at least equal to that of Y
- and any element of $X \setminus Y$ is ranked higher than all elements of $Y \setminus X$ by $\succ_m$.

Denote by P_M and I_M the strict component and the symmetric component of R_M, respectively. If XR_MY, one can say that a woman who is matched to all men in X has better matches than a woman who is match to all men in Y. We are now ready to state our result.

Proposition 2 *: Let $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ and $\mathcal{E}'^{\succ} = (N = M \cup W, s_m^{*'}, s_w^*, \succ_m, \succ_w)$ be two hierarchical mating economies such that $s_m^{*'} > s_m^*$. Let g and g' be their respective unique equilibria. Then:*

1) If $|M| \leq s_w^*$, then $\forall w \in W$, $s_w(g')I_M \; s_w(g)$; in addition, $\mathcal{F}(g) = \mathcal{F}(g') = 0$

2) If $|M| > s_w^*$, then $\forall w \in W$, $s_w(g')R_M \; s_w(g)$, and $\exists w_0 \in W$ such that $s_{w_0}(g')P_M \; s_{w_0}(g)$; in addition, $\mathcal{F}(g') < \mathcal{F}(g) < 0$.

To illustrate this proposition, consider the first and second economy in Example 6, as well as their respective pairwise stable networks g and h represented by Figure 5-1 and Figure 5-2. We note that female discrimination is greater in the second economy relative to the first economy, and that woman w_3 improves her match by switching from m_2 (in g) to m_1 (in h); no woman becomes worse off as a result of increasing female discrimination. Further, women concentrate more information than men in h compared to g: $\mathcal{F}(g) = -\frac{8}{64}$ and $\mathcal{F}(h) = -\frac{12}{64}$.

The following result states that inequality increases information concentration in small and large economies if and only if $s_w^* = 1$, that is, if female discrimination is sufficiently severe.

Proposition 3 *: Let $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ a hierarchical mating economy and $\mathcal{E} = (N = M \cup W, s_m^*, s_w^*)$ the corresponding mating economy without hierarchy.*

1) If $|M| \leq s_w^*$, $\forall g \in \mathcal{PS}(\mathcal{E})$, $\forall g' \in \mathcal{PS}(\mathcal{E}^{\succ})$, $\mathcal{P}(g) = \mathcal{P}(g')$.

2) If $|M| > s_w^*$, $\forall g \in \mathcal{PS}(\mathcal{E})$, $\forall g' \in \mathcal{PS}(\mathcal{E}^{\succ})$, $\mathcal{P}(g) \leq \mathcal{P}(g')$ if and only if $s_w^* = 1$.

When information is the AIDS virus, the first result says that an increase in female discrimination increases the quality of women's matches, but it increases their relative vulnerability to HIV/AIDS as well. The second result says that unless discrimination against women is sufficiently severe, economic inequality does not necessarily lead to higher HIV prevalence in a population.

6.2.3 Rank, Number of Partners and Information Concentration

In this section, we analyze the effect of an individual's social rank on the number of partners that he/she obtains, and on his/her likelihood of being informed. We also study the difference in the likelihood of being informed between a man and a woman of the same rank.

To state the problem more precisely, consider a man m_v and a woman w_v, both of rank v. What is the relationship between s_{m_v} and v, and between s_{w_v} and v?

Now assume that an agent z is drawn at random to receive a piece of information that spreads to his/her direct and indirect neighbors. Denote by $p(m_v)$ the probability that m_v be informed (either by being the first to receive the information or after the information spreads). What is the relationship $p(m_v)$ and v, and between $p(w_v)$ and v? Is w_v more likely to be informed than m_v?

On the first question, we find that higher-ranked men and women have more partners than their lower-ranked counterparts. We formally express this as s_{m_v} and s_{w_v} weakly increase in v. Note, however, that in keeping with the notation in the proof of Theorem 5, $v = 1$ is the highest rank, $v = 2$ is the second highest, etc. So when we say that s_{m_v} weakly increases in v, we mean that s_{m_v} increases as rank v goes from a lower rank (e.g., $v = 2$) to a higher rank (e.g., $v = 1$). The same interpretation is valid for other functions of v.

On the second question, we find that the probability that an individual be informed weakly increases with his/her rank. On the third question, we find that a woman is more likely to be informed than a man of the same rank. However, there is no clear relation between $p(m_v)-$ $p(w_v)$ and v. We state these results below.

Proposition 4 *: Let $\mathcal{E}^{\succ} = (N = M \cup W, s^*_m, s^*_w, \succ_m, \succ_w)$ be a hierarchical mating economy, and m_v and w_v a man and a woman of rank v.*

1) s_{m_v} and s_{w_v} weakly increase in v.

2) $p(m_v)$ and $p(w_v)$ weakly increase in v.

3) $p(m_v) - p(w_v) \leq 0$.

4) $p(m_v)-$ $p(w_v)$ is non-monotonic in v in general.

In the context of HIV/AIDS, Proposition 4 tells us that richer or more educated men and women have more partners and are more likely to be infected with HIV/AIDS as a result of their position in the unique pairwise stable sexual network of a hierarchical mating economy. Also, a woman is more likely to be infected than a man of the same social status. However, gender difference in prevalence is a non-monotonic function of social rank.

Throughout, we have empahsized the AIDS application for the sake of concreteness. In the next section, we will cover more applications.

7 Some Applications

We apply our theoretical findings to understand the functioning and outcomes of certain markets in the real world. Those we examine include the heterosexual market, two academic matching markets, and the country-citizen market for skilled workers.

7.1 Heterosexual Markets

Our theoretical results have testable implications for: (1) whether the distribution of sexual partners among sexually active individuals reveals the latter's preferences for sex; (2) gender difference in the timing of sexual initiation; (3) the distinct and complementary role of female discrimination, market stratification, and social inequality in gender difference in HIV/AIDS prevalence; (4) the effect of an individual's social status on his/her sexual behavior and his/her likelihood of being infected with HIV; and (5) the role of social inequality in increasing HIV/AIDS prevalence in a population.

It may be tempting to infer an individual's preferences for sex from the number of sexual partners that he/she has. In this respect, heterogeneity in the distribution of sexual partners may lead to the conclusion that sexually active individuals have heterogeneous preferences for sex. Our characterization of pairwise stable networks (Theorem 1), however, reveals that individuals with identical preferences may end up having different number of sexual partners. Heterogeneity in outcomes results from competition for women and an absence of coordination in relationship formation. Empirical observations therefore do not necessarily reveal preferences.

Our analysis also sheds light on gender differences in the timing of sexual initiation as well as explains the age gap between partners often documented in empirical studies. In fact, in a hierarchical mating economy, if we assume that rankings are determined by wealth, and that wealth is positively correlated with age, Theorem 5 essentially reveals that women match with older men. This is especially true for young women, who prefer older men to their age-mates. This shows that sexual initiation is earlier among women than men, as also documented in some countries (Zaba et al. (2004)). Theorem 5 is also consistent with the well-known "sugar daddy" and "sugar mummy" phenomena. The first consists of young women having sex with older and richer men in exchange for money, whereas the second describes an opposite pattern. However, our result suggests that the "sugar mummy" phenomenon is less prevalent, as it only involves the highest-ranked women.[27]

Most importantly, our analysis uncovers the social mechanism underlying the gender gap in the concentration of the HIV/AIDS epidemic, the biggest public health crisis of our time. In sub-Saharan Africa where the epidemic has had the most devastating impact, women account for 60% of all infected individuals (UNAIDS (2008)). Also, a recent analysis based on nationally representative surveys confirms that HIV/AIDS is more prevalent among women than men in most developing countries (Mishra et al. (2009)). Two main hypotheses have been advanced to explain the greater vulnerability of women to HIV/AIDS. The first is the assumption that the male-to-female transmission rate of the AIDS virus is greater than the female-to-male transmission rate (WHO (2003)). The second hypothesis posits that discrimination against women is a key underlying factor (WHO (2003)).

The argument behind the first hypothesis is mainly speculative, and rests on the claim that women have larger exposed surface area of mucous membrane during sexual intercourse, as well as a larger quantity of potentially infectious fluids than men (WHO (2003)). This hypothesis has been tested and invalidated in the African context in two influential studies from Uganda (Quin et al. (2000), Gray et al. (2001)). These studies use samples of monogamous heterosexual, HIV-discordant couples.[28] These couples were identified retrospectively from a population cohort in Rakai, Uganda. Frequency of intercourse within couples and HIV-1 seroconversion in the uninfected partners were assessed prospectively. Men and women independently reported similar frequencies of sexual intercourse.[29] In the first study, the male-to-female transmission rate of the AIDS virus (12.0 per 100 person-years) was not found to be significantly different from the female-to-male transmission rate (11.6 per 100

[27] The role of money is not explicit in our model. A "sugar daddy" relationship in our model would be equivalent to a relationship between a woman and a man of a higher rank, and a "sugar mummy" relationship would correspond to a relationship between a woman and a man of a lower rank. In each of figures 5-1, 5-2, 5-3 and 5-4, it can be checked that the number of sugar daddy relationships is greater than the number of sugar mummy relationships.

[28] An HIV-discordant couple is a couple in which one partner is infected with the AIDS virus and the other partner is not.

[29] This is an important feature of these data that is generally absent in most data on sexual behavior. It seems to reflect that partners were indeed faithful to each other, and thus infected individuals who were initially uninfected contracted the AIDS virus through intercourse with their initially infected partners. This makes it possible to assess gender differential transmission rates.

person-years). The second study reached a similar conclusion. The probability of the virus transmission per coital act from infected women to their initially uninfected male partners (0.0013) was not significantly different from the transmission probability per act from infected men to their initially uninfected female partners (0.0009). These findings invalidate the early hypothesis and explanation for gender differences in HIV/AIDS prevalence.[30] It is also the case that in several Western regions with low HIV/AIDS prevalence, women are not significantly more infected than men (UNAIDS (2008)). For these reasons, the focus is being shifted to the second hypothesis, which is that of societal discrimination against women.

Female discrimination has been documented in almost every society.[31] How it plays out in the determination of the gender gap in HIV/AIDS prevalence is still not well understood. Theorems 3-5 identify the *structure* of sexual markets in which discrimination against women causes a higher HIV prevalence among them compared to men. According to Theorem 3 and Corollary 1, in a large and sufficiently homogeneous population, female discrimination does not necessarily lead to higher HIV/AIDS prevalence among women than men. Female discrimination leads to higher HIV prevalence among women than men if: (a) it is sufficiently severe or the sexual market is sufficiently segmented or stratified (Theorem 3 and Corollary 1); (b) there is female-to-male subjugation (Theorem 4); or (c) there is social inequality (Theorem 5).[32] In reality, market stratification and social inequality often coexist (that is, within each social stratum, agents may be ranked). One can still show that such a coexistence will lead to higher HIV/AIDS prevalence among women than men. From an empirical point of view, it is important to note that in light of the difficulty encountered in collecting valid data on sexual networks, our focus on identifying the structure of markets, and not only the structure of networks, that result in higher HIV/AIDS prevalence among women than men is helpful, and generates easily testable implications. Conditions such as female discrimination, market stratification and social inequality can be easily observed, whereas sexual networks are extremely difficult to observe. We have proved that when those conditions are satisfied, HIV/AIDS is more prevalent among women than men in all networks that form.

Our analysis is also likely to inform the current debate on the effect of social status on HIV infection. Proposition 4 shows that higher-ranked men and women have more partners and are more likely to be infected with the AIDS virus, as a result of their position in the sexual network. This result is contrary to earlier misleading, but persistent claims, especially in the media and the policy circles, that poverty is a primary factor in the spread of HIV. Such claims generally rest on the assumption that poor women are more likely to prostitute themselves to rich men. While this assumption seems reasonable, it does not necessarily lead to the conclusion that the poor are more at risk to HIV/AIDS than the rich. In fact, if we assume that this assumption holds, then we will have to conclude that rich men are more at risk than poor men (who are less likely to find a female partner). Also, proponents of such claims generally overlook the fact that if poor and rich women all compete for a few rich men, rich women will win the competition first because they are more desired. Rich men and women thus end up having more partners than their poorer counterparts, as also shown in the unique pairwise stable network that arises in a hierarchical mating economy. Our analysis offers the first theoretical foundation to empirical studies that have found that wealthier and more educated men and women have more partners

[30] See also Powers et al. (2008) for a recent review of literature.

[31] See, e.g., Wollstonecraft (1792), Nussbaum and Glover (1995), Sen (1999)). There are several manifestations of the unfavorable treatment of women. One of these manifestations, known as the desirability bias, often appears in household surveys where women generally underreport their sexual activity (Fenton et al (2001), Zaba et al (2004), Mensch, Hewett, and Erulkar (2003), Jaya et al (2008)), consistent with the notion that women find it more difficult to admit having experienced sex outside a socially sanctioned relationship (Dare and Cleland (1994)).

[32] Social inequality should not be confused with gender inequality. Social inequality is captured in our model of a hierarchical mating economy by the rankings of agents on each side of the market. We do not compare the social status of men to that of women at all.

and are more at risk to HIV/AIDS than their poorer and less educated counterparts (e.g., Mishra et al. (2007), Lachaud (2007), Fortson (2008)).

The role of social inequality (or relative poverty) in the spread of HIV/AIDS has also received increasing attention among policymakers and is still being debated. Our analysis shows that social inequality favors the spreads of HIV/AIDS if and only if female discrimination is sufficiently severe (Proposition 3). In fact, in the presence of social inequality, a sexual network in which all sexually active individuals are directly or indirectly connected cannot arise, whereas such a network can arise if a society is homogeneous and discrimination against women is not sufficiently severe ($s_w^* > 1$). This means that if $s_w^* > 1$, social inequality can actually decrease HIV/AIDS prevalence. Social inequality therefore does not unambiguously increase HIV/AIDS prevalence in a population, although it creates a gender gap in favor of men.

7.2 Academic Markets

Our second application is to two academic markets: the department-faculty market and the faculty-student market. In general, the department-faculty market is a many-to-one matching market wherein a department can recruit several faculty whereas a faculty can be only affiliated to one department. In our application, we assume that agents within each side of the market are ranked. Indeed, within each field (e.g., Economics) or subfield (e.g., Microeconomic Theory), departments are often ranked by potential faculty (or job candidates) based on a combination of several factors including for instance work environment, productivity, quality of teaching, and the ability to place graduating students in the job market. Similarly, graduating students looking for academic jobs are often ranked based on their research and teaching ability. The department-faculty market can therefore be assimilated to a mating economy that is segmented by specialization (e.g., micro, macro, etc.), with each segment being a hierarchical mating economy. It is straightforward from Theorem 5 that such a market has a unique pairwise stable matching, wherein the best departments match with the best candidates.

In this particular market, we may also condsider the possibility of there being more candidates than slots available within each subfield. Even in this situation, one can still prove that there exists a unique pairwise stable matching, where the best departments match with the best candidates, but some good candidates may match with lower-ranked departments, and candidates on the bottom might not find a job (this outcome contrasts with that of a hierarchical heterosexual market where in general, women match with higher-ranked men). In the opposite situation where the number of slots (weakly) exceeds the number of candidates, there still exists a unique pairwise stable network in which candidates match with higher-ranked departments, and departments on the bottom are unable to hire. In this case, higher-ranked departments tend to be larger in size.

In a department-faculty market, it does not make sense to study the department-faculty difference in the concentration of information. However, one may be interested in studying the effect of being in a higher-ranked department on the probability of being informed of new ideas, say through department seminars. Proposition 4-2 would then imply that faculty in higher-ranked departments are more likely to be informed of new ideas.

We now turn to the faculty-student market. We view this market as a many-to-many asymmetric matching market wherein each faculty may have several students and each student may take several courses taught by different faculty, with the optimal number of partners being greater for faculty than for students. We are interested in the implications of such a market for how the dissemination of new ideas affect faculty and students differently. Reflecting on the role of faculty-student interactions in the diffusion of new ideas in the Economics Department at Brown University, Andrew Foster, then Chair of the Department, wrote:

"...student brings empirical or theoretical skills (perhaps learned from other faculty in the department) to a given faculty member's research... In still others, students bring the time and energy to explore an idea that the faculty member is intrigued about but not able to pursue on his or her own given other interests." (p. 15)

The question of which side of the market benefits more from the exchange of new ideas between faculty and students naturally arises. Put differently, how does a social planner design such a market so that new ideas affect students more than faculty, as this is often desired? We answer this question based on a very simple illustrative example. Suppose that a PhD program offers several courses of which each student should take 20 to complete the program. Students may not all take the same courses, as some courses, such as electives, might be of interest to some students and not to others. According to Lemma 7, if the 20 courses taken by each student are taught by 20 different faculty, the size of the market should not exceed 41 faculty and 41 students for the market to be student-information-biased. This suggests that most PhD programs where the number of faculty and students rarely exceeds 30 (within each side) are student-information-biased. In general, Lemma 7 provides a guidance as to how the social planner should choose the number of faculty and students and the number of courses to be taken by each student to complete the program in order to create student-information-bias. According to Theorem 3, within a university, the segmentation of the faculty-student market by discipline (e.g., Economics, Biology, Physics, etc.) may also result in new ideas affecting students more than faculty.

Within a department, one can also think of another faculty-student market wherein faculty play the role of advisors. Such a market can be viewed as a hierarchical mating economy wherein advisors are ranked based on their ability to advise and place students, and students are ranked based on their research potential. Theorem 5 and its proof imply that the best advisors match with the best students. Also, new ideas affect students more than faculty (Theorem 5). Further, better faculty have more students (Proposition 4-1), and higher-ranked faculty and students are more likely to be informed of new ideas (Proposition 4-2).

7.3 The Country-Citizen Market for "Brain"

The country-citizen market for "brain" is a many-to-one international matching market in which countries (M) compete for the best skilled workers (W) from all countries.[33] Each country wants to hire more than one worker, but a worker can only work for one country ($s_m^* > s_w^* = 1$). We assume that countries are ranked based on a combination of factors such as economic development and political stability and institutions ($\succ_m$), but workers are equally qualified. Each country prefers its citizen to foreigners, but values foreigners equally.[34]

We use a slightly different version of the model of a hierarchical mating economy to study this market. The difference comes from the fact that countries rank skilled workers differently. Let us study the outcomes of this market based on the following simple example.

Example 7 *An economy has 4 countries $m_1 - m_4$ and 4 skilled workers $w_1 - w_4$, with $s_m^* = 2$ and $s_w^* = 1$. Each worker w_i, $1 \leq i \leq 4$, is a native and citizen of country m_i . For each country m_i, i indicates its rank. Figure 6-1 represents the initial country-citizen matching by birth. This matching is not pairwise stable.[35] Figures 6-2, 6-3 and 6-4 represent the 3 pairwise stable matchings of this market.*

[33] For simplicity, we assume that each country contributes only its most talented citizen to the market, so that $|M| = |W|$.
[34] This assumption is consistent with hiring policies in most countries.
[35] It would have been the unique pairwise stable matching if s_m^* were 1.

A comparison between the initial matching and all the pairwise stable matchings shows that citizens born and trained in economically disadvantaged countries migrate to richer countries, a phenomenon popularly known as "brain drain".[36] We note that in two of the three pairwise stable matchings (Figures 6-3 and 6-4), the two richest countries retain their citizens and recruit the two citizens from the poorest countries. In Figure 6-2 however, the second richest country loses its citizen to the richest country, and recruits the two citizens from the poorest countries. For any probability distribution that assigns a matching a strictly positive weight if and only if that matching is pairwise stable, we can show that a country's brain drain increases with its level of underdevelopment.[37] This example also illustrates that initially economically advantaged countries become richer as a result of increased quality human capital, whereas the initially poorest countries become worse off.

One may also be interested in how a country's economic rank correlates with its likelihood of being affected by a technological shock such as a new idea or discovery that comes from the imagination of its workers. This would lead to a derivation of an analogue to item (2) of Proposition 4 for this economy. The finding would simply say that higher-ranked countries are more likely to be the prime beneficiary of new technological shocks. This is because richer countries have more skilled workers, which implies that the average number of random discoveries by these workers is greater for these countries.

These theoretical results provide qualitive insights into the patterns of international migration of skilled workers. According to the theory, the greatest economic power is the main pole of attraction for skilled workers, which captures some features of the data. Indeed, the United States is the main destination for highly qualified workers, with 40% of its foreign-born adult population having tertiary level education (Cervantes and Guellec (2002)). Between 1990 and 2002, this country attracted under its H1B temporary visa programme 900,000 highly skilled professionals, mainly IT workers, from India, China, and Russia, with a few coming from Canada, the United Kingdom and Germany (Cervantes and Guellec (2002)). Also, despite losing part of their human capital to the United States, countries such as Canada, the United Kingdom, Germany, France, Australia and New Zealand also attract massive human capital from developing regions including Africa, Latin America, and South and East Asia.[38] This is captured in our example by one of the three pairwise stable matchings that exist. Indeed, Figure 6-2 shows that the second richest country replaces its lost human capital with workers coming from the poorest countries. In general, this example qualitatively matches the data in showing that the level of a country's brain drain decreases with its level of development.

8 Conclusion

We have studied *fidelity networks*, which are networks that form in a mating economy of agents of two types (e.g., men and women), where each agent enjoys having direct relationships with the opposite type, while having multiple partners is viewed as infidelity and is punished if detected by the cheated partner. Punishment is assumed to be both retributive and restorative. There is female discrimination in that women are more severely punished than men, which results in each woman desiring fewer partners than each man. We have shown that the characterization of pairwise stable networks is sensitive to the size of the economy. In very small

[36] Brain drain is the large scale emigration of skilled workers. This term was coined by the Royal Society to describe the emigration of European "scientists and technologists" to North America after World War II.

[37] In particular, if we consider all the three pairwise stable matchings in our example to be equally likely, the average rate of brain drain over these matchings is 0% for the richest country, 33% for the second richest country, and 100% for the two poorest ones.

[38] For instance, it has been documented that a large majority of doctors trained in poor countries reside in rich countries (Chen and Boufford (2005)). Also see Easterly and Nyarko (2005) for a compelling discussion on brain drain.

economies ($n \leq 2s_w^*$), there is a unique pairwise stable network in which all men are matched to all women. In small economies ($2s_w^* < n \leq 2s_m^*$), women obtain their optimal number of partners, while each man is matched to anywhere from no woman to all the women in the economy. Finally, in large economies ($n > 2s_m^*$), each man obtains at most his optimal number of partners, and all women, except at most $s_w^* - 2$, obtain their optimum number (this implies that if $s_w^* = 1, 2$, no woman will have fewer than her optimal number of partners).

The analysis of the welfare properties of pairwise stable networks has revealed two types of tension and two types of agreement. In general, there exists a tension between pairwise stability and strong efficiency. This tension, however, is minimized in egalitrarian pairwise stable networks. There also exists a tension between male-optimality and female-optimality. About agreements, we have found that pairwise stability is compatible both with Pareto efficiency and female-optimality. Indeed, a pairwise stable network is Pareto efficient if and only if it is female-optimal, a result that relates the two notions of efficiency considered in our analysis.

We have introduced an approach to measuring contagion in a network, and have used it to analyze the gender asymmetric effect of the diffusion of a random, unexpected information shock to a network. This has resulted in a full characterization of female-information-biased economies, which are economies in which such a shock concentrates more among women than men in all pairwise stable networks. We have found that female-information-biased economies are segmented (or stratified) such that in each segment, local female discrimination is sufficiently severe or population does not exceed a certain threshold. In such economies, the size of each segment is determined by a well-defined correspondence that has some interesting properties.

We have subsequently extended the model to two important classes of economies: the class of economies characterized by female-to-male subjugation and the class of hierarchical mating economies. Economies of female-to-male subjugation generalize to the many-to-many matching environment the normative principle that underlies the formation of monogamous (one-to-one) and polygynous (many-to-one) matchings. We have found that they are female-information-biased. Hierarchical mating economies capture social inequality, and posit social rank as a determinant of an agent's desirability as a partner. Each such economies admits a unique pairwise stable network that is female-information-biased.[39]

Our application to heterosexual markets uncovers the social mechanism underlying the gender gap in HIV/AIDS prevalence in most societies. We have shown the distinct and complementary role of female discrimination, market stratification and social inequality in generating higher HIV/AIDS prevalence among women compared to men. By focusing on the *structure of markets*, and not only the *structure of networks*, that create an infection bias against women, we have generated easily testable predictions (a market structure can be easily observed, whereas collecting data on sexual networks is hard.) Also, our analysis has shed new light on the well-known "sugar daddy" and "sugar mummy" phenomena, suggesting that the former is more prevalent than the latter. Moreover, we have shown that higher-ranked men and women have more partners and are more at risk to HIV, and have derived conditions under which social inequality increases HIV prevalence in a population.

The *reduced form* of the fidelity model has proven to be applicable to non-fidelity markets as well. Our application to the the faculty-student market reveals the implications of the design of such a market for how new ideas affect faculty and students differently. Segmentation by field of specialization as well as class and faculty size matter. The application to the department-faculty market and the country-citizen market for brain uncovers a factor in the persistence of inequality between institutions: in equilibrium, initially better institutions

[39] Importantly, female-information-bias is achieved in those economies regardless of whether they are segmented or not. This also implies that a segmented economy where each segment has a different culture and structure, as it would be the case in the real world, with some segments being characterized by female-to-male subjugation and some others by hierarchy, is female-information-biased.

hire better workers, and are more likely to be the prime depository and beneficiary of new ideas and discoveries. In the latter market, we have also shown that a country's brain drain increases with its level of underdevelopment (all countries, except the most developed one, suffer brain drain, but more developed countries suffer less).

Our study suggests several possible directions for future research. One axis of research is clearly empirical. To the best of our knowledge, there is no empirical study on why women seem to be more vulnerable than men to HIV/AIDS in most societies. This is an important issue, given the damages of gender imbalance in mortality (Sen (1999)). In general, it has been assumed that greater female vulnerability to HIV/AIDS is due to biological factors, but this hypothesis has not been validated empirically (Section 7.1). Our work has yielded new hypotheses that can be easily tested using available data. As regarding the international market for talent, our analysis has also generated testable implications for how a country's economic development affects its brain drain, and how this, in turn, affects the gap between countries in the concentration of new technologies.

One can also think of several generalizations of the fidelity model. Throughout the paper, we have treated fidelity mating economies as *repugnant markets* where links cannot be sold or bought.[40] In certain environments, it is reasonable to assume that the imbalance between the supply of links by women and the demand by men will create a market for links, where buying a link means that the buyer establishes a reciprocal relationship with the seller in addition to making a monetary transfer to the latter. It will be interesting to study the equilibrium price for links and characterize the equilibrium networks, as well as examine implications for social welfare.

We have also assumed that information shocks are *unexpected*, and thus agents do not account for them when forming links. Indeed, many of the most important recent communicable shocks (e.g., the AIDS virus, the 9/11 terrorist attack, the H1N1 virus, the recent financial crisis, etc.) came as a surprise to the ordinary person.[41] At the same time, these shocks have increased awareness of the possibility of similar shocks in the future, which may affect present day behavior and social interactions. Thus, a possible extension of our analysis is to assume that agents live under the expectation of a shock, and study networks that form under this assumption. Clearly, if an expected shock is of a positive nature, the structure of the networks which form can be expected to maximize the benefits arising from a positive shock. In certain markets including the academic markets and the country-citizen market for "brain" that we have studied, the expectation of positive shocks such as new ideas or discoveries will tend to reinforce the stability of those networks that arise in the absence of the expectation of a shock. However, if it is expected that a shock will be harmful to the person who receives it and to those who are directly or indirectly connected to her, it is reasonable to think that networks which form will seek to minimize the damage of such a shock. For instance, a member of a terrorist network who is arrested may be persuaded to betray his direct connections in the network, who in turn will be arrested and will be forced to reveal their other connections and so on. If one assumes that terrorists take such a possibility into account when forming links, it might then be possible to analyze the level of coordination that it takes to form an *optimal* terrorist network, and to study the structure and properties of such a network. Financial markets are another example, where agents (e.g., banks, entrepreneurs, ordinary consumers, etc.) might envision the possibility of a financial shock and its likely contagion effects when forming links. It will then be interesting to know how such anticipation affects the formation of relationships among them, and what networks result from these relationships.

[40] See Roth (2007) for a treatment of how repugnance constraints certain markets in the real world.

[41] In the case of the AIDS virus in particular, nobody was aware of its existence when it first came to life (the virus has existed since at least 1959, but was only discovered in 1981 (Worobey et al. (2008)), and less than 1% of the urban populations had received an HIV test twenty years after the virus was discovered (Kumaranayake and Watts (2001)). This may explain why there was little sexual behavior response to the epidemic during the first twenty years after the discovery of the virus (that is, 1981-2001), which enhances the plausibility of the assumptions underlying our application of the fidelity model to HIV/AIDS.

9 Proofs

Proof of Remark 1

Proof. The proof is easy and left to the reader. ■

Proof of Lemma 1

Proof. Let g be a pairwise stable network. It is obvious that for any pair $(m, w) \in M \times W$, the inequalities $0 \leq s_m$ and $0 \leq s_w$ hold. Now assume by contradiction that there exists an agent i who is matched with more than his/her optimal number of partners. That agent will improve by unilaterally severing one of the links he/she is involved in, which implies that g is not pairwise stable, a contradiction. It follows that any pair $(m, w) \in M \times W$, $s_m \leq s_m^*$ and $s_w \leq s_w^*$. This completes the proof. ■

Proof of Lemma 2

Proof. The proof of the equivalence of (1) and (2) is straightforward and simply says tha if $|M| < s_w^*$, the unique pairwise stable network is the one in which each man is matched with all women.

Let us now show the equivalence of (1) and (3). We will proceed in two steps.

(1) $\Longrightarrow$ (3) : Assume that g is pairwise stable. Assume also that $s_w^* < |M| \leq s_m^*$. Given that the number of women is no more than the optimal number of partners for each man (because $|W| = |M| \leq s_m^*$), it is clear that each man is matched to at most $|M|$ women: $\forall m \in M,\ 0 \leq s_m \leq |M|$.

It just remains to prove that $\forall w \in W$, $s_w = s_w^*$. Assume by contradiction that there exists $w_0 \in W$ such that $s_{w_0} < s_w^*$. Then for any man m not matched with w_0, it should be the case that $s_m = |W| = |M|$ (in fact, if there exists a man m_0 not matched with w_0 such that $s_{m_0} < |W|$, it is clear that m_0 and w_0 will both improve by forming a new link, which contradicts the fact that g is pairwise stable). But this is impossible because given that $|W| = |M| \leq s_m^*$, there cannot exist any man m not matched with w_0 who has $|W|$ partners. It follows that $\forall w \in W$, $s_w = s_w^*$.

(3) $\Longrightarrow$ (1) : Assume that $s_w^* < |M| \leq s_m^*$ and that $\forall (m, w) \in M \times W,\ 0 \leq s_m \leq |M|$ and $s_w = s_w^*$. Let us show that g is pairwise stable. A man alone cannot improve by severing a link since he is at the upward sloping part of his utility function. He cannot form a new link with another woman since each woman has her optimal number of partners. And a woman cannot be part of any blocking move (either by herself or with a man) since she is at her peak. Therefore, g is a pairwise stable network. This completes the proof. ■

Proof of Lemma 3

Proof. Assume that $s_w^* \in \{1, 2\}$ and $|M| > s_m^*$ and let g be a network.

(1) $\Longrightarrow$ (2) : Assume that g is pairwise stable. Given Lemma 1, we just have to prove that $\forall w \in W$, $s_w = s_w^*$. Assume by contradiction that there exists $w_0 \in W$ such that $s_{w_0} < s_w^*$. Then for any man m not matched with w_0, it should be the case that $s_m = s_m^*$. In fact, if there exists a man m_0 not matched with w_0 such that $s_{m_0} < s_m^*$, it is clear that m_0 and w_0 will both improve by forming a new link, which contradicts the fact that g is pairwise stable.

It then follows that the number of links coming from the men's side is at least $s_m^*(|M| - s_{w_0})$. Notice that $s_m^*(|M| - s_{w_0}) \geq s_m^*(|M| - 1)$ because $s_{w_0} < s_w^* \in \{1, 2\}$ by contradiction. But $s_m^*(|M| - 1) \geq (s_w^* + 1)(|M| - 1)$ (because $s_w^* < s_m^*$), and $(s_w^* + 1)(|M| - 1) > s_w^*|M| = s_w^*|W|$ (because $|M| > s_m^* \geq s_w^* + 1$). It therefore follows that $s_m^*(|M| - s_{w_0}) > s_w^*|W|$, which means that the number of links coming from the men's side exceeds the maximal number of links that women can supply, which is impossible. We therefore conclude that $\forall w \in W$, $s_w = s_w^*$.

(2) $\Longrightarrow$ (1) : The argument here is similar to that of (3) $\Longrightarrow$ (1) of the proof of Lemma 2. ■

Proof of Lemma 4

Proof. Assume that $s_w^* > 2$ and $|M| > s_m^*$, and let g be a pairwise stable network.

1) Based on Example 1 and the network represented by Figure 1-2 in that example, we know that A is not always empty.

2) Assume that $A \neq \phi$, and assume by contradiction that there exists a woman w_0 in A who is not matched to any man. Then, it should be the case that each man has s_m^* partners, because if there exists a man m_0 who has fewer than s_m^*, then m_0 and w_0 will both improve by forming a new link, which implies that g is not a pairwise stable network, a contradiction. But given that each man has s_m^* partners, the number of links coming from the men's side is $s_m^*|M|$, which exceeds the number of links that women can supply, which is impossible.

3) Assume that $A \neq \phi$. To show that there exists a unique component h of g such that $A \subset W(h)$, it suffices to prove that $\bigcap\limits_{w \in A} g(w) \neq \phi$. In fact, if $\bigcap\limits_{w \in A} g(w) = \phi$, then $\exists w_0 \in A$ such that $g(w_0) \cap (\bigcap\limits_{w \in A \setminus \{w_0\}} g(w)) = \phi$. It is also the case that $\forall m \in M \setminus g(w_0)$, $s_m = s_m^*$. Also, we have: $\forall m \in g(w_0)$, $s_m = s_m^*$ (in fact, suppose that there exists $m_0 \in g(w_0)$ such that $s_{m_0} < s_m^*$; then, since $g(w_0) \cap (\bigcap\limits_{w \in A \setminus \{w_0\}} g(w)) = \phi$, there necessarily exists $w_1 \in A \setminus \{w_0\}$ such that $m_0 \notin g(w_1)$; but given that $s_{m_0} < s_m^*$ and $s_{w_1} < s_w^*$, it will be beneficial to both m_0 and w_1 to form a new link, contradicting the fact that g is pairwise stable). This implies that $\forall m \in g(w_0) \cup (M \setminus g(w_0)) = M$, $s_m = s_m^*$, also implying that $\sum_{m \in M} s_m = s_m^*|M| > s_w^*|W| > \sum_{w \in W} s_w$, which is impossible. Therefore, $\bigcap\limits_{w \in A} g(w) \neq \phi$.

4) We now want to show that $0 \leq |A| \leq s_w^* - 2$. Based on Example 1 and the network represented by Figure 1-1 in that example, we know that A may be empty (that is, $|A| = 0$). In addition, A being a finite set, it is the case that $|A|$ is a natural number. It follows that $|A| \geq 0$. It remains to show that $|A| \leq s_w^* - 2$. Let $h \in C(g)$ be the unique component in which the elements of A are vertices. We shall distinguish two cases: $W(h) = A$ and $W(h) \neq A$.

4-a) Suppose that $W(h) = A$. We shall first show that $M(h) = \bigcap\limits_{w \in A} g(w) = g(A)$. Since $W(h) = A$, it is obvious that $g(\bigcap\limits_{w \in A} g(w)) \subset A$, which means that no man in $\bigcap\limits_{w \in A} g(w)$ is matched with a woman outside of A, because otherwise, $W(h) \neq A$, which is a contradiction. $W(h) = A$ also obviously implies that $\bigcap\limits_{w \in A} g(w) \subset M(h)$. Now, let us assume by contradiction that $M(h)$ is not included in $\bigcap\limits_{w \in A} g(w)$. This implies that $\exists m_0 \in M(h) \setminus \bigcap\limits_{w \in A} g(w)$ such that $m_0 \in g(A)$. But since $W(h) = A$, m_0 cannot have a partner outside of A, because otherwise, $W(h) \neq A$, which is a contradiction. Also, it is necessarily the case that $s_{m_0} = s_m^*$ (because if $s_{m_0} < s_m^*$, since $\forall w \in A$, $s_w < s_w^*$, each woman in A not matched with m_0 will form a link with m_0). But given that $s_{m_0} = s_m^*$ and the fact that m_0 has all his partners in A, it follows that $|A| \geq s_m^*$. However, since by definition, $\forall (m, w) \in \bigcap\limits_{w \in A} g(w) \times A$, $(m, w) \in g$, and because of $|A| \geq s_m^*$, it is necessarily the case that $\forall m \in \bigcap\limits_{w \in A} g(w)$, $s_m = s_m^*$. This implies that $\forall m \in (\bigcap\limits_{w \in A} g(w)) \cup (M \setminus \bigcap\limits_{w \in A} g(w))$, $s_m = s_m^*$, also implying that $\sum_{m \in M} s_m = s_m^*|M| > s_w^*|W| > \sum_{w \in W} s_w$, which is impossible. Therefore, $M(h) = \bigcap\limits_{w \in A} g(w) = g(A)$.

Now, because of $\forall (m, w) \in \bigcap\limits_{w \in A} g(w) \times A$, $(m, w) \in g$ and $s_w < s_w^*$, it is necessarily the case that $|\bigcap\limits_{w \in A} g(w)| < s_w^*$. It is also the case that $\forall (m, w) \in (M \setminus \bigcap\limits_{w \in A} g(w)) \times (W \setminus A)$, $s_m = s_m^*$ and $s_w = s_w^*$, therefore implying that

$|M \setminus \bigcap_{w\in A} g(w)|s_m^* = |W \setminus A|s_w^*$. Given that $s_m^* > s_w^*$, this equation implies $|M \setminus \bigcap_{w\in A} g(w)| < |W \setminus A|$, and thus $|\bigcap_{w\in A} g(w)| > |A|$. It therefore follows from $|\bigcap_{w\in A} g(w)| < s_w^*$ that $|A| \leq s_w^* - 2$.

4-b) Now suppose that $W(h) \neq A$. Given that h is unique, it is straightforward that $A \subset W(h)$ and this inclusion is strict. This is equivalent to saying that a man in $\bigcap_{w\in A} g(w)$ is matched with a woman outside of A or ("or" here is inclusive) that a woman in A is matched wth a man in $M \setminus \bigcap_{w\in A} g(w)$ (one can show that such a man is necessarily linked to a woman outside of A). In general, let: $B = \{(m,w) \in g : (m,w) \in \bigcap_{w\in A} g(w) \times (W \setminus A)\}$, $C = \{(m,w) \in g : (m,w) \in (M \setminus \bigcap_{w\in A} g(w)) \times A)\}$, $D = \{(m,w) \in g : (m,w) \in \bigcap_{w\in A} g(w) \times A\}$ and $E = \{(m,w) \in g : (m,w) \in M \setminus \bigcap_{w\in A} g(w)) \times (W \setminus A)\}$. We have $g = B \cup C \cup D \cup E$. Each man in $M \setminus \bigcap_{w\in A} g(w)$ has s_m^* partners and each woman in $W \setminus A$ has s_w^* partners. Thus, the following equality holds: $|M \setminus \bigcap_{w\in A} g(w)|s_m^* - |C| = |W \setminus A|s_w^* - |B|$. This implies $|B| = |W \setminus A|s_w^* - |M \setminus \bigcap_{w\in A} g(w)|s_m^* + |C|$. But $|C| = \sum_{w\in A} s_w - |D|$, and $|D| = |\bigcap_{w\in A} g(w)||A|$, thus $|B| = |W \setminus A|s_w^* - |M \setminus \bigcap_{w\in A} g(w)|s_m^* + \sum_{w\in A} s_w - |\bigcap_{w\in A} g(w)||A|$. Beacuse each woman in A has at most $s_w^* - 1$ partners, the greatest possible value of $\sum_{w\in A} s_w$ is $|A|(s_w^* - 1)$. This implies that the greatest possible value of $|B|$ is $\max|B| = |W \setminus A|s_w^* - |M \setminus \bigcap_{w\in A} g(w)|s_m^* + |A|(s_w^* - 1) - |\bigcap_{w\in A} g(w)||A| = (s_m^* - |A|)|\bigcap_{w\in A} g(w)| - |A| + |W|s_w^* - |M|s_m^*$. But $\max|B| > 0$, which implies that $|A| < \frac{s_m^*|\bigcap_{w\in A} g(w)| - |M|(s_m^* - s_w^*)}{|\bigcap_{w\in A} g(w)| + 1} < s_w^* - 1$. This obviously implies that $|A| \leq s_w^* - 2$, which completes our proof.

■

Proof of Theorem 1

Proof. The proof of the equivalence of (1) on one hand, and (2) – (4) on the other hand derives directly from Lemmas 2 and 3. It remains to prove the equivalence of (1) and (5).

(1) $\Longrightarrow$ (5) : Assume that g is pairwise stable, and that $s_w^* > 2$ and $|M| > s_m^*$. Let $A = \{w \in W : s_w < s_w^*\}$ be the set of women who are matched to fewer than their optimal number of partners. We know that A may or may not be empty.

Assume that $A = \phi$. Then, it directly follows from that assumption and from Lemma 1 that $\forall(m,w) \in M \times W$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$.

Assume that $A \neq \phi$. Let $(m_1, m_2, w_1, w_2) \in \bigcap_{w\in A} g(w) \times (M \setminus \bigcap_{w\in A} g(w)) \times A \times (W \setminus A)$. m_1 is matched to all women in A; in addition, we have $0 \leq s_{m_1} \leq s_m^*$ (from Lemma 1); it thus follows that $|A| \leq s_{m_1} \leq s_m^*$.

It is also clear that $s_{m_2} = s_m^*$. In fact, if $s_{m_2} < s_m^*$, given that $m_2 \in M \setminus \bigcap_{w\in A} g(w)$, there necessarily exists a woman w_0 in A who is not matched to m_2. Both m_2 and w_0 would therefore improve by forming a new link, which contradicts the fact that g is pairwise stable.

Regarding w_1, the first inequality $1 \leq s_{w_1}$ comes from Lemma 4, and the second inequality $s_{w_1} \leq s_w^* - 1$ comes from the fact that $w_1 \in A$.

Finally, we obviously have $s_{w_2} = s_w^*$ from the fact that $0 \leq s_{w_2} \leq s_w^*$ (Lemma 1) and from $w_2 \notin A$. This completes the proof of (1) $\Longrightarrow$ $(2 - iv)$.

(5) $\Longrightarrow$ (1). Let g be a network. Suppose that $s_w^* > 2$, and let $A = \{w \in W : s_w < s_w^*\}$.

Assume that $A = \phi$ and that $\forall(m,w) \in M \times W$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$; and prove that g is a pairwise stable network. The proof is similar to that of (3) $\Longrightarrow$ (1) of Lemma 2.

Assume that $A \neq \phi$ and that $\forall (m_1, m_2, w_1, w_2) \in \bigcap_{w \in A} g(w) \times (M \setminus \bigcap_{w \in A} g(w)) \times A \times (W \setminus A)$, $|A| \leq s_{m_1} \leq s_m^*$, $s_{m_2} = s_m^*$, $1 \leq s_{w_1} \leq s_w^* - 1$, and $s_{w_2} = s_w^*$; and prove that g is a pairwise stable network. No agent will improve by unilaterally severing an existing link he/she is involved in g since he/she is either at the upward sloping part of his/her utility function or at his/her peak. No man in $\bigcap_{w \in A} g(w)$ cannot be part of a blocking move with a woman in A (since both are already matched to each other) or with a woman in $W \setminus A$ since she is at her peak. Similarly, no woman in $W \setminus A$ cannot be part of a blocking move with any man since she is at her peak. Finally, no pair of a man and a woman in $(M \setminus \bigcap_{w \in A} g(w)) \times (W \setminus A)$ (not matched in g, if any) will not improve by forming a new link since they are at their optimum. This shows that g is a pairwise stable network.

The proof of the last assertion immediately follows from items 2-4 in Theorem 1 for the case where $|M| = s_w^*$, $s_w^* < |M| \leq s_m^*$, or $|M| > s_m^*$ and $s_w^* = 1, 2$; in each of these cases, each woman obtains her optimal number of partners. When $|M| > s_m^*$ and $s_w^* > 2$, it follows from item 4 in Lemma 4; in that case, at most $s_w^* - 2$ women have fewer than their optimal number number of partners. ■

Proof of Remark 2

Proof. The proof is simple by noticing that: if $|M| \leq s_w^*$, there is a unique pairwise stable network (Lemma 2); and if $|M| > s_w^*$, then all egalitarian networks where each agent has s_w^* partners are pairwise stable. Such networks always exist and are very easy to construct. ■

Proof of Proposition 1

Proof. The proof is easy and left to the reader. ■

Proof of Theorem 2

Proof. 1) If $|M| \leq s_w^*$, the only pairwise stable network that exists in the economy is the one in which each man is matched with all women. This network is Pareto efficient because each agent being at the upward sloping part of his/her utility function has the maximal number of partners he/she can have; thus this network cannot be improved upon. It follows from the same argument that the aggregate value of this network is maximal for the entire economy, and for each side of the economy. This network is therefore strongly efficient, male-optimal and female-optimal.

2) If $|M| > s_w^*$, a Pareto efficient pairwise stable network always exists because any egalitarian pairwise stable network for instance is Pareto efficient; but Example 3 shows that a strongly efficient pairwise stable network may not exist.

2-i) Let us now prove 2-i). Let g be a pairwise stable network. Assume that g is Pareto efficient. Then, it follows from Lemma 1 that $\forall (m, w) \in g$, $0 \leq s_m \leq s_m^*$ and $0 \leq s_w \leq s_w^*$. It remains to show that $\forall w \in W$, $s_w = s_w^*$. Assume by contradiction that there exists a woman w_1 such that $s_{w_1} < s_w^*$. This implies that the set $A = \{w \in W : s_w < s_w^*\}$ is not empty. Thus by Theorem 1, $\forall (m_1, m_2, w_1, w_2) \in \bigcap_{w \in A} g(w) \times (M \setminus \bigcap_{w \in A} g(w)) \times A \times (W \setminus A)$, $s_w^* - 2 \leq s_{m_1} \leq s_m^*$, $s_{m_2} = s_m^*$, $1 \leq s_{w_1} \leq s_w^* - 1$, and $s_{w_2} = s_w^*$. It is also the case that there exists a man $m_1 \in \bigcap_{w \in A} g(w)$ such that $s_{m_1} < s_m^*$. There also exists a man $m_2 \in (M \setminus \bigcap_{w \in A} g(w))$ who is not matched with w_1 and who is matched with a woman $w_2 \in W \setminus A$ not matched with m_1. Delete the link (m_2, w_2), and add the links (m_2, w_1), (m_1, w_2), resulting in a new network g'. Note that g' is pairwise stable. In this new network, m_1 and w_1 have strictly improved relative to the network g, and no one's utility has decreased. It therefore follows that g is Pareto-dominated by g', which is a contradiction.

Thus, $\forall w \in W$, $s_w = s_w^*$.

Conversely, assume that $\forall (m, w) \in g$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$, and show that g is Pareto efficient. Assume by contradiction that it is not. Therefore, it is Pareto-dominated by another network g', which implies that for all agent i, $u_i(s_i(g)) \leq u_i(s_i(g'))$, and $u_j(s_j(g)) < u_j(s_j(g'))$ for some agent j. Note that each woman is necessarily at her peak in both g and g', which implies that j is a man. Given that g is pairwise stable, $s_j(g) < s_m^*$. Therefore, $u_j(s_j(g)) < u_j(s_j(g'))$ implies that $s_j(g) < s_j(g')$. Since no man becomes worse off in g' relative to g, the number of links coming from the men's side (or from the women's side) is strictly greater in g' than in g (that is, $|g'| > |g|$); this is impossible because $|g'| = |g| = s_w^*|W|$. Thus, g is Pareto efficient.

2-ii) A network is male-optimal if and only if each man is at his peak. In no pairwise stable network is this possible.

2-iii) A pairwise stable network that is Pareto efficient is female-optimal because each woman is at her peak in such a network, and thus its aggregate value for women is maximal. Conversely, in any pairwise stable network whose aggregate value for women is maximal, each woman is at her peak, and thus it follows from part 2-i) that such a network is always Pareto efficient. ■

Proof of Claim 1

Proof. Assume that an agent $z \in N = M \cup W$ is drawn at random to receive the piece of information γ.

1) Let $\Pr(\gamma|N, \rho(z, \gamma) = 1)$ be the proportion of agents who will receive the information given that z has received it. If z belongs to component g_i, it is obvious that the information will spread only to agents in that component. Thus, $\Pr(\gamma|N, \rho(z, \gamma) = 1) = \frac{n_i}{n}$. Given that each agent is drawn with equal probability, we have:

$$
\begin{aligned}
E[\Pr(\gamma|N)] &= \tfrac{1}{n}\textstyle\sum_{z\in N}(\Pr(\gamma|N, \rho(z,\gamma) = 1) \\
&= \tfrac{1}{n}\textstyle\sum_{z\in N_1\cup\ldots\cup N_k} \tfrac{n_i}{n} \\
&= \tfrac{1}{n}\textstyle\sum_{i\in I_k}\sum_{z\in N_i} \tfrac{n_i}{n} \\
&= \tfrac{1}{n}\textstyle\sum_{i\in I_k} n_i \tfrac{n_i}{n} \\
&= \tfrac{1}{n^2}\textstyle\sum_{i\in I_k} n_i^2.
\end{aligned}
$$

2) Still assuming that z belongs to component g_i, $|M_i|$ men and $|W_i|$ women will receive the information. Thus, the gender difference in the concentration of information is $\Pr(\gamma|M, \rho(z, \gamma) = 1) - \Pr(\gamma|W, \rho(z, \gamma) = 1) = \frac{|M_i|}{|M|} - \frac{|W_i|}{|W|} = \frac{2}{n}(|M_i| - |W_i|)$ because $|M| = |W| = \frac{n}{2}$. This implies that the expected value of $\Pr(\gamma|M) - \Pr(\gamma|W)$ is:

$$
\begin{aligned}
E[\Pr(\gamma|M) - \Pr(\gamma|W)] &= \tfrac{1}{n}\textstyle\sum_{z\in N}(\Pr(\gamma|M, \rho(z,\gamma) = 1) - \Pr(\gamma|W, \rho(z,\gamma) = 1)) \\
&= \tfrac{1}{n}\textstyle\sum_{z\in N} \tfrac{2}{n}(|M_i| - |W_i|) \\
&= \tfrac{2}{n^2}\textstyle\sum_{z\in N_1\cup\ldots\cup N_k}(|M_i| - |W_i|) \\
&= \tfrac{2}{n^2}\textstyle\sum_{i\in I_k}\sum_{z\in N_i}(|M_i| - |W_i|) \\
&= \tfrac{2}{n^2}\textstyle\sum_{i\in I_k} n_i(|M_i| - |W_i|) \\
&= \tfrac{2}{n^2}\textstyle\sum_{i\in I_k}(|M_i| + |W_i|)(|M_i| - |W_i|) \\
&= \tfrac{2}{n^2}\textstyle\sum_{i\in I_k}(|M_i|^2 - |W_i|^2).
\end{aligned}
$$

This completes the proof. ■

Proof of Lemma 5

Proof. Let g be a pairwise stable network such that $A = \phi$, and $g' \in \mathcal{J}(g)$ a non-isolated component of g.

1) We want to show that $|W(g')| = \frac{|g'|}{s_w^*}$. Given that each woman $w \in W(g')$ has s_w^* partners, the cardinality of the subgraph g' is obviously $|g'| = s_w^*|W(g')|$, which implies $|W(g')| = \frac{|g'|}{s_w^*}$.

2) We now want to show that $\max(\left\lceil \frac{|g'|}{s_m^*} \right\rceil, s_w^*) \leq |M(g')| \leq |g'| - \frac{|g'|}{s_w^*} + 1$. In the subgraph g', $|M(g')|$ men are involved in $|g'|$ relationships; and given that each man is matched to a maximum of s_m^* partners, these $|g'|$

relationships can only be shared by a minimum of $\left\lceil \frac{|g'|}{s_m^*} \right\rceil$ men, implying $\left\lceil \frac{|g'|}{s_m^*} \right\rceil \leq |M(g')|$. Also, given that each woman $w \in W(g')$ should be linked to exactly s_w^* partners, it follows that $s_w^* \leq |M(g')|$. But $\left\lceil \frac{|g'|}{s_m^*} \right\rceil \leq |M(g')|$ and $s_w^* \leq |M(g')|$ imply $\max(\left\lceil \frac{|g'|}{s_m^*} \right\rceil, s_w^*) \leq |M(g')|$.

The second inequality $|M(g')| \leq |g'| - \frac{|g'|}{s_w^*} + 1$ comes from the fact that $|g'| - \frac{|g'|}{s_w^*} + 1$ is the largest number of men that is required for all vertices of the subgraph g' to remain directly or indirectly connected. In fact, let $M(g') = \{p_1, ..., p_{|M(g')|}\}$ be the set of men, and $W(g') = \{q_1, ..., q_{|W(g')|}\}$ be the set of women. We want to construct the component g' so that $|M(g')|$ is the largest possible. Construct g' by linking w_i to $\{p_{(i-1)s_w^*-i+2}, ..., p_{is_w^*-i+1}\}$ for each $i \in \{1, ..., |W(g')|\}$. Since the function $is_w^* - i + 1$ is increasing in i, it reaches its maximum at $i = |W(g')|$, implying that $|W(g')|s_w^* - |W(g')| + 1$ is the largest possible value of $|M(g')|$; and given that $|W(g')| = \frac{|g'|}{s_w^*}$, this value is equal to $|g'| - \frac{|g'|}{s_w^*} + 1$. It is easy to see that each q_i is matched to exactly s_w^* partners and all elements of $M(g')$ and $W(g')$ are directly or indirectly linked; in fact, p_1 is linked to q_1, $p_{|W(g')|s_w^*-|W(g')|+1}$ is linked to $w_{|W(g')|}$, and each p_j such that there exists $i \in \{2, ..., |W(g')|-1\}$ such that $j = is_w^* - i + 1$ is matched with q_i and q_{i+1}, and each p_j such that there exists $i \in \{1, ..., |W(g')| - 1\}$ such that $(i-1)s_w^* - i + 2 < j < is_w^* - i + 1$ is matched with q_i. ■

Proof of Lemma 6

Proof. Let g be a k-component network with the corresponding bipartite component vector $[(|M_i|, |W_i|)]_{i \in I_k}$. Assume that $\forall g' \in \mathcal{J}(g)$, $|M(g')| \leq |W(g')|$, and let us show that $\mathcal{F}(g) \leq 0$. Assume that there are ℓ non-isolated components, l isolated components of men, and $k-\ell-l$ isolated components of women. Without loss of generality, assume that the first ℓ components of the vector $[(|M_i|, |W_i|)]_{i \in I_k}$ represent the non-isolated components of g, the l next components represent the isolated components of men, and the remaining components represent the isolated components of women. There are therefore l components $(1, 0)$ and $k - \ell - l$ components $(0, 1)$. Remark that each non-isolated component vector $(|M_i|, |W_i|)$ is such that $|M_i| + |W_i| = n_i \geq 2$ since it contains at least one man and one woman. Also, we have $\sum_{i \in I_k} |M_i| = \sum_{i \in I_\ell} |W_i| + l$ and $\sum_{i \in I_k} |W_i| = \sum_{i \in I_\ell} |W_i| + (k - \ell - l)$, which, given the fact that $\sum_{i \in I_k} |M_i| = \sum_{i \in I_k} |W_i|$, implies that $\sum_{i \in I_\ell}(|M_i| - |W_i|) = k - \ell - 2l$. Because $|M_i| \leq |W_i|$ for each $i \in I_\ell$, it thus follows that $\sum_{i \in I_\ell}(|M_i| - |W_i|) = k - \ell - 2l \leq 0$. We have the following results:

$$\begin{array}{rcl}
\mathcal{F}(g) & = & \frac{2}{n^2} \sum_{i \in I_k}(|M_i|^2 - |W_i|^2) \\
& = & \frac{2}{n^2}\{\sum_{i \in I_\ell}(|M_i|^2 - |W_i|^2) + \sum_{\ell+1 \leq i \leq \ell+l}(|M_i|^2 - |W_i|^2) + \sum_{\ell+l+1 \leq i \leq k}(|M_i|^2 - |W_i|^2)\} \\
& = & \frac{2}{n^2}\{\sum_{i \in I_\ell}(|M_i| - |W_i|)(|M_i| + |W_i|) + \sum_{\ell+1 \leq i \leq \ell+l}(1^2 - 0^2) + \sum_{\ell+l+1 \leq i \leq k}(0^2 - 1^2)\} \\
& = & \frac{2}{n^2}\{\sum_{i \in I_\ell}(|M_i| - |W_i|)n_i + l - (k - \ell - l)\} \\
& \leq & \frac{2}{n^2}\{2\sum_{i \in I_\ell}(|M_i| - |W_i|) - k + \ell + 2l\} \\
& = & \frac{2}{n^2}\{2(k - \ell - 2l) - k + \ell + 2l\} \\
& = & \frac{2}{n^2}(k - \ell - 2l) \\
& \leq & 0
\end{array}$$

Note that the last inequality is strict if at least one man is isolated. ■

Proof of Lemma 7

Proof. Let $\mathcal{E} = (N = M \cup W, s_m^*, s_w^*)$ be a mating economy.

(2) $\Longrightarrow$ (1): *a*) Assume that $s_w^* = 1$ and show that $\forall g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$. Let $g \in \mathcal{PS}(\mathcal{E})$ be a pairwise stable network and $g' \in \mathcal{J}(g)$ a non-isolated component of g. It is straightforward from Lemma 5 that $|M(g')| = 1$. Given that $|W(g')| \geq 1$, it follows that $|M(g')| \leq |W(g')|$. Thus, each non-isolated component of g is such that the number of women weakly exceeds the number of men. It therefore follows from Lemma 6 that $\mathcal{F}(g) \leq 0$.

b) Assume that $n \leq 4s_w^* + 2$ and show that $\forall g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$. Let $g \in \mathcal{PS}(\mathcal{E})$ be a pairwise stable network and A be the set of women who have less than their optimal number of partners. We shall distinguish two cases: $A = \phi$ and $A \neq \phi$.

$b-1$) Assume that $A = \phi$.

- First assume that $n < 4s_w^*$. For any $g \in \mathcal{PS}(\mathcal{E})$, let us show that there is only one non-isolated component $g' \in \mathcal{J}(g)$. Assume by contradiction that there are two such components g_1 and g_2. Then by Lemma 5, $s_w^* \leq |M_1|$ and $s_w^* \leq |M_2|$ (remember that $|M_1|$ and $|M_2|$ are respectively the number of men in g_1 and g_2), which implies that $|M| \geq |M_1| + |M_2| \geq 2s_w^*$, and $n = 2|M| \geq 4s_w^*$, thus contradicting our assumption. So there is only one non-isolated component $g' \in \mathcal{J}(g)$; since $|W(g')| = |W|$ and $\left\lceil \frac{s_w^*|W|}{s_m^*} \right\rceil \leq |M(g')| \leq |M| = |W|$, it follows that $|M(g')| \leq |W(g')|$, which by Lemma 6, implies that for any $g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$.

- Now assume that $n = 4s_w^*$. This implies that $|M| = |W| = 2s_w^*$. There exist at most two non-isolated components. If there is only one such component, then the proof follows as for the case where $|W| < 2s_w^*$. Now, suppose that there are two non-isolated components $g_1, g_2 \in \mathcal{J}(g)$; by Lemma 5, $|M_1| = |M_2| = s_w^*$. We thus have the following:

$$\begin{aligned}
\mathcal{F}(g) &= \frac{2}{n^2}\sum_{i \in I_2}(|M_i|^2 - |W_i|^2) \\
&= \frac{2}{n^2}(2s_w^{*2} - |W_1|^2 - |W_2|^2) \\
&= \frac{2}{n^2}(2s_w^{*2} - |W_1|^2 - (2s_w^* - |W_1|)^2) \\
&= \frac{2}{n^2}\{-\ 2(s_w^* - |W_1|)^2\} \\
&\leq 0
\end{aligned}$$

- Assume that $n = 4s_w^* + 2$. This implies that $|M| = |W| = 2s_w^* + 1$. There are at most two non-isolated components. If there is only one such component, then the proof is similar to that of the case where $|W| < 2s_w^*$. Suppose that there are two non-isolated components $g_1, g_2 \in \mathcal{J}(g)$; then by Lemma 5, the number of men is s_w^* in one component and $s_w^* + 1$ in the other component. Without loss of generality, assume that $|M_1| = s_w^*$ and $|M_2| = s_w^* + 1$. We thus have the following:

$$\begin{aligned}
\mathcal{F}(g) &= \frac{2}{n^2}\sum_{i \in I_2}(|M_i|^2 - |W_i|^2) \\
&= \frac{2}{n^2}(s_w^{*2} + (s_w^* + 1)^2 - |W_1|^2 - (2s_w^* + 1 - |W_1|)^2) \\
&= \frac{2}{n^2}\{-2(s_w^* - |W_1|)(s_w^* - |W_1| - 1)\} \\
&\leq 0
\end{aligned}$$

Note that the last inequality comes from the fact that the expression $-2(s_w^* - |W_1|)(s_w^* - |W_1| - 1)$ is strictly positive if and only if $|W_1| \in (s_w^* - 1, s_w^*)$, which is impossible because s_w^* and $|W_1|$ are integers.

$b-2$) Assume that $A \neq \phi$. This implies that no man is isolated, and there are at most two non-isolated components. If there is only one non-isolated component, then, it is straightforward that all men and women in the economy belong to that component, which implies that $\mathcal{F}(g) = \frac{2}{n^2}\sum_{i \in I_1}(|M_i|^2 - |W_i|^2) = 0$.

If there are two non-isolated components $g_1, g_2 \in \mathcal{J}(g)$, it should be the case that one of them, say g_1, is such that $W(g_1) = A$. Therefore, all men involved in g_1 are in the set $M(g_1) = g(A) = \bigcap\limits_{w \in A} g(w)$. From Lemma 4, we know that g_1 has at most $s_w^* - 1$ men and $s_w^* - 2$ women, leaving g_2 with at least $|M| - s_w^* + 1$ men and $|W| - s_w^* + 2$ women. Also, because each man involved in g_2 has s_m^* women (if not, all men in g_2 will have an incentive to form a link with women in g_1), it follows that $|W| - s_w^* + 2 \geq s_m^*$, which also implies that $|M| - s_w^* + 1 \geq s_m^* - 1 > s_w^* - 1$. It results from all these assertions that $|M_1| < |M_2|$ and $|W_1| < |W_2|$, which implies that $n_1 = |M_1| + |W_1| < |M_2| + |W_2| = n_2$. Also note that because no man is isolated, $|M_1| - |W_1| = -(|M_2| - |W_2|) > 0$. We therefore have the following:

$$\begin{array}{rcl} \mathcal{F}(g) & = & \frac{2}{n^2}\sum_{i\in I_2}(|M_i|^2-|W_i|^2) \\ & = & \frac{2}{n^2}\{(|M_1|-|W_1|)n_1+(|M_2|-|W_2|)n_2 \\ & = & \frac{2}{n^2}(|M_1|-|W_1|)(n_1-n_2) \\ & < & 0 \end{array}$$

This concludes the proof of (2) $\Longrightarrow$ (1).

(1) $\Longrightarrow$ (2): Assume that for any $g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$. We want to show that $s_w^* = 1$ or $n \leq 4s_w^* + 2$.

c) Assume that $s_w^* > 1$ and show that $n \leq 4s_w^* + 2$. Assume by contradiction that $n > 4s_w^* + 2$. Construct a pairwise stable network $g \in \mathcal{PS}(\mathcal{E})$ with two non-isolated components g_1 and g_2 such that $|M_1| = s_w^*$, $|M_2| = |M| - s_w^*$, $|W_1| = s_w^* + 1$, and $|W_2| = |W| - s_w^* - 1 = |M| - s_w^* - 1$. Remark that this network satisfies the bounds conditions of Lemma 5 because $s_w^* > 1$. We want to show that $\mathcal{F}(g) > 0$. We have:

$$\begin{array}{rcl} \mathcal{F}(g) & = & \frac{2}{n^2}\sum_{i\in I_2}(|M_i|^2-|W_i|^2) \\ & = & \frac{2}{n^2}(s_w^{*2}+(s_w^*+1)^2-(|M|-s_w^*)^2-(|M|-s_w^*-1)^2) \\ & = & -4s_w^*+2|M|-2 \\ & = & -4s_w^*+-2+n \\ & > & 0 \end{array}$$

$\mathcal{F}(g) > 0$ is a contradiction of our assumption, so we conclude that $n \leq 4s_w^* + 2$.

d) Assume that $n > 4s_w^* + 2$ and show that $s_w^* = 1$. Assume by contradiction that $s_w^* > 1$. Then any pairwise stable network $g \in \mathcal{PS}(\mathcal{E})$ with two non-isolated components g_1 and g_2 such that $|M_1| = s_w^*$, $|M_2| = |M| - s_w^*$, $|W_1| = s_w^* + 1$, and $|W_2| = |W| - s_w^* - 1 = |M| - s_w^* - 1$ is such that $\mathcal{F}(g) > 0$. The proof is exactly as in part *c*). This completes the proof. ∎

Proof of Theorem 3

Proof. Let $\mathcal{E}$ be a (non necessarily trivial) mating economy.

(2) $\Longrightarrow$ (1): Assume that $\mathcal{E}$ is a mating segmented economy $(\mathcal{E}^t = (N^t, s_m^{t*}, s_w^{t*}))_{t\in I_T}$ such that $\forall t \in I_T$, $s_w^{t*} = 1$ or $n^t \leq 4s_w^{t*}+2$. Let us show that any network $g \in \mathcal{PS}(\mathcal{E})$ is such that $\mathcal{F}(g) \leq 0$. Call g^t the sub-network (or sub-graph) of g that forms in the segment $\mathcal{E}^t$ of the economy. It can be shown that $\mathcal{F}(g) = \frac{2}{n^2}\sum_{i\in I_T}\frac{n^{t2}}{2}\mathcal{F}(g^t)$ where $n^t = |N^t|$. It also follows from the assumption that $\forall t \in I_T$, $s_w^{t*} = 1$ or $n^t \leq 4s_w^{t*} + 2$ that $\forall t \in I_T$, $\mathcal{F}(g^t) \leq 0$ (Lemma 7). Thus $\mathcal{F}(g) = \frac{2}{n^2}\sum_{i\in I_T}\frac{n^{t2}}{2}\mathcal{F}(g^t) \leq 0$.

(1) $\Longrightarrow$ (2): Assume that $\forall g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$. Let us show that $\mathcal{E}$ is a segmented mating economy $(\mathcal{E}^t = (N^t, s_m^{t*}, s_w^{t*}))_{t\in I_T}$ such that $\forall t \in I_T$, $s_w^{t*} = 1$ or $n^t \leq 4s_w^{t*} + 2$. Assume by contradiction that there exists a segment $\mathcal{E}^{t_0}$ such that $s_w^{t_0*} > 1$ and $n^{t_0} > 4s_w^{t_0*} + 2$. Then following Lemma 7, we can construct a pairwise stable network $g^{t_0} \in \mathcal{PS}(\mathcal{E}^{t_0})$ such that $\mathcal{F}(g^{t_0}) > 0$. Construct such a g^{t_0}. For any other segment $\mathcal{E}^t \neq \mathcal{E}^{t_0}$, construct an egalitarian pairwise stable network g^t (this is always possible and is easy); we thus have $\mathcal{F}(g^t) = 0$. Call the resulting network g. It is clear that g is a pairwise stable network of the economy $\mathcal{E}$. In addition, we have $\mathcal{F}(g) = \frac{2}{n^2}\sum_{i\in I_T}\frac{n^{t2}}{2}\mathcal{F}(g^t) = \frac{2}{n^2}\frac{n^{t_0 2}}{2}\mathcal{F}(g^{t_0}) > 0$, which is a contradiction ∎

Proof of Corollary 1

Proof. Let $\mathcal{E}$ be a mating economy that is not necessarily trivial.

(1) $\Longrightarrow$ (2): Assume that $\forall (x(g)) \in \mathcal{P}(\mathcal{E})$, $\sum_g x(g)\mathcal{F}(g) \leq 0$. Assume by contradiction that there exists a segment $\mathcal{E}^{t_0}$ such that $s_w^{t_0*} > 1$ and $n^{t_0} > 4s_w^{t_0*}+2$. It follows from Theorem 3 that $\mathcal{E}$ is not female-information-biased. This implies that there exists a pairwise stable network whose male-female difference in the contagion potential is strictly positive. Denote by $\mathcal{PS}(\mathcal{E})^+ = \{g \in \mathcal{PS}(\mathcal{E}) : \mathcal{F}(g) > 0\}$ the set of such networks, and by $\mathcal{PS}(\mathcal{E})^- = \{g \in \mathcal{PS}(\mathcal{E}) : \mathcal{F}(g) \leq 0\}$ the complement of $\mathcal{PS}(\mathcal{E})^+$. It is easy to show that $\mathcal{PS}(\mathcal{E})^-$ is not

empty and that for some element g^0 of $\mathcal{PS}(\mathcal{E})^-$, $\mathcal{F}(g^0) < 0$ (an obvious candidate for such a network g^0 is the network in which for each segment $\mathcal{E}^t$, $\left\lfloor \frac{s_w^*}{s_m^*}|M^t| \right\rfloor$ men are matched to s_m^{t*} women each if $|M^t| \geq s_m^{t*}$ or to all the women if $|M^t| < s_m^{t*}$, at most one man is matched to the remaining women (if there is such a remaining), and all other men have no partner). Let $a^+ = \sum_{g \in \mathcal{PS}(\mathcal{E})^+} \mathcal{F}(g)$ and $a^- = \sum_{g \in \mathcal{PS}(\mathcal{E})^-} \mathcal{F}(g)$. It is clear that $a^+ > 0$ and $a^- < 0$. Now, consider the probability distribution that assigns a weight $\frac{x}{|\mathcal{PS}(\mathcal{E})^+|}$ to all networks in $\mathcal{PS}(\mathcal{E})^+$ and $\frac{1-x}{|\mathcal{PS}(\mathcal{E})^-|}$ to all networks in $\mathcal{PS}(\mathcal{E})^-$, with $1 > x > \frac{-|\mathcal{PS}(\mathcal{E})^+|a^-}{|\mathcal{PS}(\mathcal{E})^-|a^+ - |\mathcal{PS}(\mathcal{E})^+|a^-}$. Such a probability distribution clearly belongs to $\mathcal{P}(\mathcal{E})$ since $\frac{x}{|\mathcal{PS}(\mathcal{E})^+|}$ and $\frac{1-x}{|\mathcal{PS}(\mathcal{E})^-|}$ are strictly positive. Yet, for this distribution, we have $\sum_g x(g)\mathcal{F}(g) = \sum_{g \in \mathcal{PS}(\mathcal{E})^+} \frac{x}{|\mathcal{PS}(\mathcal{E})^+|}\mathcal{F}(g) + \sum_{g \in \mathcal{PS}(\mathcal{E})^-} \frac{1-x}{|\mathcal{PS}(\mathcal{E})^-|}\mathcal{F}(g) = \frac{x}{|\mathcal{PS}(\mathcal{E})^+|}a^+ + \frac{1-x}{|\mathcal{PS}(\mathcal{E})^-|}a^- > 0$, which contradicts our assumption.

(2) $\Longrightarrow$ (1): Now assume that $\mathcal{E}$ is a mating segmented economy $(\mathcal{E}^t = (N^t, s_m^{t*}, s_w^{t*}))_{t \in I_T}$ such that $\forall t \in I_T$, $s_w^{t*} = 1$ or $n^t \leq 4s_w^{t*} + 2$. It follows from Theorem 3 that for any pairwise stable network $g \in \mathcal{PS}(\mathcal{E})$, $\mathcal{F}(g) \leq 0$. It therefore follows that for any probability distribution $(x(g)) \in \mathcal{P}(\mathcal{E})$, $\sum_g x(g)\mathcal{F}(g) \leq 0$.

Let us now show that the inequality in (1) is strict if and only if in (2), $n^{t_0} > 2s_w^{t_0*}$ for some segment $\mathcal{E}^{t_0}$. Assume (2) and assume in addition that $n^{t_0} > 2s_w^{t_0*}$ for some segment $\mathcal{E}^{t_0}$. Because of (2), we have $\forall (x(g)) \in \mathcal{P}(\mathcal{E})$, $\sum_g x(g)\mathcal{F}(g) \leq 0$ (this comes from the proof of (2) $\Longrightarrow$ (1)). And given that $n^{t_0} > 2s_w^{t_0*}$ for some segment $\mathcal{E}^{t_0}$, we can show that there exists $g^0 \in \mathcal{PS}(\mathcal{E})^-$ such that $\mathcal{F}(g^0) < 0$ (construct g^0 as we did in the proof of (1) $\Longrightarrow$ (2)). It therefore follows that for any probability distribution $(x(g)) \in \mathcal{P}(\mathcal{E})$, $\sum_g x(g)\mathcal{F}(g) < 0$.

Conversely, assume that the inequality in (1) is strict. Assume by contradiction that for all segment $\mathcal{E}^t$, $n^t \leq 2s_w^{t*}$. Then each segment is a very small economy, and thus the overall economy admits a unique pairwise stable network g in which all men and women within the same segment are matched (Theorem 1). It follows that $\mathcal{F}(g) = 0$, which contradicts our assumption. ■

Proof of Theorem 4

Proof. The proof of the equivalence between (1) on one hand and (2)-(5) one the other follows from that of Theorem 1, the only difference being that the constraint $(\mathcal{S})$ is taken into account in the current proof. This constraint implies for instance that if a woman is matched to her optimal number of partners s_w^* in a pairwise stable network g, each of her s_w^* male partners in that network should be matched to at least s_w^* female partners. The proof is therefore made easier by that of Theorem 1 and is left to the reader.

6) To prove that any pairwise stable network g is such that $\mathcal{F}(g) \leq 0$, it suffices to show that each non-isolated component of g has at least as many women as men. We can show this by noticing that in each non-isolated component of g, no woman has more partners than her least connected male partner. The rest then follows by invoking Lemma 6. ■

Proof of Theorem 5

Proof. 1) Let $M = \{m_1, ..., m_{|M|}\}$ and $W = \{w_1, ..., w_{|W|}\}$ be the sets of men and women, respectively. Without loss of generality, we assume that the label of each agent indicates his/her position in the social hierarchy (that is, m_1 is the highest-ranked man, m_2 the second highest-ranked man, and so on). We distinguish three cases: $|M| \leq s_w^*$, $s_w^* < |M| \leq s_m^*$, and $|M| > s_m^*$.

a) If $|M| \leq s_w^*$, then the unique pairwise stable network is the one in which each woman is matched with all men.

b) If $s_w^* < |M| \leq s_m^*$, the unique pairwise stable network is the one in which all women are matched with the s_w^* most highly ranked men, and all other men are unmatched.

c) If $|M| > s_m^*$, write $|M| = |W| = ks_m^* + r = k's_w^* + r'$ where k, r, k' and r' are integers such that $0 \leq r < s_m^*$ and $0 \leq r' < s_w^*$. Partition all women into $k+1$ sets $W_1, .., W_{k+1}$ such that for any $\ell \in I_k$, W_ℓ contains s_m^* individuals, the set W_{k+1} contains r individuals, and all women in each set W_ℓ are more highly ranked than all women in the set $W_{\ell+1}$. Similarly, partition all men into $k'+1$ sets $M_1, .., M_{k'+1}$ such that for any $\ell \in I_{k'}$, M_ℓ contains s_w^* individuals, the set $W_{k'+1}$ contains r' individuals, and all men in each set M_ℓ are more highly ranked than all men in the set $M_{\ell+1}$. So W_1 is the set of the s_m^* most highly ranked women, W_2 is the set of the next s_m^* most highly ranked women, and so on. Similarly, M_1 is the set of the s_w^* most highly ranked men, M_2 is the set of the next s_w^* most highly ranked men, and so on. It is easily shown that the unique pairwise stable network in this economy is the network in which all women in each set W_ℓ, $\ell \in I_{k+1}$, are matched with all men in the corresponding set M_ℓ. This matching is feasible because $k \leq k'$. If $r = 0$, meaning that W_{k+1} is empty, then the remaining men in the sets $M_{k+1}, .., M_{k'+1}$ (if not empty) are unmatched. If $r \neq 0$, meaning that W_{k+1} is not empty, then the remaining men in the sets $M_{k+2}, .., M_{k'+1}$ (if not empty) are unmatched.

2) *a*) If $|M| \leq s_w^*$, given that each man is matched to all women, the unique pairwise stable network g has only one component which contains all men and all women; so $\mathcal{F}(g) = \frac{2}{n^2} \sum_{i \in I_k} (|M_i|^2 - |W_i|^2) = \frac{2}{n^2}(|M|^2 - |W|^2) = 0$.

b) Assume that $|M| > s_w^*$. We shall distinguish two cases: $s_w^* < |M| \leq s_m^*$ and $|M| > s_m^*$.

$b-1$) Suppose that $s_w^* < |M| \leq s_m^*$. The unique pairwise stable network g has only one non-isolated component in which the number of men (s_w^*) strictly exceeds the number of women ($|M|$), and $|M| - s_w^* > 0$ men are isolated. It therefore follows from Lemma 6 that $\mathcal{F}(g) < 0$.

$b-2$) Suppose that $|M| > s_m^*$. Write $|M| = |W| = ks_m^* + r = k's_w^* + r'$ where k, r, k' and r' are integers such that $0 \leq r < s_m^*$ and $0 \leq r' < s_w^*$.

- If $r = 0$, then the unique pairwise stable network g described in part 1-c) is such that the number of women strictly exceeds the number of men in each non-isolated component. It follows from Lemma 6 that $\mathcal{F}(g) < 0$.

- If $r \neq 0$, it is easy to check that $k \leq k'$. Let us distinguish two cases: $k = k'$ and $k < k'$.

- Suppose that $k = k'$. This necessarily implies that there is no isolated man in the unique pairwise stable network described in part 1-c), and that $r < r' < s_w^* < s_m^*$. These inequalities in turn imply $s_w^* + s_m^* - r' - r > 0$. Also, $|M| = |W| = ks_m^* + r = k's_w^* + r'$ implies $r' - r = k(s_m^* - s_w^*)$. Note that there are s_m^* women and s_w^* men in each of the first k non-isolated components, and in the last component, we have $m_{k+1} = r'$ and $w_{k+1} = r$. It therefore follows that:

$$
\begin{aligned}
\mathcal{F}(g) &= \frac{2}{n^2} \sum_{i \in I_{k+1}} (|M_i|^2 - |W_i|^2) \\
&= \frac{2}{n^2} \{k(s_w^{*2} - s_m^{*2}) + (r'^2 - r^2)\} \\
&= \frac{2}{n^2} \{k(s_w^* - s_m^*)(s_w^* + s_m^*) + (r' - r)(r' + r)\} \\
&= \frac{2}{n^2} \{k(s_w^* - s_m^*)(s_w^* + s_m^*) + k(s_m^* - s_w^*)(r' + r)\} \\
&= \frac{2}{n^2} \{k(s_w^* - s_m^*)(s_w^* + s_m^* - (r' + r)\} \\
&= \frac{2}{n^2} \{k(s_w^* - s_m^*)(s_w^* + s_m^* - r' - r)\} \\
&< 0
\end{aligned}
$$

- Suppose that $k < k'$. This means that at least one man is isolated in the unique pairwise stable network g described in part 1-c), and that each of the first k non-isolated components of that network contains s_w^* men and s_m^* women, and the last non-isolated component contains $|M_{k+1}| = s_w^*$ men and $|W_{k+1}| = r$ women (note that we cannot resort to Lemma 6 in this case because there might be instances in which $r > s_w^*$). So g is such that $\forall g' \in \mathcal{J}(g)$, $|M(g')| > |W(g')| \Longrightarrow |M(g')| = s_w^*$, and $|M(g')| \leq |W(g')| \Longrightarrow |M(g')| \geq s_w^*$. It therefore follows from Lemma 8 that $\mathcal{F}(g) < 0$. This completes our proof. ■

Proof of Proposition 2

Proof. The proof is easy and left to the reader. ■

Proof of Proposition 3

Proof. The proof is easy and left to the reader. ■

Proof of Proposition 4

Proof. Let $\mathcal{E}^{\succ} = (N = M \cup W, s_m^*, s_w^*, \succ_m, \succ_w)$ be a hiearachical mating economy, and m_v and w_v a man and a woman of rank v.

1) The proof that s_{m_v} and s_{w_v} weakly increase and v in the unique pairwise stable networks of $\mathcal{E}^{\succ}$ directly follows from the construction of that network in the proof of Theorem 5 (note that in keeping with the notation in the proof of Theorem 5, $v = 1$ is the highest rank, $v = 2$ is the second highest, etc. So when we say that s_{m_v} weakly increases in v, we mean that s_{m_v} weakly increases as rank v goes from a lower rank (e.g., $v = 2$) to a higher rank (e.g., $v = 1$). This interpretation is valid for other functions in the rest of this proof).

2-4) To prove 2), 3) and 4), first remark that in any network (pairwise stable or not), for any individual i, $p(i) = \frac{n(i)}{n}$ where $n(i)$ is the size of the component to which i belongs in that network, and n the size of the total population. We shall now compute $p(m_v)$ and $p(w_v)$ as a function of v in the unique pairwise stable network that arises in $\mathcal{E}^{\succ}$.

a) If $|M| \leq s_w^*$, then the unique pairwise stable network is the one in which each woman is matched to all men. Therefore, $p(m_v) = p(w_v) = \frac{|M|+|W|}{n} = 1$, which implies (1) and (2).

b) If $s_w^* < |M| \leq s_m^*$, the unique pairwise stable network is the one in which all women are matched with the s_w^* most highly ranked men, and all other men are unmatched. Therefore:

- $v \leq s_w^* \Longrightarrow p(m_v) = \frac{|W|+s_w^*}{n}$ and $p(w_v) = \frac{|W|+s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = 0$.
- $v > s_w^* \Longrightarrow p(m_v) = \frac{1}{n}$ and $p(w_v) = \frac{|W|+s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{1-|W|-s_w^*}{n} < 0$.

We note that: (1) $p(m_v)$ and $p(w_v)$ weakly increase in v; (2) $p(m_v) - p(w_v) \leq 0$ for any v; and (3) $p(m_v) - p(w_v)$ weakly increases in v.

c) If $|M| > s_m^*$, write $|M| = |W| = ks_m^* + r$ such that $0 \leq r < s_m^*$. Pose $k_{\max} = \left\lceil \frac{|W|}{s_m^*} \right\rceil s_w^*$. Note that $k_{\max}$ is the rank below which all men are isolated in the unique pairwise stable network described in the proof of Theorem 5. We shall distinguish two cases: $r = 0$ and $r \neq 0$.

$c-1$) $r = 0$. In the unique pairwise stable network described in the proof of Theorem 5, w_v belongs to a component in which there are s_m^* women and s_w^* men, but this is true for m_v only if $v \leq k_{\max}$. Therefore, we have the following:

- $v \leq k_{\max} \Longrightarrow p(m_v) = \frac{s_m^*+s_w^*}{n}$ and $p(w_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = 0$.
- $v > k_{\max} \Longrightarrow p(m_v) = \frac{1}{n}$ and $p(w_v) = \frac{s_m^*+s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{1-s_m^*-s_w^*}{n} < 0$.

We note that: (1) $p(m_v)$ and $p(w_v)$ weakly increase in v; (2) $p(m_v) - p(w_v) \leq 0$ for any v; and (3) $p(m_v) - p(w_v)$ weakly increases in v.

$c-2$) $r \neq 0$. In the unique pairwise stable network described in the proof of Theorem 5, w_v belongs to the component in which there are r women and s_w^* men if $v > |W| - r$, and to a component in which there are s_m^* women and s_w^* men if $v \leq |W| - r$; m_v belongs to the component in which there are r women and s_w^* men if $k_{\max} - s_w^* + 1 \leq v \leq k_{\max}$, to a component in which there are s_m^* women and s_w^* men if $v < k_{\max} - s_w^* + 1$, and is isolated if $v > k_{\max}$. It is easy to check that $k_{\max} - s_w^* + 1 \leq |W| - r$. We shall distinguish two cases: (1) $k_{\max} - s_w^* + 1 = |W| - r$, and (2) $k_{\max} - s_w^* + 1 < |W| - r$.

$c-2-1$) Suppose that $k_{\max} - s_w^* + 1 = |W| - r$. We have the following:

- $v < k_{\max} - s_w^* + 1 \Longrightarrow p(m_v) = \frac{s_m^* + s_w^*}{n}$ and $p(w_v) = \frac{s_m^* + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = 0$.

- $v = k_{\max} - s_w^* + 1 \Longrightarrow p(m_v) = \frac{r + s_w^*}{n}$ and $p(w_v) = \frac{s_m^* + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{r - s_m^*}{n} < 0$.

- $k_{\max} - s_w^* + 1 < v \leq k_{\max} \Longrightarrow p(m_v) = \frac{r + s_w^*}{n}$ and $p(w_v) = \frac{r + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = 0$.

- $k_{\max} < v \leq |M| \Longrightarrow p(m_v) = \frac{1}{n}$ and $p(w_v) = \frac{r + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{1 - r - s_w^*}{n} < 0$.

We note that: (1) $p(m_v)$ and $p(w_v)$ weakly increase in v; (2) $p(m_v) - p(w_v) \leq 0$ for any v; and (3) $p(m_v) - p(w_v)$ is non-monotonic in v.

$c - 2 - 2)$ Suppose that $k_{\max} - s_w^* + 1 < |W| - r$. We shall distinguish three cases: $|W| - r = k_{\max}$, $|W| - r < k_{\max}$ and $|W| - r > k_{\max}$. We have the following:

* If $|W| - r = k_{\max}$, then:

- $v < k_{\max} - s_w^* + 1 \Longrightarrow p(m_v) = \frac{s_m^* + s_w^*}{n}$ and $p(w_v) = \frac{s_m^* + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = 0$.

- $k_{\max} - s_w^* + 1 \leq v \leq k_{\max} \Longrightarrow p(m_v) = \frac{r + s_w^*}{n}$ and $p(w_v) = \frac{s_m^* + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{r - s_m^*}{n} < 0$.

- $k_{\max} < v \leq |M| \Longrightarrow p(m_v) = \frac{1}{n}$ and $p(w_v) = \frac{r + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{1 - r - s_w^*}{n} < 0$.

We note that: (1) $p(m_v)$ and $p(w_v)$ weakly increase in v; (2) $p(m_v) - p(w_v) \leq 0$ for any v; and (3) $p(m_v) - p(w_v)$ weakly increases in v.

* If $|W| - r < k_{\max}$, then:

- $v < k_{\max} - s_w^* + 1 \Longrightarrow p(m_v) = \frac{s_m^* + s_w^*}{n}$ and $p(w_v) = \frac{s_m^* + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = 0$.

- $k_{\max} - s_w^* + 1 \leq v \leq |W| - r \Longrightarrow p(m_v) = \frac{r + s_w^*}{n}$ and $p(w_v) = \frac{s_m^* + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{r - s_m^*}{n} < 0$.

- $|W| - r < v \leq k_{\max} \Longrightarrow p(m_v) = \frac{r + s_w^*}{n}$ and $p(w_v) = \frac{r + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = 0$.

- $k_{\max} < v \leq |M| \Longrightarrow p(m_v) = \frac{1}{n}$ and $p(w_v) = \frac{r + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{1 - r - s_w^*}{n} < 0$.

We note that: (1) $p(m_v)$ and $p(w_v)$ weakly increase in v; (2) $p(m_v) - p(w_v) \leq 0$ for any v; and (3) $p(m_v) - p(w_v)$ is non-monotonic in v.

* If $|W| - r > k_{\max}$, then:

- $v < k_{\max} - s_w^* + 1 \Longrightarrow p(m_v) = \frac{s_m^* + s_w^*}{n}$ and $p(w_v) = \frac{s_m^* + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = 0$.

- $k_{\max} - s_w^* + 1 \leq v \leq k_{\max} \Longrightarrow p(m_v) = \frac{r + s_w^*}{n}$ and $p(w_v) = \frac{s_m^* + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{r - s_m^*}{n} < 0$.

- $k_{\max} < v \leq |W| - r \Longrightarrow p(m_v) = \frac{1}{n}$ and $p(w_v) = \frac{s_m^* + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{1 - s_m^* - s_w^*}{n} < 0$.

- $|W| - r < v \leq |M| \Longrightarrow p(m_v) = \frac{1}{n}$ and $p(w_v) = \frac{r + s_w^*}{n} \Longrightarrow p(m_v) - p(w_v) = \frac{1 - r - s_w^*}{n} < 0$.

We note that: (1) $p(m_v)$ and $p(w_v)$ weakly increase in v; (2) $p(m_v) - p(w_v) \leq 0$ for any v; and (3) $p(m_v) - p(w_v)$ is non-monotonic in v. ■

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Example 1 & **Example 4**

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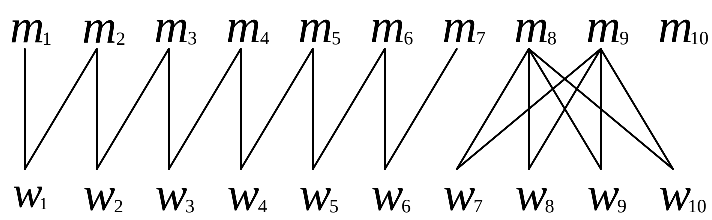

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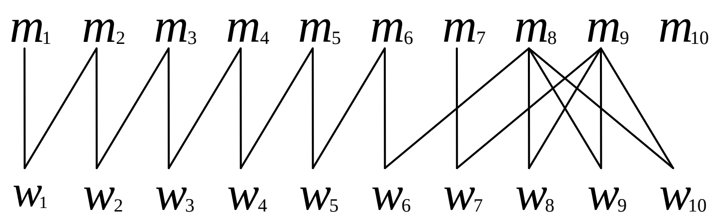

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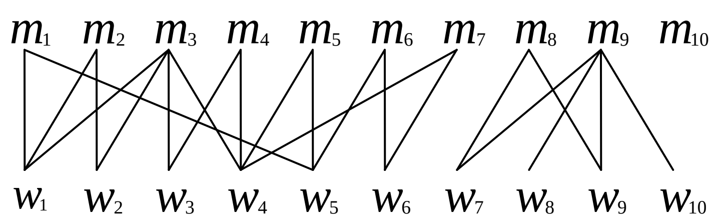

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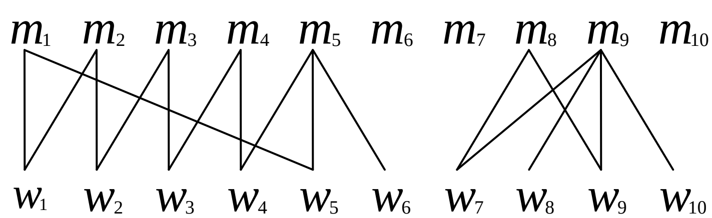

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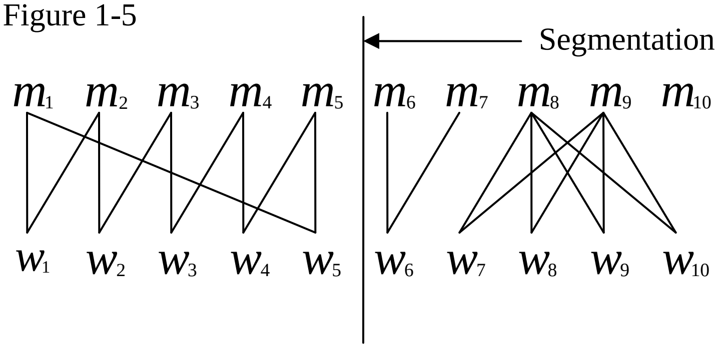

Example 2

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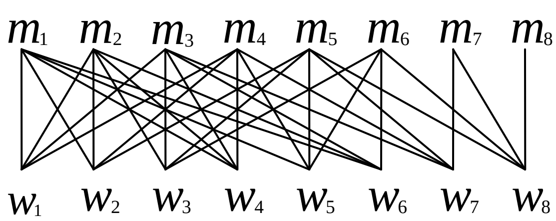

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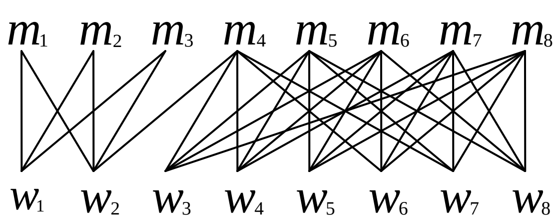

Example 3

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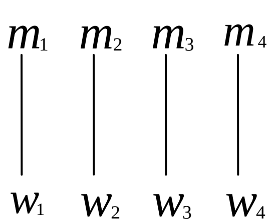

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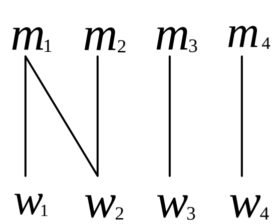
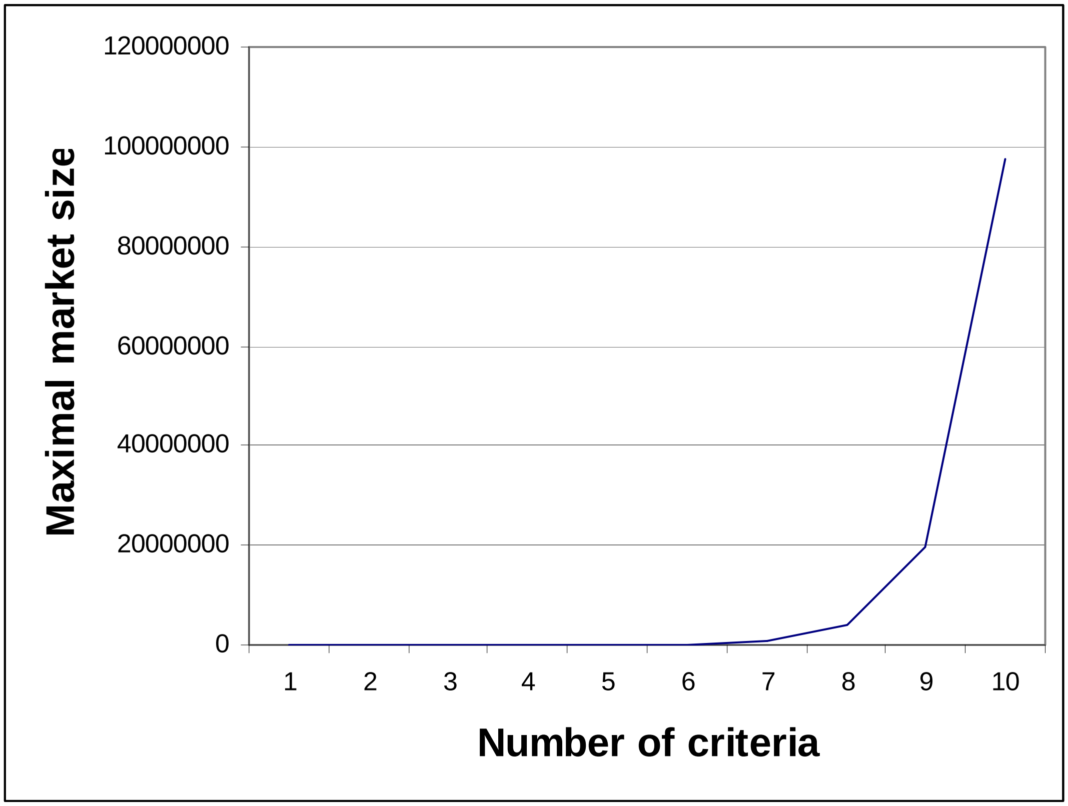


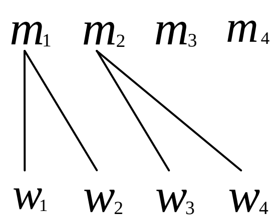
Example 5

Figure 4: Relationship between the number of segmentation criteria and the maximal size of a female-information-biased economy



Example 6

Figure 5-1



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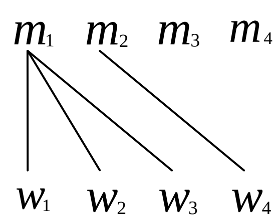

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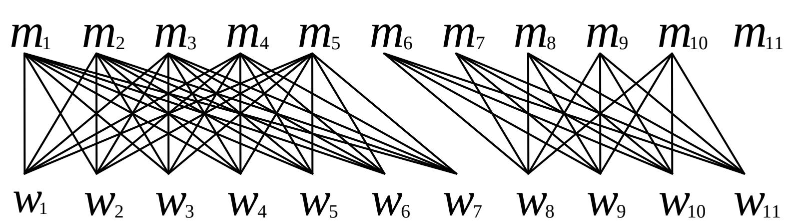

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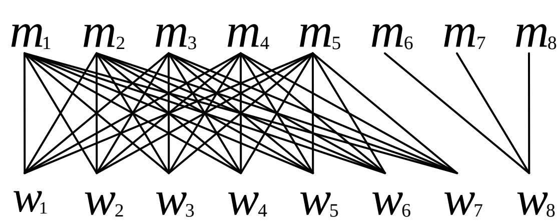
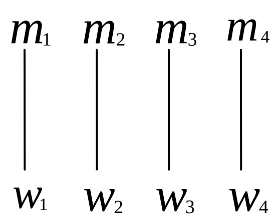


Example 7: Brain drain

Figure 6-1



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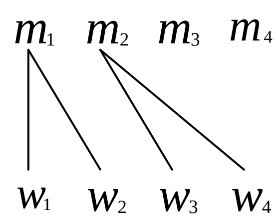

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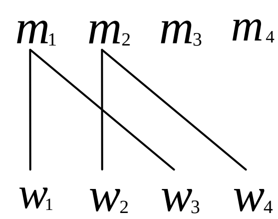

Figure 6-4