

Loss Leading as an Exploitative Practice*

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Abstract

Large retailers, enjoying substantial market power in some local markets, often compete with smaller retailers who carry a narrower range of products in a more efficient way. We find that these large retailers can exercise their market power by adopting a loss-leading pricing strategy, which consists of pricing below cost some of the products also offered by smaller rivals, and raising the prices on the other products. In this way the large retailers can better discriminate multi-stop shoppers from one-stop shoppers – and may even earn more profit than in the absence of the more efficient rivals. Loss leading thus appears as an exploitative device, designed to extract additional surplus from multi-stop shoppers, rather than as an exclusionary instrument targeting rivals – yet, these are hurt by the conduct. We show further that banning below-cost pricing increases consumer surplus and small rivals' profits as well as social welfare.

Our insights apply more generally to industries where a firm, enjoying substantial market power in one segment, competes with more efficient rivals in other segments, and single sourcing generates customer-specific benefits; they also apply to complementary products, such as platform and applications. There as well, our analysis provides a rationale for below-cost pricing based on exploitation rather than exclusion.

JEL Classification: L11, L41

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1 Introduction

The last three decades have seen the emergence of large retailers that offer a full range of groceries and other goods to attract consumers through one-stop shopping, as well as an increased concentration in retailing markets. As a result, competition between large retailers is limited in many local retailing markets,¹ and these large retailers have substantial market power over parts of the product lines, although they compete also with small retailers, such as hard-discounters and specialist retail chains, who carry much narrower product lines but may be more efficient in delivering these goods.² This raises a concern that large retailers may impede competition by leveraging their market power into the product segments that are also served by their smaller rivals.³

Large grocery retailers can exercise their market power in two ways, namely, through *buyer power* against suppliers or *seller power* against consumers and smaller rivals.⁴ While most of the recent literature has focused on buyer power,⁵ relatively little attention has been devoted

¹For instance, in its assessment of local market concentration in grocery retailing, the UK Competition Commission (2008, Section 6) defines *highly-concentrated local markets* as "local markets with three or fewer fascias in total where one of those fascias had a share of local grocery sales area that is greater than 60 per cent within a 10- or 15 minute drive-time." It finds that 27% of larger grocery stores are located in highly-concentrated local markets within a 10-minute drive time. The Commission finds moreover that the impact of rivals on a large retailer's profit and finds that of the impact of another large retailer is less than 4%, while small retailers have no statistically significant effect; see Competition Commission (2008), Appendix 4.4 at § 47.

²The rise of the hard-discount format is a new landscape in grocery-retailing. Hard discounters, popularized in the EU countries by retailers such as Aldi and Lidl, have relatively small stores and offer much fewer categories of goods – less than 10% of the lines offered by large retailers. Their assortment is dominated by private labels and their shopping environment gives priority to functionality and low distribution costs. As a result, they can offer prices up to 60% lower than those of leading name brands, and 40% lower than large retailers' private labels. See Dobson (2002) and Cleeren *et al.* (2008) for a detailed discussion.

³See for example the reports of the US Federal Trade Commission (2001, 2003), the proceedings of the FTC conference held on May 24, 2007, available at <http://www.ftc.gov/be/grocery/index.shtm>, or the groceries market enquiries of the UK Competition Commission (2000, 2008) recommending the adoption of codes of practices. In France, these concerns motivated in 1996 two Acts, aimed at curbing the expansion of large retailers as well as the exploitation of their market power.

⁴See Dobson and Waterson (1999) for detailed discussion.

⁵For example, Chen (2003) argues that buyer power results in lower prices for both retailers and consumers. While practitioners have often voiced concerns that buyer power might discourage suppliers' investment and innovation – see for example European Commission (1999) at p. 4 –, Inderst and Wey (2007) develop a model in which buyer power may instead increase suppliers' investment and enhance welfare.

to the analysis of seller power and its impact on retail competition.⁶ Yet, as argued by Paul Dobson (2009), it is in regard to how large retailers can distort retail competition that we might see the most profound market effects. This paper sheds a new light on the exercise of seller power and finds that it can induce large retailers to adopt a below-cost pricing strategy known as *loss leading*, which consists of pricing below cost some of the competitive products (leader products) and charging higher prices for the other goods. This practice is indeed widely adopted by large retailers: in its groceries market investigation, the UK Competition Commission notes for example that most large retailers in the UK engage in loss leading, mainly for staples such as milk and dairy, alcohol, bread and bakery products that consumers purchase repeatedly and regularly – and which constitute the core product lines of small retailers such as hard-discounters; it finds that the sales of loss leader products represent up to 6% of a retailer’s total sales.⁷

Antitrust enforcement and regulations against loss leading have stirred hot debates. For instance, in 2000 the German Federal Cartel Office ordered Wal-Mart, Aldi, and Lidl to stop selling below cost staples including milk and butter, arguing that this could impair competition and force smaller retailers to exit the market. By contrast, OECD (2007) argues that rules against loss leading are likely to protect inefficient competitors and harm consumers. There are also conflicting judgements on loss leading in US case law. For example, in *American Drugs vs. Wal-Mart Stores* (1993), Wal-Mart was sued under Arkansas’ *Unfair Practice Act* for below-cost pricing on certain pharmaceuticals. Wal-Mart lost the initial trial, but however successfully appealed before the Supreme Court of Arkansas, which found that "the loss-leader strategy employed by Conway Wal-Mart is readily justifiable as a tool to foster competition and to gain a competitive edge as opposed to simply being viewed as a stratagem to eliminate rivals all

⁶The recent literature on seller power has mainly focused on its interaction with buyer power through the so-called "waterbed effect". Dobson and Inderst (2007) and Inderst and Valletti (2008) argue for example that large retailers, who possess more bargaining power than their smaller rivals, can obtain better terms when negotiating with suppliers, which in turn may lead suppliers to increase the prices they charge to smaller retailers. While such waterbed effect could cause a self-perpetuating process widening the gap in the terms obtained by large and small retailers, some of the latter ones, such as hard discounters, belong to large retail networks who have developed their own private labels and business formats designed to reduce their operational costs. This paper studies such asymmetric competition, where large retailers face smaller but more efficient retailers, and ignores the role of buyer power in order to focus specifically on how large retailers can use their seller power at the expense of consumers and smaller rivals.

⁷See Competition Commission (2008); Dobson (2002) also provides a detailed economic analysis of loss-leading pricing in the UK grocery retailing, with particular concerns on the bakery retailers.

together."⁸ A similar discrepancy appears in the statutes dealing with below-cost sales.⁹ In the US, 22 states are equipped with general sales-below-cost laws, and 16 additional states prohibit below-cost sales on motor fuel. In the EU, below-cost resale is banned in Belgium, France, Ireland, Luxembourg, Portugal, and Spain, and is restricted in other countries including Austria, Denmark, Germany, Greece, Italy, Sweden and Switzerland, whereas it is generally allowed in the Netherlands and the UK.

In the absence of specific regulations, practitioners tend to tackle loss leading with predatory-pricing approaches.¹⁰ However, loss leading is a persistent below-cost pricing strategy, and in most cases courts and competition authorities are unlikely to show the feasibility that the predator could recoup the losses incurred during the predation phase by raising the prices after driving the rival out of the market.¹¹ For instance, in its 1997 report, the UK Office of Fair Trading argued that, in the analysis of alleged predation in retailing cases, a price-cost comparison is of little use, since pricing below cost on individual items may be profitable without being predatory. This begs several related questions: what is the rationale for loss leading if it is not predatory? What is then the impact on competition, consumers and society? Competition authorities face a dilemma in answering these questions.¹²

In the economic literature, loss leading has been viewed as an advertising strategy adopted to

⁸See Boudreaux (1996) for details. Yet in *Star Fuel Marts v. Murphy Oil (2003)*, a preliminary injunction was granted under Oklahoma's Unfair Sales Act, prohibiting below-cost sales of gasoline by Sam's East, a Wal-Mart subsidiary selling groceries in a wholesale club format. The court ruled that pricing below cost was prima facie evidence of intent to harm competitors, as well as of a tendency to dampen competition.

⁹See Skidmore *et al.* (2005). Calvani (2001) also discusses below-cost sales statutes in the U.S.

¹⁰See e.g., Bolton, Brodley and Riordan (2000) and Eckert and West (2003) for detailed discussions of how predatory-pricing tests should be designed.

¹¹The feasibility of recoupment is often a necessary condition for a case of predation; in the U.S., for example, this approach was by the Supreme Court in the *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*, which involved allegations of predatory pricing by Brown & Williamson against a smaller rival in an effort to discipline the pricing of generic cigarettes. The Court noted that predatory pricing was generally implausible without recoupment conditions, and further stated that intent ought to play no role in assessing whether conduct is predatory.

¹²For instance, in its most recent report, the UK Competition Commission concludes: "We find that the pattern of below-cost selling that we observed by large grocery retailers does not represent behavior that was predatory in relation to other grocery retailers." (See Competition Commission (2008) at p. 98). However, it also argues that below-cost pricing by large retailers might disproportionately squeeze smaller rivals' profit margins and even force them to exit (See p. 96-97).

attract consumers facing imperfect information of prices;¹³ below-cost pricing may then compensate consumers for their imperfect information and thereby improve consumer surplus.¹⁴ Loss leading has also been interpreted as an optimal cross-subsidizing strategy by a multi-product firm facing different demand elasticities across products.¹⁵ By contrast, little attention has been devoted to the often-voiced concerns that small retailers' profits are squeezed by large retailers' loss-leading strategies, and that consumers may end-up facing higher prices for non-staple products.¹⁶

This paper aims at filling this gap. We develop a model of asymmetric competition between large and small retailers, reflecting the characteristics of concentrated local markets where a few large retailers compete with smaller retailers who carry a narrower product range but in a more effective way. We moreover abstract away from the above-mentioned efficiency justifications by assuming that consumers are perfectly informed of all prices and by allowing for homogeneous consumer valuations for the goods. Our key modelling feature is to account for the heterogeneity in consumers' shopping costs: some consumers face higher shopping costs, e.g., because of tighter time constraints or lower taste for shopping, and thus have a stronger preference for one-stop shopping, whereas others have lower shopping costs and can therefore benefit from multi-stop shopping.

We first present the main insights in a stylized setting where a large retailer enjoys a monopoly position over some product lines (the monopolized segment) and faces a competitive fringe of smaller but more efficient rivals on other goods (the competitive segment). For simplicity, in this

¹³Lal and Matutes (1994), for example, consider a situation where multi-product firms compete for consumers who are initially unaware of prices, and find that in equilibrium firms may indeed choose to advertise a few loss leaders in order to increase store traffic. Ellison (2005) develops the model to analyze add-on pricing, and shows that loss leading can be optimal when firms advertise base goods while add-on prices are unobserved.

¹⁴Walsh and Whelan (1999) show that, in the presence of imperfect information, loss leading can generate the same long-run equilibrium outcomes as those observed under a *laissez-faire* full information scenario.

¹⁵Bliss (1988) may be the first paper viewing loss leading as a cross-subsidizing strategy, but does not formally establish existence conditions. Beard and Stern (2008) build on this model and incorporate continuous rather than unit consumer demands; they show that loss leading can indeed arise although for rather specific demand functions. Ambrus and Weinstein (2008) study Bertrand competition among symmetric firms competing for one-stop shoppers. They first show that loss leading cannot occur when consumers have inelastic demand. When demand is elastic, loss leading can occur but only under rather specific forms of demand complementarity; in particular, loss leading cannot arise when consumer demand is sufficiently diverse. The scope for loss leading in these settings, as well as its impact on consumers and welfare, still need to be assessed.

¹⁶See, for instance, Dobson (2002), at p.13.

setting all consumers have homogeneous valuations for the goods. If the rivals were excluded from the competitive segment, the large retailer would charge monopoly prices for both segments, based on consumer valuations and the distribution of their shopping costs. When more efficient rivals are present in the competitive segment, however, consumers with low shopping costs engage in multi-stop shopping: they buy the competitive goods from a more efficient rival, who offers better value, while still purchasing the monopolized goods from the large retailer. In contrast, consumers with higher shopping costs, who thus favor one-stop shopping, keep buying both types of products from the large retailer as long as its broader range of products delivers overall a greater value. The presence of more efficient rivals thus exerts a competitive pressure on the large retailer, but at the same time it opens a door for screening multi-stop shoppers from one-stop shoppers. We show that this is optimally achieved by adopting a loss-leading strategy, that is, by pricing the competitive goods below cost and raising instead the price for the monopolized goods: keeping constant the total margin charged to one-stop shoppers, this pricing strategy, which entails a negative margin in the competitive segment, allows the large retailer to charge a greater margin to multi-stop shoppers in the monopolized segment.

We show that loss leading indeed arises whenever the additional value generated by the large retailer's broader lines of products (the monopolized segment) exceeds the efficiency advantage of its smaller rivals in the competitive segment. In any such case, loss leading allows the large retailer to increase its profit, at the expense of consumer surplus, market efficiency (small rivals' market share being artificially reduced) and social welfare. When its broader range generates a large enough comparative advantage, the large retailer can even obtain in this way more profit than in the absence of the smaller rivals. We then extend the analysis to the case where the large retailer faces a strategic rival rather than a fringe in the competitive segment, in which case loss leading also hurts the rival by reducing the market share and squeezing the profit margin that the small retailer would otherwise obtain. However, this margin squeeze appears here more as the by-product of exploitation rather than as the result of exclusion; indeed, it is the very presence of a more efficient retailer with narrower product range that allows the large retailer to screen consumers according to their shopping costs. In other words, loss leading emerges here as an exploitative practice, adopted by the large retailer to extract rents from consumers, rather than as an exclusionary device aimed at foreclosing the market. Yet, the lack of exclusionary intent, as well as the fact that the small retailers remain active, should not lead to the conclusion that loss leading is an innocuous strategy, since its use as an exploitative device hurts consumers

as well as rivals.¹⁷ We show that a ban on loss leading would discipline the large retailer and benefit consumers as well as the small rival, and would also increase social welfare by improving the distribution efficiency in the competitive segment.

Finally, we show that loss leading still arises in more general settings with heterogeneous consumer valuations for the goods and/or (imperfect) competition among large retailers (in a symmetric Hotelling fashion). While retail competition among large retailers may limit their overall margins, the presence of smaller but more efficient rivals still opens a door for discriminating multi-stop shoppers from one-stop shoppers, and again this is optimally achieved through loss leading. The exploitative use of loss leading thus appears to be a robust feature in market environments where a few large retailers enjoy substantial market power over one-stop shoppers and compete with more efficient rivals carrying narrower lines of products.

To summarize, this paper provides a new rationale for the adoption of loss leading and highlights its harmful impact on competition and consumers in the absence of efficiency justifications, thus giving support to small rivals' complaints and competition concerns.¹⁸ The analysis also supports the above-mentioned suspicion about the exclusionary nature of the practice, and finds instead an exploitation rationale: the primary purpose is to extract additional rents from consumers rather than to foreclose the market, the harmful impact on the smaller rivals and retail efficiency arising only as a by-product of the strategy. Yet, the exploitative use of loss leading harms consumers and society as well as the small rivals, which may provide a rationale for antitrust enforcement.¹⁹

While this research was motivated by the use of loss leading in retailing markets, its insights

¹⁷In his report prepared on behalf of the Federation of Bakers, Dobson (2002) argues that the structure of the UK retail market, and the mix of different retail formats, is particularly conducive to the emergence of loss leading, as a form of competitive price discrimination which could lead to higher prices on other products, thus harming consumers as well as squeezing smaller rivals' profits.

¹⁸Chambolle (2005) also studies asymmetric competition between a large retailer and a smaller one, in a different setting in which both retailers are equally efficient but a majority of consumers is closer to the smaller store, and travel costs are too large for multi-stop shopping; the large retailer then never use the competitive good as a loss leader, but can instead use in this way the monopolized good, in which case this can benefit consumers as well as society. This is in line with the observation that in practice, concerns are voiced when loss leaders are chosen among the staples offered by the smaller retailers.

¹⁹Allain and Chambolle (2005) and Rey and Vergé (2010) note however that below-cost pricing regulations can allow manufacturers to impose price floors on their retailers, in which case they can be used to better exert market power or to reduce interbrand as well as intrabrand competition; banning loss leaders may then have a perverse effect on consumer welfare.

apply to a variety of situations where: (i) a firm enjoys substantial market power in one market and faces tougher competition in other markets; (ii) dealing with a single supplier gives customers some benefits (due e.g. to scale economies, lower adoption or maintenance costs, ...), which vary across customers. Pricing below cost in the competitive markets can then allow the larger firm to screen customers more effectively and extract some of these benefits. This insight can in particular shed a new light on antitrust cases such as the *IBM* and *Microsoft* cases;²⁰ while the debates have mainly focused on exclusionary motives, our analysis suggests an alternative rationale based on exploitation rather than exclusion.

The rest of the paper is organized as follows. Section 2 presents a simple model of asymmetric retail competition between a large retailer and smaller rivals, where consumers only differ in their shopping costs. Section 3 shows that loss leading arises as an exploitative device whenever the large retailer enjoys substantial market power over some product segments, and competes in other segments with a fringe of smaller but more efficient retailers; section 4 extends this insight to the case where the large retailer competes instead with a strategic smaller retailer. Section 5 the analyze the welfare impact of a ban on loss leading. Section 6 discusses some applications and presents the extension to more general settings that allow imperfect competition and heterogeneous consumer valuations. Finally, section 7 concludes.

2 The model

2.1 Market structure and consumer choice

A large retailer (denoted by L), who supplies a broad range of products, competes in a local market with one or several homogeneous small retailers (denoted by S) who offer much narrower product lines. For the sake of exposition, we simply assume that there are two markets (which can be interpreted as different goods or different lines of products), A and B . Product A is monopolized by L , whereas L and S offer different varieties of product B , denoted B_L and B_S ; in what follows, we will refer to A as the "monopolistic segment" and to B as the "competitive segment". L incurs respectively a unit cost c_A and c_L for supplying A and B_L , while S faces a unit cost c_S for B_S .

Each consumer desires at most one unit of A and one unit of B ;²¹ consuming A or B_i (for

²⁰See e.g. *United States v. International Business Machines Corporation*, Docket number 69 Civ. DNE (S.D. NY) and *United States of America v. Microsoft Corporation*, Civil Action No. 98-1232 TPJ (D.C.).

²¹The assumption of unit demands appears reasonable for groceries and other day-to-day consumer purchases.

$i = L, S$) brings a utility u_A or u_i , whereas consuming both A and B_i yields $u_{Ai} \leq u_A + u_i$.²² Assuming homogeneous valuations for A , B_L and B_S allows us to avoid cross-subsidization motives stemming from differences in demand elasticities, as studied by Bliss (1988).²³ For the analysis, it is convenient to use the social values $w_i \equiv u_i - c_i$ (for $i = A, L, S$) and $w_{Ai} \equiv u_{Ai} - c_A - c_i$ (for $i = L, S$). We are interested in the case where it is socially efficient for L to supply both products rather than one: $w_{AL} > w_A, w_L$;²⁴ in particular, its broader range of products enables L to bring an additional value $w_{AL} - w_L > 0$ on product A . We are moreover interested in the case where small retailers are more efficient in distributing B :²⁵ $w_S > w_L$. For the sake of exposition, we assume that the efficiency advantage of small retailers does not affect the added value of A : $w_{AS} - w_S = w_{AL} - w_L$.

Finally, we build on Armstrong and Vickers (2010) and assume that consumers incur a shopping cost for visiting a store.²⁶ This shopping cost may reflect the opportunity cost of the time spent in traffic, parking, selecting products, checking out, and so forth; it may also account for the consumer's taste for shopping. To highlight the fact that consumers may be more or less time-constrained, or value their shopping experience in different ways, we assume that the shopping cost, denoted by t , varies across consumers and is distributed according to a cumulative distribution function $F(\cdot)$, with density function $f(\cdot)$; we assume that the inverse hazard rate, $h(\cdot) \equiv F(\cdot)/f(\cdot)$, is strictly increasing.²⁷

We model retail competition as follows: (i) L and S simultaneously set their prices, respectively (p_A, p_L) and p_S ;²⁸ (ii) consumers then observe all prices and make their shopping

To be sure, price changes affect the composition of consumer baskets, but are less likely to have a large impact on the volume of purchases for staples.

²²This allow for (partial) substitution between A and B ; the analysis however readily applies to the case of complementary goods – see section 7.2.

²³To show the robustness of the analysis, we relax this assumption in section ??.

²⁴These conditions imply $c_A < u_{AL} - u_L \leq u_A$ and $c_L < u_{AL} - u_A \leq u_L$. It is thus indeed *a fortiori* efficient for L to supply either product rather than none: $w_A, w_L > 0$.

²⁵For instance, small retailers could be discount stores with lower distribution costs, or specialist stores that bring higher value for B .

²⁶Armstrong and Vickers (2010) consider a symmetric duopoly à la Hotelling in which consumers have heterogeneous and elastic demands for two products and incur an additional shopping cost when dealing with both suppliers; they show the existence of an equilibrium in which firms price all products above (or at) cost but offer conditional discounts (mixed bundling).

²⁷This assumption ensures that profit functions are single-peaked.

²⁸We first consider stand-alone prices, and show later that allowing for bundled discounts cannot increase L 's

decisions. When making these decisions, consumers are thus fully aware of all prices and take also into account the value of the proposed assortments as well as their shopping costs.

We will successively consider several scenarios. In a first scenario, B_S is competitively supplied by a fringe of small retailers, who offer it at cost; this scenario allows us to develop our main insight in the simplest way, by focussing on L 's strategy. In a second scenario, a single small retailer acts instead as a strategic player. Studying the (pure strategy) equilibria of this scenario allows us to show the robustness of the main insight and to discuss margin squeeze issues. Finally, we extend the analysis to (imperfectly) competitive large retailers (and heterogeneous valuations for the goods). Before considering these scenarios, we conclude this section with a benchmark case in which L faces no competition from any rival.

2.2 Benchmark: monopoly

We suppose here that L is a monopolist for both products. By assumption, it is more profitable to sell both products rather than one.²⁹ Purchasing both products yields a net surplus $u_{AL} - p_{AL} - t$. Consumers will therefore buy as long as $t \leq v_{AL} \equiv u_{AL} - p_{AL} = w_{AL} - r_{AL}$, where v_{AL} denotes the consumer value from purchasing both A and B_L , while $r_{AL} \equiv p_{AL} - c_A - c_L$ denotes L 's total margin. The monopolist thus faces a demand $F(v_{AL})$ and makes a profit

$$r_{AL}F(v_{AL}) = r_{AL}F(w_{AL} - r_{AL}).$$

This profit function is quasi-concave in r_{AL} (see Appendix A) and the first-order condition is given by:

$$r_{AL} = h(v_{AL}). \tag{1}$$

The monopoly outcome is thus such that $r_{AL}^m = w_{AL} - v_{AL}^m$ and:

$$v_{AL}^m \equiv l^{-1}(w_{AL}), \tag{2}$$

profit; see the remark in section 3.

²⁹Since consumers have homogeneous valuations, all active consumers behave in the same way. Suppose that they buy B only (that is, $p_A \geq u_{AL} - u_A$); then reducing p_A slightly below $u_{AL} - u_A$ would ensure that consumers buy A as well, bringing an additional revenue (almost) equal to $w_{AL} - w_A$ from each of them; a similar reasoning applies to the case where active consumers would only buy A .

where the function $l(x) \equiv x + h(x)$ is increasing in x . L 's monopoly profit is then given by:³⁰

$$\Pi_{AL}^m \equiv F(v_{AL}^m)h(v_{AL}^m). \quad (3)$$

3 Loss leading as an exploitative device

We suppose in this section that a competitive fringe of small retailers supplies B_S at cost: $p_S = c_S$. One-stop shoppers can thus obtain w_S by patronizing a small retailer, or $v_{AL} = w_{AL} - r_{AL}$ by buying both products from L .

If one-stop shoppers favor L ($v_{AL} \geq w_S$), which we will refer to as "regime L ", small retailers can only attract multi-stop shoppers, who buy A from L and B_S from them. Multi-stop shopping involves double shopping costs, $2t$, but yields a value $v_{AS} \equiv u_{AS} - p_A - c_S = w_{AS} - r_A$, where $r_A \equiv p_A - c_A$ denotes L 's margin on A , and consumers are willing to do so if $v_{AS} - 2t \geq v_{AL} - t$, that is, if the additional shopping cost is offset by the extra gain from multi-stop shopping (denoted by τ), i.e.,

$$t \leq \tau \equiv v_{AS} - v_{AL} = w_S - w_L + r_L,$$

where $r_L \equiv r_{AL} - r_A$ denotes L 's margin on B_L . Thus, in regime L consumers are willing to visit L as long as $t \leq v_{AL}$, but prefer patronizing both stores if $t \leq \tau$. L therefore attracts a demand $F(v_{AL}) - F(\tau)$ for both products (from one-stop shoppers) and an additional demand $F(\tau)$ for product A only (from multi-stop shoppers);³¹ it thus obtains a profit equal to:

$$r_{AL}(F(v_{AL}) - F(\tau)) + r_A F(\tau) = r_{AL}F(v_{AL}) - r_L F(\tau),$$

which, using $v_{AL} = w_{AL} - r_{AL}$ and $\tau = w_S - w_L + r_L$, can be expressed as a function of r_{AL} and r_L as:

$$\Pi_L(r_{AL}, r_L) = r_{AL}F(w_{AL} - r_{AL}) - r_L F(w_S - w_L + r_L). \quad (4)$$

$\Pi_L(r_{AL}, r_L)$ is additively separable and moreover quasi-concave in r_{AL} and r_L (see Appendix A). To attract one-stop shoppers, L must however offer a better value than its rival:³² $v_{AL} \geq w_S$,

³⁰We implicitly assume here any relevant upper bound on shopping costs. If t is instead distributed over a range $[0, T]$, where $T \leq l^{-1}(w_{AL})$, then the optimal (monopoly) value is $v_{AL}^{m'} = T$ and the corresponding profit is $(w_{AL} - T)F(T)$.

³¹In Appendix B, it is shown that any pricing strategy leading to $\tau < 0$ (resp., $\tau > v_{AL}$) is equivalent to a pricing strategy yielding $\tau = 0$ (resp., $\tau = v_{AL}$); therefore, without loss of generality, we can restrict attention to prices such that $\tau \in [0, v_{AL}]$.

³²This condition also ensures that prospective multi-stop shoppers are indeed willing to buy A on a stand-alone basis: $w_S \leq v_{AL} = w_{AL} - r_A - r_L$ implies $r_A \leq w_{AL} - w_S - r_L = w_{AL} - w_L - \tau < w_{AL} - w_L = w_{AS} - w_S$.

or

$$r_{AL} \leq w_{AL} - w_S. \quad (5)$$

>From the expression (4), it is clearly optimal for L to price B_L below cost: setting $r_L \geq 0$ is indeed dominated by $r_L < 0$.³³ The intuition is straightforward. Keeping r_{AL} – and thus the total price paid by one-stop shoppers – constant, subsidizing B_L allows L to increase its margin on A ($r_A > r_{AL}$) and reap in this way a higher profit from multi-stop shoppers, who buy only A from it. Since the margin r_L does not affect (5), its optimal value is moreover characterized by the first-order condition:

$$r_L^* = -h(w_S - w_L + r_L^*) = -h(\tau^*) < 0. \quad (6)$$

Using $r_L^* = \tau^* - (w_S - w_L)$, the optimal threshold τ^* thus satisfies:

$$\tau^* \equiv l^{-1}(w_S - w_L) > 0. \quad (7)$$

Therefore, L obtains a profit of the form:

$$\Pi_L = r_{AL}F(v_{AL}) + h(\tau^*)F(\tau^*),$$

where the first term represents the base profit achieved from both types of customers, whereas the second term represents the additional profit that are extracted from multi-stop shoppers through loss leading.

In the absence of any restriction on its total margin, L would charge $r_{AL} = r_{AL}^m$ and thus offer one-stop shoppers a value $v_{AL} = v_{AL}^m = l^{-1}(w_{AL})$. Conversely, this strategy satisfies (5) and thus attracts indeed one-stop shoppers as long as $v_{AL}^m \geq w_S$, or $w_{AL} \geq l(w_S)$ ($> w_S$); therefore, when L derives a sufficiently large comparative advantage from its broader line of products, the optimal strategy consists of charging the monopoly margin r_{AL}^m for the bundle, and $r_L^* = -h(\tau^*)$ for B_L .³⁴ The loss-leading strategy then gives L a profit equal to:

$$\Pi_L^* = r_{AL}^m F(v_{AL}^m) - r_L^* F(\tau^*) = \Pi_{AL}^m + h(\tau^*) F(\tau^*),$$

which *exceeds* the monopolistic profit Π_{AL}^m .

³³More precisely, any $r_L > 0$ is dominated by $r_L = 0$, which in turn is dominated by any slightly negative r_L ; pricing way below cost (namely, $r_L < -(w_S - w_L)$) would however eliminate multi-stop shopping ($\tau < 0$) and thus yield the same profit as $r_L = 0$.

³⁴Note that τ^* then satisfies $\tau^* < v_{AL}^m$. To see this, take instead v_{AL} and τ as control variables and rewrite L 's profit as $\Pi_L(v_{AL}, \tau) = r_{AL}F(v_{AL}) - r_L F(\tau) = (w_{AL} - v_{AL})F(v_{AL}) + (w_S - w_L - \tau)F(\tau)$. Then we have $v_{AL}^m = \arg \max_v (w_{AL} - v)F(v) > \arg \max_v (w_S - w_L - v)F(v) = \tau^*$, since $w_{AL} \geq l(w_S)$ ($> w_S \geq w_S - w_L$).

When instead L 's comparative advantage is not large enough (namely, $w_{AL} < l(w_S)$), L must improve its offer in order to keep attracting one-stop shoppers. It is then optimal for L to match the value offered by the competitive fringe: $\tilde{v}_{AL}^* = w_S$, or $\tilde{r}_{AL}^* = w_{AL} - w_S (< r_{AL}^m)$.³⁵ The loss-leading strategy then gives L a profit equal to:

$$\tilde{\Pi}_L^* \equiv (w_{AL} - w_S) F(w_S) + h(\tau^*) F(\tau^*).$$

Alternatively, L can leave one-stop shoppers to the small retailers ("regime S ") and focus instead on multi-stop shoppers, who are willing to buy A from L as long as the added value $v_A \equiv w_{AL} - w_L - r_A$ exceeds the extra shopping cost t . In this way, L obtains:

$$\Pi_L = r_A F(v_A) = r_A F(w_{AL} - w_L - r_A).$$

It is then optimal for L to adopt the monopoly margin r_A^m which, together with the corresponding value $v_A^m = w_{AL} - w_L - r_A^m$, is characterized by:

$$r_A^m = h(v_A^m), \quad v_A^m = l^{-1}(w_{AL} - w_L).$$

This gives L a profit equal to:

$$\Pi_A^m \equiv r_A^m F(v_A^m).$$

The loss-leading strategy is clearly preferable when $v_{AL}^m \geq w_S$, since it then gives L more profit than the monopolistic profit $\Pi_{AL}^m (> \Pi_A^m)$.³⁶ We show in Appendix B that it remains preferable as long as L enjoys a comparative advantage over S (that is, $w_{AL} \geq w_S$), which yields:

Proposition 1 *Suppose the large retailer (L) faces a competitive fringe of small retailers (S). Then:*

- *When L enjoys a comparative advantage over S (i.e., $w_{AL} > w_S$), its unique optimal pricing strategy involves loss leading: L prices the competitive product B_L below cost. Furthermore, when its comparative advantage is large (namely, $v_{AL}^m \geq w_S$), L keeps the total price for the two products at the monopoly level ($r_{AL} = r_{AL}^m$) and earns a higher profit than in the absence of any rivals; otherwise L simply charges a total price reflecting its comparative advantage ($r_{AL} = w_{AL} - w_S$).*

³⁵If needed, L can slightly enhance its offer to make sure that it attracts all one-stop shoppers.

³⁶For the sake of exposition, throughout the paper we refer to loss leading as *selling* a product below cost. Here, for instance, L may keep *offering* B below cost when $w_{AL} < w_S$, but it then only sells A (to multi-stop shoppers, who buys B from S).

- When instead L faces a comparative disadvantage (i.e., $w_{AL} < w_S$), its unique optimal pricing strategy consists of monopolizing the non-competitive product and leaving the market of the competitive product to the small retailers.

Proof. See Appendix B. ■

Whenever L can attract one-stop shoppers as well as multi-stop shoppers, loss leading provides an exploitative device, which allows L to discriminate more effectively these two categories of consumers: by using the competitive segment as a loss leader, L keeps attracting one-stop shoppers, but raises the price that multi-stop shoppers pay on the non-competitive segment. As long as $w_S \leq v_{AL}^m$, L can keep the total price at the monopoly level and earns in this way more profit than in the absence of any rival; in this range, an increase in w_S actually benefits L , who can exploit the efficiency gain of its rivals ($h(\tau^*)F(\tau^*)$ increases with w_S). However, this also mitigates L 's comparative advantage and reduces the parameter region in which L can benefit from loss leading. Moreover, when $v_{AL}^m < w_S \leq w_{AL}$, an increase in w_S forces L to reduce its total margin ($r_{AL} = w_{AL} - w_S$ decreases); and when $w_S > w_{AL}$, L loses its comparative advantage and can only monopolize market A .

Remark: Bundled discounts. In principle, L might offer three prices: one for A , one for B_L and one for the bundle. But, since L sells A to every consumer who visits its store, only two prices matter here: the price p_A when buying A only, and the total price p_{AL} when buying both A and B_L . Alternatively, these prices can be implemented through stand-alone prices, p_A for A and $p_L \equiv p_{AL} - p_A$ for B_L . Therefore, offering additional bundled discount based on two stand-alone prices p_A and p_L could not improve L 's profit here.

Illustration: Uniform density of shopping costs. Suppose that the shopping cost is uniformly distributed: $F(t) = t$. The optimal r_L and optimal threshold τ are then given by:

$$r_L^* = -\tau^*, \tau^* = \frac{w_S - w_L}{2}.$$

Then, whenever $w_{AL} \geq 2w_S$, the optimal margin r_{AL} is set to the monopoly level

$$r_{AL}^m = v_{AL}^m = \frac{w_{AL}}{2},$$

and in this way L obtains more profit than monopoly level:

$$\Pi_L^* = \Pi_{AL}^m + \frac{(w_S - w_L)^2}{4} = \frac{(w_{AL})^2}{4} + \frac{(w_S - w_L)^2}{4}.$$

When instead $w_S \leq w_{AL} < 2w_S$, L maintains the same margin r_L^* but charges $\tilde{r}_{AL}^* = w_{AL} - w_S$ and its profit reduces to:

$$\tilde{\Pi}_L^* = (w_{AL} - w_S)w_S + \frac{(w_S - w_L)^2}{4},$$

which coincides with

$$\Pi_A^m = \frac{(w_{AL} - w_L)^2}{4}$$

when $w_{AL} = w_S$. Finally, whenever $w_{AL} < w_S$, L leaves the competitive segment to its smaller rivals and earns Π_A^m by exploiting its monopoly power on A .

Remark: asymmetric shopping costs. In practice, a consumer may incur different costs when visiting L or S – visiting a larger store may for example be more time consuming. Our analysis easily extends to such situations. Suppose for example that consumers bear a cost αt when patronizing L (and t , as before, when visiting S). The threshold τ remains unchanged,³⁷ while one-stop shoppers are now willing to patronize L as long as $t < v_{AL}/\alpha$. As long as one-stop shoppers favor L , its profit is now:

$$\Pi_L = r_{AL} \left(F \left(\frac{v_{AL}}{\alpha} \right) - F(\tau) \right) + r_A F(\tau) = r_{AL} F \left(\frac{v_{AL}}{\alpha} \right) - r_L F(\tau),$$

which leads L to adopt the same loss-leading strategy as before $r_L^* = -h(\tau^*)$, where $\tau^* = l^{-1}(w_{AS} - w_{AL})$.

4 Loss leading and margin squeeze

Focusing on the case where the small retailer is a competitive fringe allows us to highlight the pure exploitative effect of loss leading without considering its impact on smaller rivals, since competition among them dissipate their margins anyway. Yet, in many antitrust cases, small retailers have complained that their profit were squeezed as a result of large retailers' loss-leading strategies. We thus consider here the case where L competes against a single smaller rival S ; this allows us to analyze the margin-squeeze effect on S caused by loss leading.

S can now earn a positive margin $r_S > 0$ for the product B_S , which leaves a value $v_S = w_S - r_S$ for the consumers. The analysis of L 's behavior remains however valid, replacing the competitive value w_S with the net value $v_S = w_S - r_S$. We will focus here on the regime where L attracts one-stop shoppers, by offering strictly more value than its rival ($v_{AL} > v_S$). L faces a demand $F(v_{AL}) - F(\hat{\tau})$ for both products from one-stop shoppers, and an additional demand $F(\hat{\tau})$ for product A from multi-stop shoppers, where the shopping cost threshold is now given by:

$$\hat{\tau} \equiv v_{AS} - v_{AL} = w_S - w_L + r_L - r_S. \quad (8)$$

³⁷A consumer favours multi-stop shopping if $v_{AS} - (1 + \alpha)t > v_{AL} - \alpha t$, which amounts as before to $t < \tau = v_{AS} - v_{AL}$.

In this way, L therefore earns a profit:

$$\begin{aligned}\Pi_L &= r_{AL}F(v_{AL}) - r_L F(\hat{\tau}) \\ &= r_{AL}F(w_{AL} - r_{AL}) - r_L F(w_S - w_L + r_L - r_S).\end{aligned}\tag{9}$$

The optimal margins are then determined implicitly by the first-order conditions

$$r_{AL} = h(v_{AL}), \text{ and } r_L = -h(\hat{\tau}).$$

Since S only attracts multi-stop shoppers, its profit is equal to

$$\Pi_S = r_S F(\hat{\tau}) = r_S F(w_S - w_L + r_L - r_S).\tag{10}$$

Thus, its best response to r_L is given by the first-order condition:

$$r_S = h(\hat{\tau}).$$

These first-order conditions forms a candidate equilibrium in which L : (i) charges the monopoly margin for the bundle of products ($\hat{r}_{AL}^* = r_{AL}^m$); and (ii) prices the competitive good below cost: $\hat{r}_L^* = -\hat{r}_S^* = -h(\hat{\tau}^*)$. The equilibrium margin \hat{r}_L^* and \hat{r}_S^* and the resulting threshold $\hat{\tau}^*$ satisfy:

$$\hat{\tau}^* = w_S - w_L + \hat{r}_L^* - \hat{r}_S^* = w_S - w_L - 2h(\hat{\tau}^*),$$

or equivalently:

$$\hat{\tau}^* \equiv j^{-1}(w_S - w_L),\tag{11}$$

where $j(x) \equiv x + 2h(x)$ is strictly increasing. It follows that in equilibrium S earns:

$$\hat{\Pi}_S^* \equiv h(\hat{\tau}^*) F(\hat{\tau}^*),$$

while L obtains:

$$\hat{\Pi}_L^* \equiv \Pi_{AL}^m + h(\hat{\tau}^*) F(\hat{\tau}^*).$$

Since $\hat{\tau}^* = j^{-1}(w_S - w_L) < l^{-1}(w_S - w_L) = \tau^*$, L 's profit is lower than when facing a competitive fringe of similar small retailers.

For the above margins to form an equilibrium, two conditions must be satisfied: first, L must indeed attract one-stop shoppers; second, while L has no incentive to exclude its rival, since it earns more profit than a pure monopolist, S may want to attract one-stop shoppers by offering a higher value than v_{AL}^m . We show in Appendix C that these two conditions are satisfied when L enjoys a significant comparative advantage, namely, when $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$, where the threshold $\hat{w}_{AL}(w_S, w_L)$ lies above w_S and increases with w_S . We also show that loss leading does not arise when $w_{AL} < \hat{w}_{AL}(w_S, w_L)$:

Proposition 2 *Suppose that the large retailer, L , faces a strategic smaller rival, S . Then loss leading arises in a unique Nash equilibrium if and only if L enjoys a significant comparative advantage (namely, $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$). In that equilibrium, L sells the competitive product below-cost while keeps the total price for both products at the monopoly level, and earns a profit higher than that absent the rival.*

Proof. See Appendix C. ■

Loss leading thus constitutes a robust exploitative device, which still allows L to discriminate multi-stop shoppers from one-stop shoppers even when competing with a strategic smaller rival. As before, adopting loss leading allows L to earn even more profit than a pure monopolist if its comparative advantage is large enough. Compared with the case of a competitive fringe, loss leading is now adopted in equilibrium only when it allows L to earn the full monopoly margin from one-stop shoppers, but it does so in a broader range of circumstances: it is shown in Appendix C that the equilibrium conditions $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$ is less stringent than the similar condition for the case of a competitive fringe ($v_{AL}^m \geq w_S$).

Compared with the case of a competitive fringe of smaller retailers, whose profit is not affected by L 's behavior, the loss-leading strategy now hurts S 's profit, not only by reducing its market share, but also by squeezing its margin: S 's best response is such that $\hat{\tau} = l^{-1}(w_S - w_L + r_L)$ and $r_S = h(\hat{\tau})$ decrease as r_L decreases. Yet, this appears here as a side effect of the exploitative motive rather than as the result of exclusionary motive. In particular, foreclosing the market through strategic tying or (pure) bundling would not be profitable here, since L could obtain at most the monopoly profit in the case of exclusion.

Remark: Strategic margin squeeze. Although margin squeeze appears here as a by-product of the use of loss leading as an exploitative device, the large retailer has an incentive to manipulate rivals' prices: the lower S 's price for B_S , the more L can extract from multi-stop shoppers. As a result, and in contrast to the standard case where firms usually benefit from higher rival prices, here L would want S to *decrease* its own price. Thus, if L could move first and act as a Stackelberg leader, it would *decrease* even further its price for B_L (in contrast with the standard Stackelberg insight), so as to induce S to respond by decreasing its own price, and allow L to raise the price it charges (on A) to multi-stop shoppers.

Since L benefits from the presence of S , it may however want to limit its loss-leading strategy in order to maintain that presence. Suppose for example that the entry of S is uncertain. It is then profitable for L to adopt a loss-leading strategy in case of entry, in order to extract additional rents from multi-stop shoppers, but this also reduces the likelihood of entry. Thus, while L would

not gain from committing itself to never adopting a loss-leading strategy (since then it would extract no additional rent from multi-stop shoppers), it would benefit from limiting its extent. We develop a simple model along this line in Appendix E, which yields the following insights:

Proposition 3 *If L and S compete as Stackelberg leader and follower; then, whenever L 's comparative advantage leads it to adopt a loss-leading strategy, it sells the competitive product B further below-cost, compared with that it would do in the absence of a first-mover advantage: its margin on B , r_L^S , satisfies $r_L^S < \hat{r}_L^*$. However, if the entry of S depends on the realization of a random entry cost then, when L 's comparative advantage leads it to adopt a loss-leading strategy, it limits the subsidy on B so as to increase the likelihood of entry: its new margin, \hat{r}_L^S , satisfies $\hat{r}_L^S > r_L^S$.*

Proof. See Appendix D. ■

5 Banning loss leading

We now show that loss leading reduces consumer surplus and social welfare as well as smaller rivals. For the sake of exposition, we consider here the case when L faces a strategic rival, and focus on the case when L would attract one-stop shoppers, and thus engage in loss leading (that is, $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$), and analyze the impact of a ban on below-cost pricing.

We show in Appendix E that L keeps attracting one-stop shoppers in equilibrium. Since the profit expression (9) is quasi-concave and separable in r_{AL} and r_L , L then maintains the total margin at the monopoly level (r_{AL}^m) but now sells B_L at cost ($r_L = 0$); as a result, its profit reduces to $\Pi_{AL}^m = r_{AL}^m F(v_{AL}^m)$.

Since L no longer subsidizes the competitive segment, S faces more demand from multi-stop shoppers: the shopping cost threshold increases from $\tau = w_S - w_L + r_L^* - r_S$ to $\tau = w_S - w_L - r_S$. Maximizing its profit $\Pi_S = r_S F(\tau)$ then leads S to charge a margin satisfying $r_S = h(\tau) = h(w_S - w_L - r_S)$; the equilibrium threshold is thus:

$$\tau^* = l^{-1}(w_S - w_L) > j^{-1}(w_S - w_L) = \hat{\tau}^*.$$

That is, S increases its market share (from $\hat{\tau}^*$ to τ^*) as well as its margin (from $\hat{r}_S^* = h(\hat{\tau}^*)$ to $\hat{r}_S^b \equiv h(\tau^*)$); banning loss leading thus increases S 's profit by

$$\Delta_{\Pi_S} = h(\tau^*) F(\tau^*) - h(\hat{\tau}^*) F(\hat{\tau}^*) > 0.$$

Banning loss leading does not affect the value of one-stop shopping, since L maintains the same total margin, r_{AL}^m . It however encourages consumers to take advantage of multi-stop shopping: banning loss leading forces L to compete "on the merits", which induces those consumers with a shopping cost lower than τ^* to patronize both stores; in contrast, subsidizing B_L (and overcharging A by the same amount) discourages consumers with a shopping cost exceeding $\hat{\tau}^*$ from visiting S . The ban on loss leading thus benefits consumers whose shopping cost lies between $\hat{\tau}^*$ and τ^* , since the resulting lower price for A allows them to save $\tau^* - t$. Using a revealed preference argument, it also benefits genuine multi-stop shoppers (those with a shopping cost $t < \hat{\tau}^*$), by increasing the value of multi-stop shopping from $\hat{v}_{AS}^* \equiv v_{AL}^m + \hat{\tau}^*$ to $v_{AS}^* \equiv v_{AL}^m + \tau^*$. Overall, a ban on loss leading thus increases total consumer surplus by:

$$\Delta_{CS} = (\tau^* - \hat{\tau}^*) F(\hat{\tau}^*) + \int_{\hat{\tau}^*}^{\tau^*} (\tau^* - t) dF(t) > 0.$$

Finally, the increase in multi-stop shopping also enhances total welfare, since more consumers benefit from a better distribution of B . The gain in social welfare is equal to:

$$\Delta_W = \int_{\hat{\tau}^*}^{\tau^*} (w_S - w_L - t) dF(t),$$

which is indeed positive since $\hat{\tau}^* < \tau^* < w_S - w_L$. Therefore, we have:

Proposition 4 *Assume that L faces a strategic rival S , and would engage in loss leading; banning below-cost pricing then leads L to maintain the same total margin but sell the competitive good at cost; as a result, this ban increases consumer surplus as well as the rival's profit and social welfare.*

Proof. See Appendix E. ■

A similar analysis applies when L faces a competitive fringe: while loss leading has then no effect on rivals' profit, it still reduces their market share and hurts consumers as well as it distorts efficiency. By stressing the large retailer's use of loss leading as an exploitative device extracting rents from multi-stop shoppers, rather than as an exclusionary or predatory practice, our analysis sheds a new light on the ongoing debate on the adoption of below-cost pricing rules, beyond the general laws against predatory pricing, and can help placing the evaluation of anticompetitive effects on firmer grounds.

6 Extensions: heterogeneous valuations and competition among large retailers

The use of loss leading as an exploitative device, designed to extract additional surplus from multi-stop shoppers, has been so far established in a relatively simple setting where a large retailer enjoys local monopoly power on some product segments and consumers have moreover homogeneous valuations in all segments. We now discuss the robustness of our insights when these assumptions are relaxed.

Note first that introducing heterogeneous valuations for B does not affect our analysis of loss leading as long as consuming B_L remains efficient (that is, $u_L > c_L$ for all consumers): since L prices B_L below cost in equilibrium, the consumer value from B_L is always positive ($v_L = u_L - p_L > 0$), and so is the value from B_S as $v_S > v_L$; therefore, one-stop shoppers would still buy B_L from L and likewise multi-stop shoppers would buy B_S from the smaller stores. By contrast, heterogeneous valuations for A makes its demand elastic, which limits L 's ability to raise prices in this segment; by the same token, this may makes loss leading less attractive, since the purpose of the exploitative device is precisely to charge more (to multi-stop shoppers) on this segment. Likewise, (imperfect) competition among large retailers curbs their capacity to charge high prices on A and may thus also challenge the use of loss leading as an exploitative device.

To check the robustness further, we extend here the basic setting to allow for an elastic demand for A and also possibly for (imperfect) competition among large retailers. More precisely, we now assume that two large retailers are present, L_1 and L_2 , who incur the same costs in distributing A and B . On the B market, they offer the same variety B_L and as before face competition from one or several identical small retailer(s). For the sake of exposition we will still assume that consumers have homogeneous valuations for B_L and B_S , and moreover focus on the case of independent products (that is, the utility provided by the assortment AB_i is simply the sum of the utilities derived by consuming A and B_i on a stand-alone basis). The large retailers however offer different varieties of A , A_1 and A_2 , over which consumers have heterogeneous preferences à la Hotelling: each consumer's preference is characterized by $x \in [0, 1]$, where x represents the consumer's "distance" from A_1 ($1 - x$ representing the distance from A_2), and the product differentiation parameter is $1/\sigma$; a consumer with preference x thus obtains a utility $u_A - \frac{x}{\sigma} - p_{A_1} = w_A - r_{A_1} - \frac{x}{\sigma}$ from buying A_1 and a utility $w_A - r_{A_2} - \frac{1-x}{\sigma}$ from buying A_2 . Varying the differentiation parameter σ and the distribution of the distance parameter x allows

for quite general demand functions. In particular, a low value of σ corresponds to the case of "local monopolies", which amounts to introducing an arbitrarily elastic demand in the previous analysis, whereas higher values of σ represent to market environments where the large retailers exert a competitive pressure on each other; we will investigate both cases in turn.

As before, we also allow for general distributions of shopping costs (including bounded ones – see below), and now allow as well for quite general distributions of the distance parameter x . We will however restrict attention to symmetric distributions (that is, the density $g(\cdot)$ satisfies $g(x) = g(1-x)$) and, to keep in line with the previous analysis, we will assume that retailers' profits are strictly quasi-concave in prices, so as to ensure the existence of interior optima. In particular, we will assume that the distribution of x has a monotonic (inverse) hazard rate $k(\cdot) \equiv G(\cdot)/g(\cdot)$, where $G(\cdot)$ denotes the cumulative distribution, increases with x .

Finally, throughout this section we will focus on (symmetric) equilibria in which: (i) small retailers attract some multi-stop shoppers, by offering a value v_S that exceeds the value v_L that large retailers offer on the B market; and (ii) large retailers attract some one-stop shoppers, by offering them a value v_{AL} that exceeds v_S , as well as the value v_A that they offer on the A market alone.

6.1 Local monopolies

Consider first the case where σ is small, so that L_1 and L_2 do not compete against each other. Each large retailer then enjoys a local monopoly power in market A and only competes with small retailer(s) in market B . It is thus in the same situation as in our basic framework, except that it now faces an elastic demand for A . We briefly characterize here L_1 's sales, dropping the subscript 1 for expositional purposes.

One-stop shoppers are willing to patronize L if:

$$t \leq v_{AL} - \frac{x}{\sigma} \iff x \leq \sigma(v_{AL} - t),$$

where $v_{AL} = w_{AL} - r_{AL}$, and prefer this to patronizing S as long as:

$$v_{AL} - \frac{x}{\sigma} \geq v_S = w_S - r_S \iff x \leq \hat{x} \equiv \sigma(v_{AL} - v_S).$$

The potential one-stop shoppers are thus the consumers for whom:

$$x \leq x_{AL}(t) \equiv \sigma(v_{AL} - \max\{t, v_S\}).$$

Consumers however prefer multi-stop shopping to patronizing L only if:

$$t \leq \tau = v_S - w_L + r_L,$$

and prefer this to buying B_S if:

$$t \leq v_A - \frac{x}{\sigma} \iff x \leq x_A(t) \equiv \sigma(v_A - t),$$

where $v_A \equiv w_A - r_A$. Therefore, as long as L attracts some one-stop shoppers ($v_{AL} > v_S$) and S attracts some multi-stop shoppers ($\tau > 0$), then (see Figure 1):

- consumers with $t < \tau$ buy A from L and B_S from S if $x < x_A(t)$ (region D_{A_1S}), and only B_S otherwise;
- consumers with $\tau < t < v_{AL}$ and $x < x_{AL}(t)$ buy both A and B_L from L (region $D_{A_1L_1}$), and otherwise buy either B_S only (if $t \leq v_S$) or nothing (if $t > v_S$).

The demands for assortments A_2L_2 and A_2S can be obtained by symmetry and are also portrayed in Figure 1.

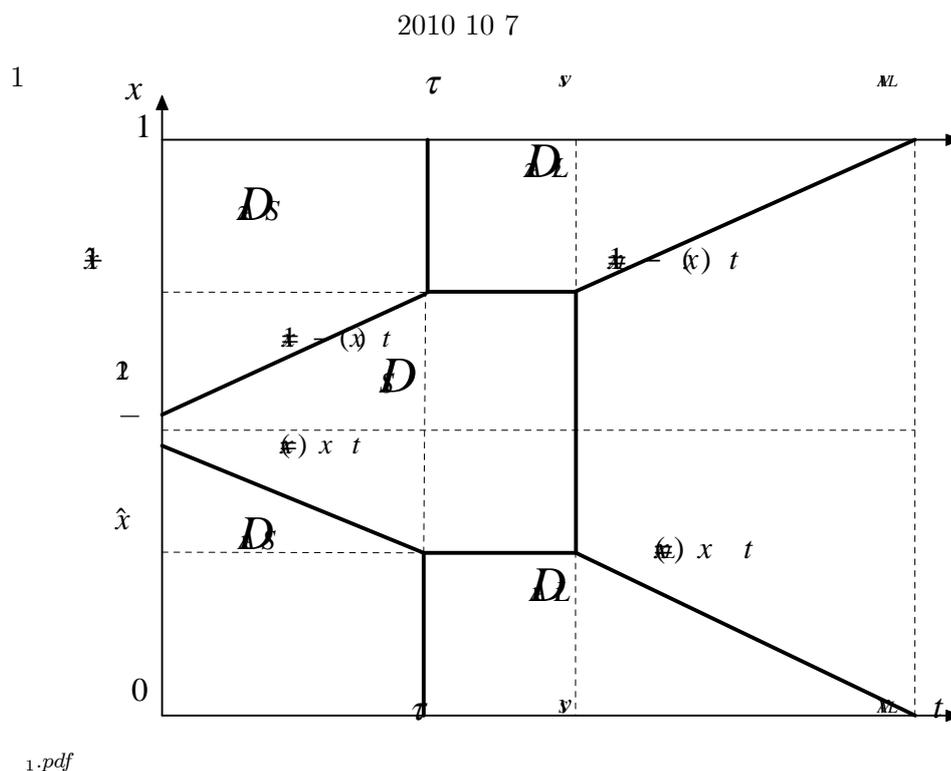


Figure 1: Local monopolies

This description applies as well when the shopping cost t is bounded, truncating if needed the relevant interval for t . For example, if the shopping cost is distributed over $[0, T]$, where $T < v_S$, then all consumers are willing to buy B_S from S ; therefore, market B is always entirely

served, by either a small or a large retailer. In addition, some consumers (those with a higher taste for A and/or lower shopping cost) will also buy A from L . More precisely, a consumer will buy A from L when $x \leq x_A(t)$ if $t < \tau$, and when $x \leq x_{AL}(t) (< x_A(t))$ if $t \in [\tau, T]$ (in which case it will also buy B_L from L).

We show in Appendix F that, in all these cases, introducing an elastic demand does not preclude large retailers from adopting a loss-leading strategy, so as to extract additional surplus from multi-stop shoppers:

Proposition 5 *Suppose that large retailers are so differentiated (σ small) that they do not compete against each other, but only compete against their smaller rivals. Then, as long as they attract some one-stop shoppers in equilibrium, the large retailers adopt a loss-leading pricing strategy to exploit extra surplus from multi-stop shoppers.*

Proof. See Appendix F. ■

As before, keeping constant the total price for the assortment AB_L offered to one-stop shoppers, subsidizing B_L allows a large retailer to increase the price it charges to multi-stop shoppers on market A . By contrast with the previous case, however, increasing the price for A not only discourages multi-stop shopping, but also results in fewer sales, since the demand for A is now elastic. Yet, the analysis shows that multi-stop shoppers' demand is relatively less price-sensitive and, as a result, subsidizing B to increase the price of A remains a profitable strategy. More precisely:

- In the range $t \in [0, \tau]$, the marginal consumer is a multi-stop shopper located at $x = x_A(t) = \sigma(v_A - t)$; an increase in the relevant margin r_A thus generates a loss $-\sigma g(x_A(t))$ but increases the profit achieved on the mass $G(x_A(t))$ of consumers that actually buy: thus, if the retailers could charge customized margins, tailored to the shopping cost, they would adopt $r_A(t) = G(x_A(t)) / \sigma g(x_A(t)) = k(x_A(t)) / \sigma$.
- Similarly, in the range $t \in [\tau, v_{AL}]$, the marginal consumer is a one-stop shopper located at $x = x_{AL}(t) = \sigma v_{AL} - \sigma \max\{t, v_S\}$ and the optimal customized margin would thus be $r_{AL}(t) = k(x_{AL}(t)) / \sigma$.

By construction, $x_A(\cdot)$ and $x_{AL}(\cdot)$ decrease as t increases³⁸ (and coincide for $t = \tau$ – see Figure 1); the monotonicity of the hazard rate thus implies that the retailers want to charge

³⁸That is, consumers who face a higher shopping cost are less likely to buy and/or to visit multiple stores.

higher margins to multi-stop shoppers ($t < \tau$) than to one-stop shoppers ($t > \tau$), which requires subsidizing B_L .

6.2 Competition among large retailers

We now turn to (symmetric) equilibria in which the large retailers compete against each other as well as against their smaller rivals. Large retailers may then compete for one-stop and/or for multi-stop shoppers. In the former case, in a symmetric equilibrium (of the form $r_{A_1L_1} = r_{A_2L_2} = r_{AL}$ and $r_L = r_{L_2} = r_L$) some consumers (with $x = 1/2$) are indifferent between buying both goods from either L_1 or L_2 , and prefer doing so to patronizing S only; this implies (using $x = 1/2$, and dropping the subscripts 1 and 2 for ease of exposition):

$$\hat{v}_{AL} \equiv v_{AL} - \frac{1}{2\sigma} \geq v_S,$$

which is equivalent to:

$$\hat{v}_A \equiv v_A - \frac{1}{2\sigma} \geq \tau = v_S - v_L.$$

Therefore, consumers with preference $x = 1/2$ and shopping cost $t < \tau$, who thus prefer multi-stop shopping (that is, buying B_S from S and A from either L_1 or L_2) to visiting L_1 or L_2 only, also prefer multi-stop shopping to patronizing S only (since $t < \tau$ then implies $t < \hat{v}_A$). In other words, if large retailers compete for one-stop shoppers, they will also compete for multi-stop shoppers. This observation allows us to classify the (symmetric) candidate equilibria in two types:

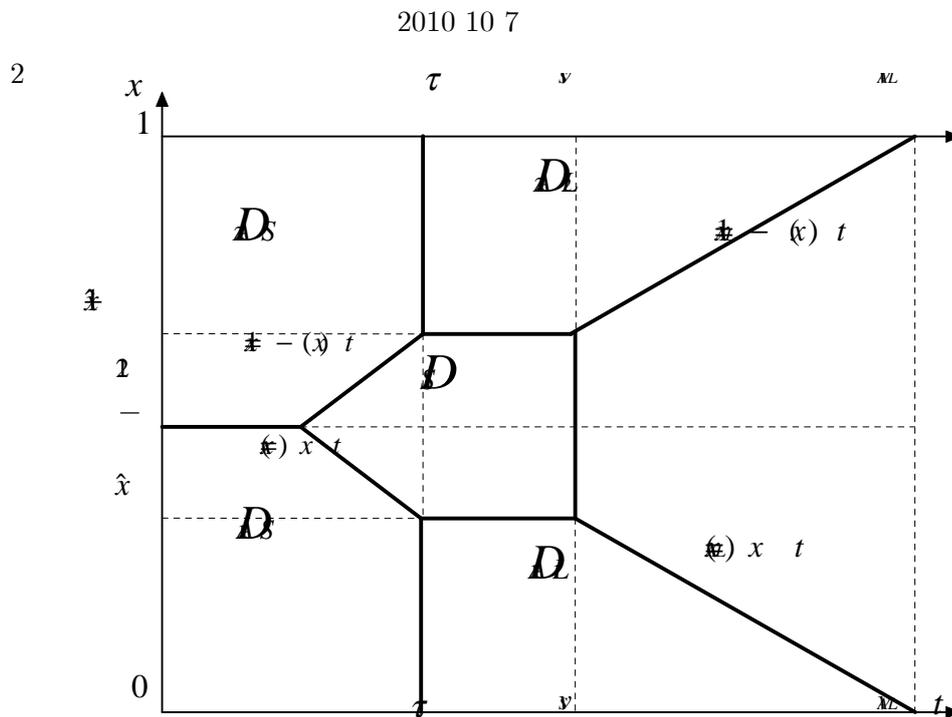
- Type M : large retailers compete only for *multi-stop* shoppers;
- Type O : large retailers compete for *one-stop* shoppers as well as for multi-stop shoppers.

In the first type of equilibria (which is illustrated in Figure 2), for $x = 1/2$ some consumers with low shopping costs are indifferent between assortments A_1S and A_2S , and prefer those assortments to any other option, whereas consumers with higher shopping costs patronize S only; the relevant threshold for the shopping cost is such that

$$\hat{v}_A + v_S - 2t = v_S - t,$$

that is, $t = \hat{v}_A$. Consumers with $t < \hat{v}_A$ thus buy B from S and A from either L_1 or L_2 (depending on x is smaller or higher than $1/2$). Conversely, consumers whose shopping cost exceeds v_{AL} buy nothing. As for consumers whose shopping cost lies between \hat{v}_A and v_{AL} :

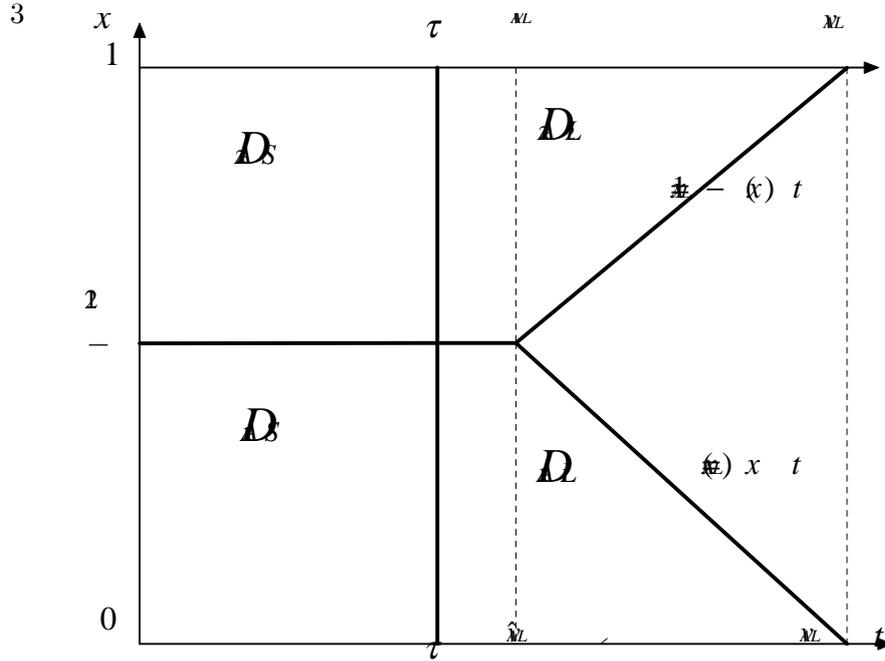
- when $t < \tau$, consumers still buy B_S from S ; they also buy A from L_1 if $x < x_A(t) = \sigma(v_A - t)$, or from L_2 if $x > 1 - x_A(t)$;
- when $t > \tau$:
 - if $x < x_{AL}(t)$, consumers buy both goods from L_1 ;
 - if $x > 1 - x_{AL}(t)$, consumers buy both goods from L_2 ;
 - if $x_{AL}(t) < x < 1 - x_{AL}(t)$, consumers patronize S if $t < v_S$, and buy nothing otherwise.



^{2.pdf} Figure 2: Large retailers competing for multi-stop shoppers

In the second type of equilibria (as demonstrated in Figure 3), all consumers with a shopping cost $t < \tau$ buy B_S from S and A from either L_1 (if $x < 1/2$) or L_2 (if $x > 1/2$), while consumers with $t > v_{AL}$ buy nothing. For consumers with $\tau < t < v_{AL}$, then:

- if $t < \hat{v}_{AL}$, consumers will buy both goods from either L_1 (if $x < 1/2$) or L_2 (if $x > 1/2$);
- if $\hat{v}_{AL} < t < v_{AL}$, consumers will buy both goods from L_1 if $x < x_{AL}(t)$ or from L_2 if $x > 1 - x_{AL}(t)$, and buy nothing otherwise.



3.pdf
Figure 3: Large retailers competing for both types of consumers

A similar description applies when the shopping cost t is bounded, truncating as needed the interval for t . We show in the Appendix that loss leading is still used as an exploitative device:

Proposition 6 *Suppose that large retailers are close enough substitutes (σ large) to compete against each other as well as against their smaller rivals. Then, large retailers adopt a loss-leading pricing strategy in any symmetric equilibrium in which they attract some one-stop shoppers.*

Proof. See Appendix G. ■

While competition here limits large retailers' margins (on A as well as on the assortment AL), subsidizing B still allows them to better discriminate consumers according to their shopping costs: pricing B_L below cost, and increasing the price of A so as to maintain r_{AL} unchanged, does not affect one-stop shoppers, who are still willing to buy A , but allows large retailers to extract more surplus from multi-stop shoppers who only buy product A from them. While this may also encourage some multi-stop shoppers to switch to the other large retailer as well as to stop buying A , the analysis shows that multi-stop shoppers remain less price-sensitive than one-stop shoppers; as a result, large retailers aim again at charging greater margins on them, and the

loss-leading strategy remains profitable. The use of loss leading as an exploitative device thus appears quite robust in market environments where large retailers compete imperfectly against each other and face smaller rivals who are more efficient in distributing a narrower range of products.

7 Applications

7.1 Competition versus acquisition

In practice, the retail chains operating large stores have often entered smaller scale grocery retailing markets, either by setting-up their own discount or specialist stores or by merging with existing chains of small stores. For instance, the French leading retailer, Carrefour, has created the discount chain LeaderPrice, which provides a short range of staples with lower prices and competes face to face with traditional discounters such as Lidl in some local markets, and more recently has started to open smaller stores (under the name "Carrefour city" and "Carrefour markets"). We analyze here the impact of such entry on retail competition, by assuming that L can either open (at no cost) a smaller but more efficient format similar to S , or acquire such a store. We consider several initial situations.

In local markets where L faces a competitive fringe of small rivals, opening yet another store would have no effect on firms' profits and consumer surplus. By contrast, in markets where L initially enjoys a monopoly position, opening a smaller store generates extra profit through a better screening of consumers. As long as L enjoys a comparative advantage for one-stop shoppers (i.e., $w_{AL} > w_S$), it is optimal to induce them to patronize L , and use S to cater to multi-stop shoppers. The total profit is then of the form:

$$\Pi_L + \Pi_S = r_{AL} (F(v_{AL}) - F(\tau)) + (r_A + r_L) F(\tau),$$

which, using $r_A = r_{AL} - r_L$, can be rewritten as:

$$\Pi_L + \Pi_S = r_{AL} F(v_{AL}) + (r_S - r_L) F(\tau).$$

It is thus optimal to charge $r_{AL} = r_{AL}^m$ and $r_L - r_S = -h(\tau^*)$, where $\tau^* = l^{-1}(w_S - w_L)$, and in this way L and S generate a joint profit equal to $\Pi_L^* = \Pi_{AL}^m + h(\tau^*) F(\tau^*)$.³⁹ Since it does

³⁹This profit corresponds to what L would obtain when facing a competitive fringe of small stores, provided it benefits from a large enough comparative advantage (namely, if $v_{AL}^m \geq w_S$); otherwise competition would partly dissipate this profit.

not not affect the value of one-stop shopping, but transforms some consumers into multi-stop shoppers, opening the small store enhances consumer surplus and total welfare as well as it improves profit.

In local markets where instead L faces a single small store, then it adopts again a loss-leading strategy if its comparative advantage is large enough (namely, if $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$), but only obtains in this way $\hat{\Pi}_L^* = \Pi_{AL}^m + h(\hat{\tau}^*)F(\hat{\tau}^*)$, as the strategic response of S reduces the extra profit that L can extract from multi-stop shoppers. Opening a small store to compete head to head with S would reduce the margin r_S down to zero, thus restoring L 's ability to extract $h(\tau^*)F(\tau^*)$ from multi-stop shoppers. However, as the resulting competition may also constrain L 's pricing policy towards one-stop shoppers (if $w_{AL} < l(w_S)$, L must lower its total margin below r_{AL}^m so as to match the value that one-stop shoppers would get from S), this is profitable only when L 's comparative advantage is strong enough.⁴⁰ As the competition fosters multi-stop shopping (the shopping cost threshold increases from $\hat{\tau}^*$ to τ^*), and can only have a positive impact on one-stop shopping, it also enhances consumer surplus as well as total welfare. Alternatively, L may instead acquire S , in which case L and S could together generate again a total profit of Π_L^* . This scenario is equivalent to opening a new store if L 's comparative advantage is particularly large (namely, $w_{AL} \geq l(w_S)$), otherwise the merger is more profitable as it avoids the competitive constraint on the price charged to one-stop shoppers. In both cases, however, consumers and society would benefit from such a merger, which would again foster one-stop shopping (if $w_{AL} < l(w_S)$, then consumers and society would however benefit even more from the opening of an additional store competing with S).

The following proposition summarizes this discussion:

Proposition 7 *In local markets in which there is initially imperfect competition in the B segment, then whenever it enjoys a large enough comparative advantage, the large retailer can benefit from either opening or acquiring a smaller but more efficient store, and this also enhances consumer surplus and total welfare.*

7.2 Complementary goods and adoption costs

While we have focused here on the case where A and B are independent goods or partial substitutes, the analysis applies also – even more straightforwardly – to the case of complements.

⁴⁰This is clearly the case when $w_{AL} \geq l(w_S)$, since then $v_{AL}^m = l^{-1}(w_{AL}) \geq w_S$ and L thus obtains $\Pi_L^* = \Pi_{AL}^m + h(\tau^*)F(\tau^*)$ when smaller rivals charge $r_S = 0$; by continuity, this is still the case when w_{AL} is not excessively lower than $l(w_S)$.

Suppose for example that A is a prerequisite for using B (as in the case of CD players and speakers): product B has no value on a stand-alone basis ($u_L = u_S = 0$), and must be used together with product A (with $w_{AS} = u_{AS} - c_A - c_S > w_{AL} = u_{AL} - c_A - c_L$).⁴¹ Denoting by w_S (resp. w_L) the additive value for using B_S (resp. B_L) on top of product A , the above analysis goes through, except that one-stop shoppers necessarily favor L (since there is no value in patronizing S only). Regime L thus systematically prevails and, as a result, L always engages in loss leading: it charges the monopoly margin r_{AL}^m for the bundle and a negative margin, $r_L^* = -h(\tau^*)$, for B_L .

Also, while we have focused so far on retailing markets, the insights apply to industries in which the costs of adopting a technology, of learning how to use a product, of maintaining equipment, ..., play a role similar to the shopping cost that consumers incur to visit an additional store. These insights can therefore shed a new light on famous antitrust cases such as the *Microsoft* saga, in which Microsoft has been accused of excluding rivals in adjacent markets – e.g., the markets for browsers or media players. While the arguments mainly focused there on the rationality of an exclusionary conduct, our analysis suggests an alternative motivation for subsidizing or otherwise encouraging customers to adopt the platform developer's own application, to the detriment of its rivals'.

To see this, suppose that L runs a platform A and offers an application B_L that competes with a fringe of rivals' applications B_S , and consider first a simple example where: (i) A and B are perfect complements (that is, $u_A = u_L = u_S = 0$), and (ii) rivals offer a better product; ($w_{AS} > w_{AL}$), but (iii) adopting a rival application (which may involve a different environment, or switching and learning costs in case of entry) involves a cost t that varies across customers according to the distribution $F(\cdot)$. Our analysis then carries through. By construction, customers purchase either AL or AS , and favor "mix-and-match" when $t < \tau = w_{AS} - w_{AL} + r_L$; as long as L sells its application (i.e., $r_{AL} \leq w_{AL}$), it obtains a profit equal to:

$$\Pi_L = r_A F(\tau) + r_{AL} (1 - F(\tau)) = r_{AL} - r_L F(\tau).$$

L 's optimal pricing policy thus consists in charging the full price for the bundle ($r_{AL} = w_{AL}$) and to subsidize its application: $r_L = -h(\tau^*)$, where as before $\tau^* = l^{-1}(w_{AS} - w_{AL})$.

While the cost of adopting L 's application was for simplicity assumed to be constant, the insight carries over to situations where both adoption costs vary across customers, as long as

⁴¹The analysis applies irrespective of whether A generates or not a value on a stand-alone basis, as long as combining it with B generates a higher value.

adopting a rival application involves a higher cost. If for example L 's and the rivals' applications involve adoption costs at and $(1 + a)t$, then the mix-and-match threshold τ remains unchanged becomes and the analysis parallels that of asymmetric shopping costs (see the remark at the end of section 3).

Similar insights also apply to industries in which procuring several categories of products from the same supplier allows a customer to save on operating costs. For example, in its decision on the proposed merger between *Aerospatiale-Alenia* and *De Havilland*,⁴² the European Commission mentions that the new entity would benefit from being the only one to offer regional aircraft in all three relevant sizes, thus allowing "one-stop shopper" airlines to save on maintenance and spare parts as well as on pilot training and certification. To see how the analysis can be transposed in such industries, suppose for instance that L covers both segments A and B while S covers B only, that procuring both products from the same supplier involves a maintenance cost f , while dealing with different suppliers increases the maintenance cost to $f + t$, where t is customer-specific. Then, whenever active customers prefer procuring both products (e.g., because the products are complements, or because airlines cannot be viable without operating aircraft in all relevant sizes), the same analysis as above applies and L subsidizes again the competitive product (and charges for example the full value for the bundle if f is constant, or mimics the pricing policy with asymmetric shopping costs if f is proportional to t).

8 Conclusion

Large retailers, enjoying substantial market power in some local markets, often compete with smaller retailers who carry a narrower range of products in a more efficient way. We find that these large retailers can exercise their market power by adopting a loss-leading pricing strategy, which consists of pricing below cost some of the products also offered by smaller rivals, and raising the prices on the other products. In this way the large retailers can better discriminate multi-stop shoppers from one-stop shoppers – and may even earn more profit than in the absence of the more efficient rivals. Loss leading thus appears as an exploitative device, designed to extract additional surplus from multi-stop shoppers, rather than as an exclusionary instrument targeting rivals – yet, these are hurt by the conduct. We show further that banning below-cost pricing increases consumer surplus and small rivals' profits as well as social welfare.

⁴²See the decision of the European Commission of 2 October 1991 in case No. IV/M053 - *Aerospatiale-Alenia/de Havilland*.

Our analysis sheds a new light on the potential harm of loss leading and identifies the key factors underlying it: asymmetry in the product range and heterogeneity of consumers' shopping costs. While the insights are shown to be quite robust to alternative specifications of cost and demand conditions, policy measures should however also take into account potential efficiency justifications, and empirical studies are needed to assess the resulting balance.

Finally, while we mainly focus on retailing markets, our insights apply as well to industries where a firm, enjoying substantial market power in one segment, competes with more efficient rivals in other segments, and single sourcing generates customer-specific benefits. They also apply to complementary products, such as platform and applications. While some of these industries have been the subject of heated debates in cases of alleged predatory pricing or related conduct, our analysis provides an alternative rationale for below-cost pricing based on exploitation rather than exclusion.

Appendix

A Quasi-concavity of profit functions

We check here the quasi-concavity of the profit functions. In the monopoly case, it is optimal for L to choose $r_{AL} < w_{AL}$ (otherwise, it would make no profit), which yields a profit:

$$\Pi(r_{AL}) = r_{AL}F(w_{AL} - r_{AL}).$$

Differentiating with respect to r_{AL} yields:

$$\Pi'(r_{AL}) = f(w_{AL} - r_{AL})\phi(r_{AL}),$$

where the function $\phi(r_{AL}) \equiv h(w_{AL} - r_{AL}) - r_{AL}$ is strictly decreasing; therefore, the first-order condition, which boils down to $\phi(r_{AL}) = 0$, has a unique solution $r_{AL} = l(w_{AL})$ and the profit function Π is strictly quasi-concave in the relevant range $r_{AL} \leq w_{AL}$. The solution $r_{AL} = l(w_{AL})$ thus constitutes a global optimum.

In regime L , as long as $\tau = w_S - w_L + r_L - r_S$ lies between 0 and $v_{AL} = w_{AL} - r_{AL}$, L 's profit is equal to:

$$\Pi_L(r_{AL}, r_L) = r_{AL}F(w_{AL} - r_{AL}) - r_LF(w_S - w_L + r_L - r_S),$$

which is thus additively separable with respect to r_{AL} and r_L . Using the same argument as above, the terms $r_{AL}F(w_{AL} - r_{AL})$ and $-r_LF(w_S - w_L + r_L - r_S)$ are moreover quasi-concave in, respectively, r_{AL} and $-r_L$. It follows that L 's unique best response to r_S is characterized by $r_{AL}^m = h(w_{AL} - r_{AL}^m)$ and $r_L^* = -h(w_S - w_L + r_L^* - r_S)$. A similar reasoning applies to regime S . Likewise, when the small retailer is a strategic player, its best response maximizes $\Pi_S = r_SF(w_S - w_L + r_L - r_S)$, which is quasi-concave in r_S , and is thus the solution to $r_S^* = h(w_S - w_L + r_L - r_S^*)$.

B Proof of Proposition 1

We first show that, without loss of generality, we can focus on $\tau \in [0, v_{AL}]$. If $\tau > v_{AL}$ (i.e., $w_S - w_L + r_L > w_{AL} - r_{AL}$, or $r_L > r_L' \equiv (w_{AL} - r_A - (w_S - w_L))/2$), there are no one-stop shoppers: active consumers buy A from L and B_S from S , and do so as long as $2t < v_{AS}$. However, keeping r_A constant, decreasing r_L to r_L' such that $\tau' = v_{AL}$ does not affect the number of active consumers (since v_{AS} does not change), who still visit both stores as before. If instead

$\tau < 0$ (i.e., $r_L < -w_S - w_L$), there are no multi-stop shoppers: active consumers only visit L , and do so as long as $t < v_{AL}$; however, keeping r_A constant, increasing r_L to $r'_L = -(w_S - w_L)$ yields $\tau' = 0$ without affecting consumer behavior.

The optimal margins and profits for the regimes L and S are characterized in the text, and the loss-leading strategy is clearly preferable when $v_{AL}^m \geq w_S$, since it then gives L more profit than the monopolistic profit Π_{AL}^m , which exceeds the monopoly profit that can be achieved in market A only (Π_A^m): $\Pi_{AL}^m = \max_r rF(w_{AL} - r) > \max_r rF(w_A - r) = \Pi_A^m$ since $w_{AL} > w_A$. We now show that the loss-leading strategy remains more profitable when $w_{AL} \geq w_S > v_{AL}^m$, where it involves $r_L^* < 0$ and $\tilde{r}_{AL}^* = w_{AL} - w_S$. To see this, fix \tilde{r}_{AL}^* but use r_A rather than r_L as the optimization variable; the margin on B_L and the shopping cost threshold are then given by:

$$r_L = r_{AL} - r_A = w_{AL} - w_S - r_A, \tau = w_S - w_L + r_L = w_{AL} - w_L - r_A.$$

The maximum profit $\tilde{\Pi}_L^*$ can then be written as

$$\begin{aligned} \tilde{\Pi}_L^* &= \tilde{r}_{AL}^* (F(v_{AL}^*) - F(\tau^*)) + r_A^* F(\tau^*) \\ &= (w_{AL} - w_S) (F(w_S) - F(\tau^*)) + r_A^* F(\tau^*) \\ &= \max_{r_A} \{ (w_{AL} - w_S) (F(w_S) - F(w_{AL} - w_L - r_A)) + r_A F(w_{AL} - w_L - r_A) \} \\ &\geq (w_{AL} - w_S) (F(w_S) - F(w_{AL} - w_L - r_A^m)) + r_A^m F(w_{AL} - w_L - r_A^m) \\ &= (w_{AL} - w_S) (F(w_S) - F(v_A^m)) + \Pi_A^m. \end{aligned}$$

Since $w_S > v_{AL}^m = l^{-1}(w_{AL}) > l^{-1}(w_{AL} - w_L) = v_A^m$, it follows that $\tilde{\Pi}_L^* \geq \Pi_A^m$ whenever $w_{AL} \geq w_S$.

Conversely, when $w_{AL} < w_S$, we have:

$$\begin{aligned} \tilde{\Pi}_L^* &= (w_{AL} - w_S) (F(w_S) - F(w_{AL} - w_L - \tilde{r}_A^*)) + \tilde{r}_A^* F(w_{AL} - w_L - \tilde{r}_A^*) \\ &< \tilde{r}_A^* F(w_{AL} - w_L - \tilde{r}_A^*) \\ &\leq \Pi_A^m, \end{aligned}$$

where the first inequality stems from $w_S > w_{AL}$ ($> w_{AL} - w_L - \tilde{r}_A^*$).

Finally, in the limit case where $w_{AL} = w_S$, using B_L as a loss leader amounts to monopolizing product A . Notice that offering $v_{AL} = w_S$ requires $r_{AL} = w_{AL} - v_{AL} = 0$, or $r_A = -r_L$, thus the margin on A reflects the subsidy on B_L . In this case, the optimal subsidy strategy maximizes $-r_L F(\tau) = -r_L F(w_S - w_L + r_L) = r_A F(w_{AL} - w_L - r_A)$. Consumers are also indifferent between these two strategies: in both cases they face the same price for A . While the loss-leading

strategy may yield a lower price for B_L (in the monopolization scenario, L may actually stop carrying B_L), this does not affect multi-stop shoppers (who do not buy B_L from L), whereas one-stop shoppers are indifferent between buying A and B_L from L or B_S only from S .

C Proof of Proposition 2

We derive here the conditions under which the loss leading outcome, $\hat{r}_{AL}^* = r_{AL}^m$ and $\hat{r}_L^* = -\hat{r}_S^* = -h(\hat{\tau}^*)$, where $\hat{\tau}^* = j^{-1}(w_S - w_L)$, forms a Nash equilibrium, before checking that the equilibrium is unique. To attract one-stop shoppers, first L must offer a better value than S :⁴³

$$v_{AL}^m \geq \hat{v}_S^* \equiv w_S - h(\hat{\tau}^*). \quad (12)$$

This condition implies $v_{AL}^m \geq \hat{v}_S^* > \hat{v}_S^* - \hat{v}_L^* = \hat{\tau}^*$, which in turn implies $w_{AL} > w_S$:

$$w_{AL} = l(v_{AL}^m) \geq l(\hat{v}_S^*) = \hat{v}_S^* + h(\hat{v}_S^*) = w_S - h(\hat{\tau}^*) + h(\hat{v}_S^*) > w_S.$$

Second, while L has no incentive to exclude its rival, since it earns more profit than a pure monopolist, S may want to attract one-stop shoppers by reducing r_S so as to offer $v_S \geq v_{AL}^m$. Such a deviation allows S to attract all consumers (one-stop or multi-stop shoppers) with shopping costs $t \leq v_S$ and thus yields a profit $\Pi_S^d(v_S) \equiv r_S F(v_S) = (w_S - v_S) F(v_S)$. It is easy to check that the best such deviation is to offer $v_S^d = v_{AL}^m$ (or slightly above v_{AL}^m , if one-stop shoppers are indifferent between two stores in this case). To see this, note that $\Pi_S^d(v_S)$ is quasi-concave in v_S and let v_S^m denote the value of v_S . Since the candidate equilibrium margin, \hat{v}_S^* , maximizes $(w_S - w_L + \hat{r}_L^* - v_S) F(v_S)$, where $w_S - w_L + \hat{r}_L^* < w_S$, a simple revealed argument yields $v_S^m < \hat{v}_S^*$. Thus, increasing v_S further above $v_{AL}^m > \hat{v}_S^*$ would reduce S 's profit monotonically and it is then optimal for S to offer precisely $v_S^d = v_{AL}^m$, which gives S a profit equal to $\Pi_S^d(v_{AL}^m) = (w_S - v_{AL}^m) F(v_{AL}^m)$. Thus, the loss-leading outcome is immune to such a deviation if

$$\hat{\Pi}_S^* \equiv h(\hat{\tau}^*) F(\hat{\tau}^*) \geq \hat{\Pi}_S^d \equiv (w_S - v_{AL}^m) F(v_{AL}^m). \quad (13)$$

This condition can be further written as:

$$\Psi(w_{AL}; w_S) \equiv (w_S - v_{AL}^m) F(v_{AL}^m) \leq \hat{\Pi}_S^*, \quad (14)$$

⁴³As before, this is equivalent to $w_{AL} - w_L - \hat{r}_L^* = v_{AL}^m - \hat{v}_L^* \geq \hat{v}_S^* - \hat{v}_L^* = \hat{\tau}^* (> 0)$, which implies that multi-stop shoppers are indeed willing to buy A when visiting L . Moreover, this condition also implies $v_{AL}^m > \hat{v}_S^* - \hat{v}_L^* = \hat{\tau}^* (> 0)$.

where $v_{AL}^m = l^{-1}(w_{AL})$ is thus such that $v_{AL}^m + h(v_{AL}^m) = w_{AL}$. Therefore:

$$\begin{aligned} \frac{\partial \Psi}{\partial w_{AL}}(w_{AL}; w_S) &= ((w_S - v_{AL}^m) f(v_{AL}^m) - F(v_{AL}^m)) \frac{dv_{AL}^m}{dw_{AL}} \\ &= (w_S - v_{AL}^m - h(v_{AL}^m)) \frac{f(v_{AL}^m)}{1 + h'(v_{AL}^m)} \\ &= (w_S - w_{AL}) \frac{f(v_{AL}^m)}{1 + h'(v_{AL}^m)}. \end{aligned}$$

It follows that, in the range $w_{AL} \geq w_S$, $\Psi(w_{AL}; w_S)$ decreases with w_{AL} (and strictly so for $w_{AL} > w_S$). Thus, condition (13) amounts to $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$, where $\hat{w}_{AL}(w_S, w_L)$ is the unique solution to $\Psi(w_{AL}; w_S) = \hat{\Pi}_S^*$. To show that this solution exists and lies above w_S , note first that Ψ becomes negative for $w_{AL} > l(w_S)$ (since then $v_{AL}^m = l^{-1}(w_{AL}) > w_S$) and that, for $w_{AL} = w_S$, $\Psi(w_{AL}; w_S) = (w_{AL} - v_{AL}^m) F(v_{AL}^m) = \Pi_{AL}^m = \max_v (w_{AL} - v) F(v)$; since $w_{AL} > w_S - w_L + \hat{r}_L^*$, this exceeds $\hat{\Pi}_S^* = \max_\tau (w_S - w_L + \hat{r}_L^* - \tau) F(\tau)$.

Finally, in the range $w_{AL} > w_S (> w_S - \hat{v}_L^*)$, a simple revealed argument yields:

$$\hat{\tau}^* = \arg \max_v (w_S - \hat{v}_L^* - \tau) F(\tau) < v_{AL}^m = \arg \max_v (w_{AL} - v) F(v).$$

Therefore, (13), which is equivalent to:

$$v_{AL}^m \geq w_S - \frac{h(\hat{\tau}^*) F(\hat{\tau}^*)}{F(v_{AL}^m)}, \quad (15)$$

implies (12). The two conditions (12) and (13) thus boil down to $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$.

It remains to show that $\hat{w}_{AL}(w_S, w_L)$ increases with w_S . Differentiating $\hat{w}_{AL}(w_S, w_L)$ with respect to w_S yields:

$$\frac{\partial \hat{w}_{AL}}{\partial w_S} = \frac{\frac{\partial \Psi}{\partial w_S} - \frac{\partial \hat{\Pi}_S^*}{\partial w_S}}{-\frac{\partial \Psi}{\partial w_{AL}}},$$

where the denominator is positive in the relevant range whereas the numerator is equal to:

$$\begin{aligned} \frac{\partial \Psi}{\partial w_S} - \frac{\partial \hat{\Pi}_S^*}{\partial w_S} &= F(v_{AL}^m) - \frac{d(h(\hat{\tau}^*) F(\hat{\tau}^*))}{d\hat{\tau}^*} \frac{\partial \hat{\tau}^*}{\partial w_S} \\ &= F(v_{AL}^m) - \frac{1 + h'(\hat{\tau}^*)}{1 + 2h'(\hat{\tau}^*)} F(\hat{\tau}^*), \end{aligned}$$

which is positive since $v_{AL}^m > \hat{\tau}^*$.

We now show that no other equilibrium exists when $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$. First, we turn to regime S , in which one-stop shoppers patronize S ($v_{AL} < v_S$) and show that there is no such equilibrium when $w_{AL} > w_S$. In this regime, L faces only a demand $F(v_A)$ for A from multi-stop shoppers, where $v_A = w_{AL} - w_L - r_A$, and thus makes a profit equal to $r_A F(v_A)$. L could however deviate and attract one-stop shoppers by reducing r_L (keeping r_A and thus

v_A constant) so as to offer $v'_{AL} = v_S$ (or slightly above v_S). Doing so would not change the number of multi-stop shoppers, since $\tau' = v_S - v'_L = v'_{AL} - v'_L = v'_A = v_A$, and L would obtain the same margin, r_A , from those consumers. But it would now attract one-stop shoppers (those for which $v_A \leq t \leq v_{AL} = v_S$), from which L could earn a total margin $r'_{AL} = w_{AL} - v'_{AL} = w_{AL} - v_S = w_{AL} - w_S + r_S$. Since any candidate equilibrium requires $r_S \geq 0$, the deviation would be profitable when $w_{AL} > w_S$.

Second, consider the boundary between the two regimes, in which one-stop shoppers are indifferent between visiting L or S ($v_{AL} = v_S$). Note that there must exist some active consumers, since either retailer can profitably attract consumers by charging a small positive margin; therefore, we must have $v_{AL} = v_S > 0$. Suppose that all active consumers are multi-stop shoppers (in which case L only sells A while S sells B_S to all consumers), which requires $v_{AL} = v_S \leq \tau$. Applying the same logic as in the beginning of Appendix B, we can without loss of generality focus on the case $v_{AL} = v_S = \tau$. It is then profitable for L to transform some multi-stop shoppers into one-stop shoppers, by reducing its margin on B_L to $r'_L = w_L - \varepsilon > 0$ and increasing r_A by ε , so as to keep v_{AL} constant: doing so does not affect the total number of active consumers, but transforms those whose shopping cost lies between $\tau' = v_S - v'_L = \tau - \varepsilon$ and τ into one-stop shoppers. While L obtains the same margin on them (since $r'_{AL} = r_{AL}$), it now obtains a higher margin $r'_A < r_A$ on the remaining multi-stop shoppers.

Therefore, some consumers must visit a single store, and by assumption must be indifferent between visiting either store ($v_{AL} = v_S$). Suppose now some one-stop shoppers visit S . Since S can avoid making losses, we must then have $r_S \geq 0$. But then, $v_{AL} = v_S$ implies $r_{AL} = r_S + w_{AL} - w_S > 0$ and, thus, it would be profitable for L to reduce r_{AL} slightly, so as to attract all one-stop shoppers. Therefore, all one-stop shoppers must go to L if $r_{AL} > 0$. Conversely, we must have $r_S \leq 0$, otherwise S would benefit from slightly reducing its margin, so as to attract all one-stop shoppers. Therefore, in any candidate equilibrium such that $v_{AL} = v_S > 0$, either:

- There are some multi-stop shoppers (i.e. $\tau > 0$) and thus $r_S = 0$; but then, slightly increasing r_S would allow S to keep attracting some multi-stop shoppers and obtain a positive profit, a contradiction.
- Or all consumers buy both products from L , which requires $r_L \leq r_S - (w_S - w_L) \leq -(w_S - w_L) < 0$. But then, increasing r_L to $r'_L = r_S - (w_S - w_L) + \varepsilon$ and reducing r_A by the same amount (so as to keep r_{AL} constant) would lead those consumers with $t < \tau' = \varepsilon$ to buy B_S from S , allowing L to avoid granting them the subsidy r_L .

It follows that there is no equilibrium such that $v_{AL} = v_S$.

Finally, since loss leading (in which L not only offers, but actually *sells* below cost) can only arise when L sells to one-stop shoppers, which thus requires $v_{AL} \geq v_S$. But this cannot be an equilibrium when $w_{AL} < \hat{w}_{AL}(w_S, w_L)$, since: (i) in the range $v_{AL} > v_S$, the only such candidate is the above described loss-leading outcome, which requires $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$; and (ii) as just discussed, no equilibrium exists in the boundary case $v_{AL} = v_S$.

D Proof of Proposition 3

Stackelberg leadership. Suppose first that L benefits from a first-mover advantage: it sets its prices first, and then, having observed these prices, S sets its own price. Retail prices are often strategic complements, and it is indeed the case here for S in the B segment: as noted before, S 's best response, $\hat{r}_S(r_L)$, increases with r_L . Thus, in the case of "normal competition" in the B market, L would exploit its first-mover advantage by *increasing* its price for B_L , so as to encourage its rival to increase its own price and relax the competitive pressure. In contrast, here L has an incentive to *decrease* r_L even further. This leads S to decrease its own price, which allows L to raise the price for A . To see this, note that L 's Stackelberg profit from a loss-leading strategy can be written as:

$$\Pi_L^S(r_L) = \Pi_{AL}^m - r_L F(\hat{\tau}(r_L)) = \Pi_{AL}^m - r_L F(w_S - w_L + r_L - \hat{r}_S(r_L)).$$

Denoting by r_L^S the optimal Stackelberg margin and using $\hat{r}_S(\hat{r}_L^*) = \hat{r}_S^*$, we have:

$$\begin{aligned} -r_L^S F(w_S - w_L + r_L^S - \hat{r}_S(r_L^S)) &\geq -\hat{r}_L^* F(w_S - w_L + \hat{r}_L^* - \hat{r}_S(\hat{r}_L^*)) \\ &\geq -r_L^S F(w_S - w_L + r_L^S - \hat{r}_S^*), \end{aligned}$$

where the second inequality stems from the fact that \hat{r}_L^* constitutes L 's best response to r_S^* . Since $-r_L^S > 0$ and $F(\cdot)$ and $\hat{r}_S(\cdot)$ are both increasing, this in turn implies $r_L^S \leq \hat{r}_L^*$. This inequality is moreover strict, since (using $\hat{\tau}(\hat{r}_L^*) = \hat{\tau}^*$):

$$(\Pi_L^S)'(\hat{r}_L^*) = -F(\hat{\tau}^*) - \hat{r}_L^* f(\hat{\tau}^*) (1 - \hat{r}_S'(\hat{r}_L^*)) = \hat{r}_L^* f(\hat{\tau}^*) \hat{r}_S'(\hat{r}_L^*) < 0.$$

Thus, L sells the competitive product B_L further below-cost, compared with what it would do in the absence of a first-mover advantage: $r_L^S < \hat{r}_L^*$.

Entry accommodation. Suppose now that the presence of S is uncertain. To capture this possibility, assume that S must incur a fixed cost of entry, γ , which is ex ante distributed according to a cumulative distribution function $F_\gamma(\cdot)$, and consider the following timing:

- In stage 1, L chooses its prices.
- In stage 2, the entry cost is realized and S chooses whether to enter; if it enters, it then sets its own price.

If entry were certain, maximizing its Stackelberg profit would lead L to adopt r_L^S . But now, S enters only when its best response profit, $\hat{\Pi}_S(r_L)$, exceeds the realized cost γ , which occurs with probability $\rho(r_L) \equiv F_\gamma(\hat{\Pi}_S(r_L))$; L 's ex ante profit is therefore equal to:

$$\hat{\Pi}_L^S(r_L) = \Pi_{AL}^m + \rho(r_L) \Pi_L^S(r_L).$$

The optimal margin, \hat{r}_L^S , thus satisfies:

$$\rho(\hat{r}_L^S) \Pi_L^S(\hat{r}_L^S) \geq \rho(r_L^S) \Pi_L^S(r_L^S) \geq \rho(r_L^S) \Pi_L^S(\hat{r}_L^S),$$

which implies:

$$\rho(\hat{r}_L^S) \geq \rho(r_L^S).$$

Since F_γ and $\hat{\Pi}_S$ are both increasing in r_L , so is ρ and thus $\hat{r}_L^S \geq r_L^S$. This inequality is moreover strict, since:

$$\left(\hat{\Pi}_L^S\right)'(r_L^S) = \rho'(r_L^S) \Pi_L^S(r_L^S) + \rho(r_L^S) \left(\Pi_L^S\right)'(r_L^S) = \rho'(r_L^S) \Pi_L^S(r_L^S) > 0.$$

Therefore, when L 's comparative advantage leads it to adopt a loss-leading strategy, it limits the subsidy on B so as to increase the likelihood of entry: $\hat{r}_L^S > r_L^S$.

E Proof of Proposition 4

If L attracts one-stop shoppers in the absence of a ban, then it must offer $v_{AL} = v_{AL}^m > \hat{v}_S^* = w_S - \hat{r}_S^*$ and S must moreover not be tempted to deviate and attract one-stop shoppers, which boils down to $\hat{\Pi}_S^* = h(\hat{\tau}^*) F(\hat{\tau}^*) \geq \hat{\Pi}_S^d = (w_S - v_{AL}^m) F(v_{AL}^m)$. If L keeps attracting one-stop shoppers (i.e., $v_{AL} > v_S$) when loss leading is banned, then the unique candidate equilibrium is $r_{AL} = r_{AL}^m$, $r_L = 0$ and $\hat{r}_S^b = h(\tau^*)$, where $\tau^* = l^{-1}(w_S - w_L)$. But then, since S increases its price (i.e., $\hat{r}_S^b = h(\tau^*) > \hat{r}_S^* = h(\hat{\tau}^*)$), it offers less value ($v_S = \hat{v}_S^b \equiv w_S - \hat{r}_S^b < \hat{v}_S^*$), and thus L indeed attracts one-stop shoppers: $v_{AL} = v_{AL}^m > (\hat{v}_S^* >) \hat{v}_S^b$. Furthermore, as S must again offer at least $v_S = v_{AL}$ to attract one-stop shoppers, it still cannot obtain more than $\hat{\Pi}_S^d$ by deviating in this way; therefore, since S now obtains more profit ($\Pi_S^* \equiv h(\tau^*) F(\tau^*) > \hat{\Pi}_S^* = h(\hat{\tau}^*) F(\hat{\tau}^*)$), it is less tempted to deviate: $\Pi_S^* > (\hat{\Pi}_S^* >) \hat{\Pi}_S^d$. Therefore, the above candidate equilibrium is indeed an equilibrium, in which L keeps attracting one-stop shoppers and adopts a loss-leading strategy.

F Proof of Proposition 5

We focus on large retailers' strategies, taking the strategies of the smaller retailer(s) as given; thus, whether the smaller rival is a strategic player or a competitive fringe does not matter here.

A large retailer's profit can be written as (see Figure 1):

$$\Pi_L = r_{AL}D_{AL} + r_A D_{AS} = r_{AL} \int_{\tau}^{v_{AL}} G(x_{AL}(t)) f(t) dt + r_A \int_0^{\tau} G(x_A(t)) f(t) dt.$$

Optimizing this profit with respect to r_A and r_{AL} (keeping r_L constant, and using $v_{AL} = w_{AL} - r_{AL}$ and $\tau = v_S - w_L + r_L$, where $r_L = r_{AL} - r_A$) yields:

$$\begin{aligned} \int_0^{\tau} [G(x_A(t)) - \sigma r_A g(x_A(t))] f(t) dt + r_L G(\hat{x}) f(\tau) &= 0, \\ \int_{\tau}^{v_{AL}} [G(x_{AL}(t)) - \sigma r_{AL} g(x_{AL}(t))] f(t) dt - r_L G(\hat{x}) f(\tau) &= 0. \end{aligned}$$

Therefore, if in equilibrium r_L were non-negative, we would have:

$$\int_0^{\tau} [\sigma r_A - k(x_A(t))] g(x_A(t)) f(t) dt \geq 0 \geq \int_{\tau}^{v_{AL}} [\sigma r_{AL} - k(x_{AL}(t))] g(x_{AL}(t)) f(t) dt,$$

that is, r_A would exceed a weighted average of $k(x_A(t))/\sigma$ for $t \in [0, \tau]$, whereas r_{AL} would be lower than a weighted average of $k(x_{AL}(t))/\sigma$ for $t \in [\tau, v_{AL}]$. But since $k(x_A(t))$ and $k(x_{AL}(t))$ decrease as t increases ($k(\cdot)$ increases by assumption, and both $x_A(t)$ and $x_{AL}(t)$ decrease by construction), this would imply $r_A > r_{AL}$, a contradiction. Therefore, in equilibrium, $r_L < 0$.

If the shopping cost t is distributed over some interval $[0, T]$, where $T > \tau$ to ensure that large retailers still attract some one-stop shoppers, the first-order conditions become:

$$\begin{aligned} \int_0^{\tau} [\sigma r_A - k(x_A(t))] g(x_A(t)) f(t) dt &= r_L G(\hat{x}) f(\tau), \\ \int_{\tau}^{\min\{v_{AL}, T\}} [\sigma r_{AL} - k(x_{AL}(t))] g(x_{AL}(t)) f(t) dt &= -r_L G(\hat{x}) f(\tau), \end{aligned}$$

and it thus suffices to replace v_{AL} with $\min\{v_{AL}, T\}$ in the above reasoning.

G Proof of Proposition 6

Consider first (symmetric) equilibria of type M , in which large retailers compete only for multi-stop shoppers. In the absence of any bound on shopping costs, the demands for assortments $A_1 L_1$ and $A_1 S$ in such equilibrium, where $r_{A_1 L_1} = r_{A_2 L_2} = r_{AL}$ and $r_{L_1} = r_{L_2} = r_L$ (and thus $r_{A_1} = r_{A_2} = r_A$), can be expressed as:

$$D_{AS} = \int_0^{\tau} G(\hat{x}_A(t)) f(t) dt \text{ and } D_{AL} = \int_{\tau}^{v_{AL}} G(x_{AL}(t)) f(t) dt,$$

where as before $\tau = v_S - v_L$ and $x_{AL}(t) = \sigma(v_{AL} - \max\{t, v_S\})$, and $\hat{x}_A(t) \equiv \sigma(v_A - \max\{t, \hat{v}_A\}) = \min\{1/2, x_A(t) = \sigma(v_A - t)\}$. Following a small change dr in r_{A_1} , adjusting r_{L_1} by $-dr$ so as to keep $r_{A_1 L_1}$ constant, we have:

- for $t < \hat{v}_A$, the consumer indifferent between buying A from L_1 or L_2 is such that:

$$w_A - (r_A + dr) - \frac{x}{\sigma} = w_A - r_A - \frac{1-x}{\sigma},$$

or:

$$x = \frac{1}{2} - \frac{\sigma dr}{2};$$

- for $\hat{v}_A < t < \tau$, the consumer indifferent between buying A from L_1 or patronizing S becomes $x = x_A(t) - \sigma dr$;
- in addition, those consumers for which $t \in [\tau - dr, \tau]$ and $x \leq \hat{x}_A(t)$ turn to one-stop shopping and now buy B as well as A from L_1 .

Therefore, optimizing L_1 's profit with respect to $r_{A_1 L_1}$ (keeping r_{L_1} constant) yields, at a symmetric equilibrium:

$$\int_0^\tau G(\hat{x}_A(t)) f(t) dt - \int_0^{\hat{v}_A} \frac{\sigma}{2} r_A g(\hat{x}_A(t)) f(t) dt - \int_0^{\hat{v}_A} \sigma r_A g(\hat{x}_A(t)) f(t) dt + r_L G(\hat{x}) f(\tau) = 0,$$

or:

$$\int_0^\tau [\sigma r_A - \eta_A(t)] g(\hat{x}_A(t)) f(t) dt = r_L G(\hat{x}) f(\tau), \quad (16)$$

where:

$$\eta_A(t) \equiv \begin{cases} 2k(\hat{x}_A(t)) & \text{for } t < \hat{v}_A, \\ k(\hat{x}_A(t)) & \text{for } t > \hat{v}_A. \end{cases}$$

Likewise, following a small change dr in $r_{A_1 L_1}$, keeping r_{A_1} constant (and thus changing r_{L_1} by dr as well), we have:

- for $t > \tau$, the marginal (one-stop) shopper becomes $x = x_{AL}(t) - \sigma dr$;
- in addition, those consumers for which $t \in [\tau, \tau + dr]$ and $x \leq x_{AL}(t)$ become multi-stop shoppers: they stop buying B from L_1 .

We must therefore have:

$$\int_\tau^{v_{AL}} [G(x_{AL}(t)) - \sigma r_{AL} g(x_{AL}(t))] f(t) dt - r_L G(\hat{x}) f(\tau) = 0, \quad (17)$$

or:

$$\int_{\tau}^{v_{AL}} [\sigma r_{AL} - \eta_{AL}(t)] g(x_{AL}(t)) f(t) dt = -r_L G(\hat{x}) f(\tau),$$

where $\eta_{AL}(t) \equiv k(x_{AL}(t))$.

Thus, if r_L were non-negative, the two conditions (16) and (17) would imply:

$$\int_0^{\tau} [\sigma r_A - \eta_A(t)] g(\hat{x}_A(t)) f(t) dt \geq 0 \geq \int_{\tau}^{v_{AL}} [\sigma r_{AL} - \eta_{AL}(t)] g(x_{AL}(t)) f(t) dt,$$

where η_A and η_{AL} decrease as t increases, and coincide for $t = \tau$; this, in turn, would imply $r_A > r_{AL}$, a contradiction. A similar argument applies when the shopping cost t is distributed over some interval $[0, T]$.

The same approach can be used for (symmetric) equilibria of type O , in which large retailers compete as well for one-stop shoppers. In the absence of any bound on shopping costs, the demands for assortments $A_1 L_1$ and $A_1 S$ in such equilibrium can be expressed as:

$$D_{AS} = \int_0^{\tau} G(1/2) f(t) dt \text{ and } D_{AL} = \int_{\tau}^{v_{AL}} G(\hat{x}_{AL}(t)) f(t) dt,$$

where $\hat{x}_{AL}(t) \equiv \sigma(v_A - \max\{t, \hat{v}_{AL}\}) = \min\{1/2, x_{AL}(t) = \sigma(v_{AL} - t)\}$.

Following a small change dr in r_{A_1} , adjusting r_{L_1} by $-dr$ so as to keep $r_{A_1 L_1}$ constant, we have:

- for $t < \tau$, the consumer indifferent between buying A from L_1 or L_2 becomes $1/2 - \sigma dr/2$;
- in addition, those consumers for which $t \in [\tau - dr, \tau]$ and $x \leq \hat{x}_A(t)$ become one-stop shoppers.

Therefore, optimizing L_1 's profit with respect to $r_{A_1 L_1}$ (keeping r_{L_1} constant) yields, at a symmetric equilibrium:

$$\int_0^{\tau} G(1/2) f(t) dt - \int_0^{\tau} \frac{\sigma}{2} r_A g(1/2) f(t) dt + r_L G(1/2) f(\tau) = 0,$$

or:

$$\int_0^{\tau} [\sigma r_A - \hat{\eta}_A] g(1/2) f(t) dt = r_L G(1/2) f(\tau),$$

where $\hat{\eta}_A \equiv 2k(1/2)$. Likewise, following a small change dr in $r_{A_1 L_1}$, keeping r_{A_1} constant (and thus changing r_{L_1} by dr as well), we have:

- for $\tau < t < \hat{v}_{AL}$, the marginal (one-stop) shopper becomes $x = x_{AL}(t) - \sigma dr/2$;

- for $\hat{v}_{AL} < t < v_{AL}$, the marginal (one-stop) shopper becomes $x = x_{AL}(t) - \sigma dr$;
- in addition, those consumers for which $t \in [\tau, \tau + dr]$ and $x \leq \hat{x}_{AL}(t)$ become multi-stop shoppers: they stop buying B from L_1 .

We must therefore have:

$$\int_{\tau}^{v_{AL}} G(\hat{x}_{AL}(t)) f(t) dt - \int_{\tau}^{\hat{v}_{AL}} \frac{\sigma}{2} r_{AL} g(\hat{x}_{AL}(t)) f(t) dt - \int_{\hat{v}_{AL}}^{v_{AL}} \sigma r_{AL} g(\hat{x}_{AL}(t)) f(t) dt - r_L G(\hat{x}) f(\tau) = 0,$$

or:

$$\int_{\tau}^{v_{AL}} [\sigma r_{AL} - \hat{\eta}_{AL}(t)] g(\hat{x}_{AL}(t)) f(t) dt = -r_L G(\hat{x}) f(\tau),$$

where:

$$\eta_{AL}(t) \equiv \begin{cases} 2k(\hat{x}_{AL}(t)) & \text{for } t < \hat{v}_{AL}, \\ k(\hat{x}_{AL}(t)) & \text{for } t > \hat{v}_{AL}. \end{cases}$$

Thus, if r_L were non-negative, the two conditions (16) and (17) would imply:

$$\int_0^{\tau} [\sigma r_A - \hat{\eta}_A] g(1/2) f(t) dt \geq 0 \geq \int_{\tau}^{v_{AL}} [\sigma r_{AL} - \hat{\eta}_{AL}(t)] g(\hat{x}_{AL}(t)) f(t) dt,$$

and a contradiction follows, since $\hat{x}_{AL}(t) \leq 1/2$, with a strict inequality for $t > \hat{v}_{AL}$, and thus $\hat{\eta}_{AL}(t) \leq 2k(\hat{x}_{AL}(t)) \leq \hat{\eta}_A$, with again a strict inequality for $t > \hat{v}_{AL}$. A similar argument applies again when the shopping cost t is distributed over some interval $[0, T]$.

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