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Dynamic Optimal Taxation in Open Economies

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Abstract

This paper analyzes optimal capital taxation in open economies with strategic interaction in a neo-classical growth model. With a territorial or source-based tax system, I show that optimal capital taxes in steady state are zero for a large open economy, thereby generalizing the result previously established only for the special cases of a closed and a small open economy. If the steady-state assumption is relaxed, optimal capital taxes are still zero when marginal utilities of private and public consumption are bounded, or when the utility function is quasi-linear in consumption. Moreover, in the latter case the solution is also time-consistent. For the residential or world-wide tax principle, however, countries are not able to tax all factors of production, so capital income taxes are generally non-zero except in the limiting cases of a closed or small open economy. Allowing for both residential and territorial taxes restores zero capital taxes.

JEL Classification: H21, E62

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1 Introduction

The taxation of capital is an important and hotly debated policy issue, also within the economics profession. Diamond and Saez (2011) for instance argue in favor of significantly positive capital

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taxes, whereas Mankiw, Weinzierl, and Yagan (2009) contend that they should be set to zero. The argument for zero capital taxes is mostly based on the findings of Chamley (1986) and Judd (1985) for a closed economy. The world today, though, is not well described by a closed economy; capital is highly mobile between countries. Does the argument for zero capital taxes change in an open economy? Correia (1996a) extends the result to a small open economy. But the literature so far has abstracted "from interesting strategic issues that arise when more than one authority sets taxes, and [...] from general equilibrium linkages between an economy's fiscal policy and world prices." (Chari and Kehoe, 1999, p.66)

A closed and a small open economy are the limiting cases of a large open economy, where the relative mass of the home economy compared to the world economy is one and zero, respectively. If optimal capital taxes are zero in both the extremes of a closed and a small open economy, one could think that the intermediate case of a large open economy would yield the same result. But there is a reason why capital taxes could be zero only in the limiting cases: a large economy can influence the world interest rate and thus shift some of the tax burden on foreigners as well as directly tax their capital employed in the home country, see for example Ha and Sibert (1997).[1] Furthermore, the observation by (Chari and Kehoe, 1999, pp.40-41) that "zero capital income taxation in the steady state is optimal if the extra constraints do not depend on the capital stock" does not necessarily hold in a large open economy with strategic interaction.

In this paper, I take the conventional framework of a neo-classical economy and perfect commitment to analyze optimal capital taxation in a large open economy. The common primal approach of finding optimal allocations by eliminating taxes from the problem is not easily applicable here, since taxes abroad are taken as given (see the appendix for further details). I use best-response functions of a one-shot game between governments for a given belief of policies abroad to determine optimal policy. *I find that with a territorial tax system, in which capital is taxed according to where it is employed and not where it originates from, optimal capital taxes are zero in the long run. With a residential tax system, where capital taxes are paid where the owner resides, optimal capital taxes are generally not zero.*[2]

[1] They use an overlapping generations framework with time-consistent taxes and inelastic labor supply and find that corporate taxes (territorial taxes in this model) are optimally positive for capital importers and negative for exporters, in line with earlier results from static models.

[2] In this model best-response functions are not easily interpreted in steady state for residential taxes, since these need to equalize across countries in the long run, as shown by Razin and Sadka (1991). I therefore limit the analysis to symmetric countries when studying residential taxes.

The zero capital tax result hinges on a complete tax system, i.e. that all factors of production can be taxed, see for instance Correia (1996b) and Jones, Manuelli, and Rossi (1997). This condition is not met when taxes are paid according to where the owner resides (as foreign capital investment cannot be taxed) and the optimal tax is indeed no longer zero in this case. When one allows for both residential and territorial taxes at the same time, the tax system is complete again and effective capital taxes are zero.[3] Neither territorial nor residential taxes need to be zero, but the after-tax returns are optimally set to be equal to the pre-tax returns. That is, as long as governments can levy source-based taxes (whether in combination with other taxes or not), the zero-tax result holds in an open economy.

The interaction between governments is modeled as a one-shot generalized game. In consequence, only the equilibrium is feasible. In order to allow off-equilibrium behavior to be feasible, I propose an alternative framework where each country's labor taxes (or capital taxes) adjust to balance the government budget constraint for given bond issuance and capital tax (or labor tax) decisions. Capital taxes are still zero in the long run, even when other governments do not behave optimally. This framework also permits to consider mixed-strategy equilibria; these are all characterized by zero capital taxes. To ensure that governments are not restricted by the Laffer curve, government expenditures can be endogenized (so that the set of feasible strategies is always non-empty for all foreign strategies) without changing results.

Results are robust with respect to (i) the size of the economy compared to the rest of the world and the number of countries; (ii) differences between countries regarding the production technology, government outlays, and utility function parameters;[4] (iii) the inclusion of capital adjustment costs, which one could interpret as barriers to capital mobility; (iv) agent heterogeneity regarding initial wealth and labor productivity, coupled with non-linear labor income taxation; (v) the availability of consumption taxes.

The steady-state assumption can also be relaxed: Following Judd (1999), I show that if the marginal utilities of public and private consumption are bounded, then average intertemporal distortions will be zero in the long run. Another alternative to a steady state is to assume that the marginal utility of consumption cannot be influenced, for example with a quasi-linear utility

[3]This corresponds to the empirically relevant case where corporate income taxes are source-based and capital income taxes are residence-based.

[4]The time discount factor is an exception. When discounting differs across countries, a steady state or stable long-run average does not exist.

function. In this case the problem of time inconsistency, as pointed out by Kydland and Prescott (1977), also disappears.

In contrast to the tax competition literature, starting with Wilson (1986), zero capital taxes are not caused by a race to the bottom, since they are independent of the size of the country or the barriers to capital mobility. In a related paper, Gross (2012), I discuss tax competition in a neo-classical framework of optimal taxation in detail. I study cases where it is optimal to tax capital in a closed economy and how these optimal taxes are affected when the economy is open. I find that with an endogenous capital stock, capital in the long run is an intermediate good, and therefore does not generate any rents which a government would like to capture. If however the global capital stock is exogenously given, as commonly assumed in the literature, then it is optimal to tax capital as much as possible in a closed economy (this is equivalent to a lump-sum tax) and the degree of openness limits a government's ability to capture the rents from capital.

The organization of the rest of the paper is as follows: in the next section I set up the model and provide a proof of the zero capital tax result in a large open economy. The third section discusses the robustness of the result. In the subsequent section, I analyze residential taxation and a hybrid system. The last section concludes. The appendix discusses the primal approach and consumption taxes in a large open economy and contains proofs.

2 Territorial tax system

The model consists of two countries,[5] between which capital can be freely shifted, whereas labor is immobile. Barriers to capital mobility do not affect results, see section 3.6. Without loss of generality, I abstract from population and productivity growth. A stationary transformation is possible here, as in Mendoza and Tesar (2005). As in their paper, exogenous growth means that tax policy can only have level effects but does not affect long-term growth. For simplicity, depreciation is not considered but could easily be incorporated. I will first analyze one country and discuss some of the conditions that have to hold in the other country later. Variables with a star denote foreign quantities or prices. In this section, I first briefly describe the agents of the model, then define the game between governments and agents, and last prove that optimal capital taxes are zero in steady state.

[5] It is easy to extend the model to any number of countries. For ease of exposition, I will limit myself to the case of two countries.

2.1 Agents

The setup is standard, there is a continuum of identical households[6] and firms and a government populating each country. Heterogeneous agents who differ in labor productivity and wealth do not affect results, see section 3.7.

2.1.1 Households

The representative agent takes prices as given and maximizes lifetime utility over an infinite horizon:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{1}$$

where $u(c_t, l_t)$ is a well-behaved utility function over consumption c_t and leisure l_t. $\beta \in (0,1)$ is the discount factor. It should be noted that most parameters can differ between countries, but the discount factor has to be the same for a steady state to exist. For now I assume that a steady state does exist, as in Atkeson, Chari, and Kehoe (1999) and Jones, Manuelli, and Rossi (1997). This assumption can be relaxed - see sections 3.1 and 3.2. The household has one unit of time at its disposal every period, which can be used for labor n_t and leisure. The per-period budget constraint is:

$$\begin{aligned} c_t =& (1-\tau_t^n) w_t n_t + [1 + (1-\tau_t^k) r_t] k_t - k_{t+1} + \\ & [1 + (1-\tau_t^{k*}) r_t^*] a_t - a_{t+1} + (1+R_t) b_t - b_{t+1}. \end{aligned} \tag{2}$$

b_t are government bonds and R_t is the interest rate on them.[7] k_t is the amount of capital owned domestically, a_t is the capital that the household owns abroad, w_t and r_t are the domestic wage and interest rate. a_0, k_0, and b_0 are exogenously given. Finally, τ_t^n and τ_t^k are the tax rates on wages and capital, respectively. Taxes on capital are territorial, that is they are paid where the capital is employed, independent of the citizenship of the owner.

Optimal behavior implies a no-arbitrage condition, that the returns on government bonds and

[6] Without loss of generality I assume that the measure of households is one for the home country. The relative size of the other country is χ.

[7] For notational simplicity, I assume that the income on government bonds remains untaxed; in this context, it is equivalent to the assumption that all bonds are held domestically. Allowing them to be held by foreigners (and that income on bonds is taxed) does not change results.

capital in both countries must be equal after taxes,

$$R_{t+1} = r^*_{t+1}(1 - \tau^{k*}_{t+1}) = r_{t+1}(1 - \tau^k_{t+1}), \tag{3}$$

as well as the familiar conditions concerning the trade-off between consumption versus leisure and consumption today versus tomorrow:

$$u_l(t) = u_c(t)(1 - \tau^n_t)w_t \tag{4}$$

$$u_c(t) = \beta u_c(t+1)[(1 - \tau^k_{t+1})r_{t+1} + 1] \tag{5}$$

$$u_c(t) = \beta u_c(t+1)[(1 - \tau^{k*}_t)r^*_{t+1} + 1]. \tag{6}$$

Subscripts refer to derivatives with respect to that variable, e.g. $u_c(t)$ is the derivative of the utility function with respect to consumption at time t.

The foreign agent's household optimization problem is a mirror image of the domestic one's. a^*_t is the total amount of capital employed abroad that is owned by foreigners and k^*_t is capital from abroad that is in use domestically. In order to keep all variables in per-capita terms, I divide them by country size χ.

$$\begin{aligned} c^*_t/\chi =& (1 - \tau^{n*}_t)w^*_t n^*_t/\chi + [1 + (1 - \tau^{k*}_t)r^*_t]a^*_t/\chi - a^*_{t+1}/\chi + \\ & [1 + (1 - \tau^k_t)r_t]k^*_t/\chi - k^*_{t+1}/\chi + (1 + R^*_t)b^*_t/\chi - b^*_{t+1}/\chi. \end{aligned} \tag{7}$$

2.1.2 Firms

Output is produced by a representative firm with the private inputs labor n_t and capital $K_t = k_t + k^*_t$ according to a production function $f(K, n)$ with constant returns to scale that satisfies the Inada conditions (total capital employed abroad is given by $A^*_t = a_t + a^*_t$). Output is the numeraire and is used for consumption, capital, and government expenditures. The firm rents capital from private agents (no matter from which country) and hires labor from domestic citizens, taking factor prices as given. The maximization of profit, along with constant returns to scale implies zero profits and the following remunerations for the inputs:

$$r_t = F_K(t) \tag{8}$$
$$w_t = F_n(t). \tag{9}$$

2.1.3 The Government

The benevolent government's objective is to maximize the utility of its citizens. It needs to finance an exogenous stream of unproductive expenditures $\{g_t\}_\infty$, which converges to a constant g after some finite time to allow for a steady state. Revenue is generated by distortionary taxes on capital earnings τ_t^k and wages τ_t^n. Capital taxes have to be announced one period in advance, so τ_0^k is given.[8] I assume that the government is always able to finance its expenditures with these taxes and also by labor taxes alone, i.e. that it is not at a corner solution. This assumption is relaxed in section 3.5 when government expenditures are endogenous. The government may trade in one-period bonds, with b_t denoting the total outstanding government debt. A no-Ponzi condition has to hold to avoid an infinite debt build-up. The government's per-period budget constraint can be written as

$$g_t + b_t(1 + R_t) = \tau_t^k r_t K_t + \tau_t^n w_t n_t + b_{t+1}. \tag{10}$$

2.2 Game between Agents

As is standard in the taxation literature, the government moves first and announces its tax schedule, whereupon households (and firms) react to this. Each single household and firm has measure zero, so a competitive equilibrium defines the second stage. In an open economy, there are not only domestic households, but also their counterparts abroad, which are affected by domestic policies. Agents in both countries act simultaneously.

The first stage consists of a one-shot game between the two governments which announce their tax schedules at the same time. This sub-game can be seen as a generalized game (much like the second stage if the competitive equilibrium is viewed as the outcome of a game between infinitely

[8] The government is otherwise potentially able to finance its complete stream of expenditures by expropriating the initial capital stock. This acts as a lump-sum tax, since capital at time zero is taken as given and was accumulated without regard to the newly announced tax structure. However, even if one allowed for lump-sum taxes, optimal steady state capital taxes would not be affected (although the result then becomes trivial).

many agents) in the spirit of Debreu (1952).[9] That is, feasible strategies depend on the actions taken by the other player. The government chooses its bond issues and taxes for all periods subject to its budget constraint. Foreign taxes and bond issues are taken as given (that is, each government decides upon its optimal plan of action for a given belief of what the other does). All other variables (the households' and firms' choices) are reaction functions to the tax schedule announced by the home and foreign government.

Definition 1 (Admissible Policy) *An admissible policy is a sequence of capital taxes* $\{\tau_t^k\}_{t=1}^{\infty}$*, labor taxes* $\{\tau_t^n\}_{t=0}^{\infty}$*, and bond issues* $\{b_t\}_{t=1}^{\infty}$ *for a given belief of foreign policy* $\{\tau_{t+1}^{k*}, \tau_t^{n*}, b_{t+1}^*\}_{t=0}^{\infty}$ *such that the government budget constraint holds every period and that the allocations of consumption, labor supply, and bond and capital holdings at home and abroad satisfy*

1. *agents maximizing utility subject to their budget constraints, taking prices and taxes as given;*
2. *firms maximizing profits, taking prices as given.*

The government can consider the households' choice variables as control variables when it incorporates the optimality conditions and budget constraints of the households.[10] I can therefore reformulate the government's problem as a social planner's function, subject to a set of constraints as laid out above. These are the government budget constraint (10), the household's budget constraint (2), and the household's conditions (4), (5) and (6). Additionally, it is kept in mind that input prices r and w are equal to their marginal products, taking into account firms' profit maximization. These conditions have to be satisfied not only for the home country, but also abroad (except for the foreign government's budget constraint). The set of control variables is

$$X = \{c_t, c_t^*, n_t, n_t^*, k_{t+1}, k_{t+1}^*, a_{t+1}, a_{t+1}^*, b_{t+1}, \tau_{t+1}^k, \tau_t^n\}_{t=0}^{\infty}. \tag{11}$$

The government's Lagrangean (with multipliers ψ, θ, μ, ζ and γ respectively for the constraints mentioned above, with an asterisk for the constraints abroad) is then (not showing the transversality

[9] The original paper considers only finite spaces, but a more general proof is, for instance, provided in Tian (1992).

[10] I assume an interior solution to the household's problem here: This is true when the utility function is strictly increasing and concave in both of its arguments, consumption and leisure, when the production function satisfies the Inada conditions, and when taxes are restricted to be bounded and strictly smaller than one. I also assume that initial government debt and the stream of government expenditures are low enough so that the government is able to finance its expenditures at any time through labor taxes, so that it is not restricted by the top of the Laffer curve. Instead of assuming this, one can also endogenize government expenditures, see section 3.

conditions)

$$
\begin{aligned}
L = &\sum_{t=0}^{\infty} \beta^t \{u(c_t, l_t) \qquad (12)\\
&+\psi_t[\tau_t^k r_t K_t + \tau_t^n w_t n_t - b_t(1+(1-\tau_t^k)r_t) + b_{t+1} - g_t]\\
&+\theta_t[(1-\tau_t^n)w_t n_t + [1+(1-\tau_t^k)r_t]k_t - k_{t+1} + (1+(1-\tau_t^{k*})r_t^*)a_t - a_{t+1}+\\
&\quad (1+(1-\tau_t^k)r_t)b_t - b_{t+1} - c_t]\\
&+\mu_t[u_c(t)(1-\tau_t^n)w_t - u_l(t)]\\
&+\zeta_{t|t>0}\left[(r_t(1-\tau_t^k)+1) - u_c(t-1)/(\beta u_c(t))\right]\\
&+\gamma_{t|t>0}\left[(r_t^*(1-\tau_t^{k*})+1) - u_c(t-1)/(\beta u_c(t))\right]\\
&+\theta_t^*/\chi[(1-\tau_t^{n*})w_t^* n_t^* + (1+(1-\tau_t^{k*})r_t^*)a_t^* - a_{t+1}^* + (1+(1-\tau_t^k)r_t)k_t^* - k_{t+1}^*+\\
&\quad (1+(1-\tau_t^{k*})r_t^*)b_t^* - b_{t+1}^* - c_t^*]\\
&+\mu_t^*[u_c^*(t)(1-\tau_t^{n*})w_t^* - u_l^*(t)]\\
&+\zeta_{t|t>0}^*\left[(r_t^*(1-\tau_t^{k*})+1) - u_c^*(t-1)/(\beta u_c^*(t))\right]\\
&+\gamma_{t|t>0}^*\left[(r_t(1-\tau_t)^k+1) - u_c^*(t-1)/(\beta u_c^*(t))\right]\}.
\end{aligned}
$$

Optimal policy for each government is when it chooses the best action that is admissible for any belief of what the other does:

Definition 2 (Optimal Response Function) *An optimal response function is the admissible policy maximizing the agent's discounted lifetime utility for each belief of foreign policy.*

A strategy specifies the action taken at each information node of a game; since it is a one-shot game, a strategy corresponds to choosing a policy. Following from this definition, I will now turn to an equilibrium for an open economy. For the moment I only consider pure strategies, since off-equilibrium feasibility is not guaranteed (see below). In section 3.4 I discuss how the game can be defined differently, so that off-equilibrium behavior is feasible, allowing for mixed-strategy equilibria. In section 3.5 I endogenize government spending to ensure that an optimal response always exists.

Definition 3 (Pure Strategy Open Economy Equilibrium) *A pure strategy open economy equilibrium is a sequence of prices* $\{w_t, r_t, w_t^*, r_t^*, R_t, R_t^*\}_{t=0}^{\infty}$*, government policies* $\{\tau_t^n, \tau_t^{n*}, \tau_{t+1}^k, \tau_{t+1}^{k*}, b_{t+1}, b_{t+1}^*\}_{t=0}^{\infty}$*,* *and allocations* $\{c_t, c_t^*, n_t, n_t^*, k_{t+1}, k_{t+1}^*, a_{t+1}, a_{t+1}^*\}_{t=0}^{\infty}$ *that follows from a set of equilibrium strategies and private sector equilibrium conditions such that:*[11]

1. *each government's equilibrium strategy is an optimal response to the other governments' equilibrium strategy;*

2. *agents in all countries choose consumption, labor supply, and bond and capital holdings to maximize their utility subject to their budget constraint, taking prices and taxes as given;*

3. *firms in all countries maximize profits, taking prices as given.*

The set of admissible policies depends on the beliefs of the other player's policy. An admissible policy for one government for a given belief of the other government's policy can result in the latter not being admissible. The corresponding allocation is thus not feasible, i.e. it does not satisfy the global resource constraint. This is not a problem from a game-theoretic point of view, it is a generalized game as in Debreu (1952) and preferences are naturally defined over non-feasible outcomes. However, in equilibrium beliefs and actual choices coincide, so equilibrium actions are always feasible. In more technical terms, let $x_i = \{b_{t+1}, \tau_{t+1}^k, \tau_t^n\}_{t=0}^{\infty}$ be an action for country i. Call $f_i(x_j)$ the set of admissible actions for country i when country j plays x_j. Let $x_i(x_j)$ be the optimal response function for country i given that it believes country j will play x_j. In equilibrium, beliefs and actual choices are equal; let $x_i^* = x_i(x_j^*)$ be the equilibrium action. If country j plays some $\hat{x}_j \in f_j(x_i^*)$, then x_i^* is probably not optimal anymore. However, it is also likely that it is not admissible (and thus feasible) anymore, i.e. that $x_i^* \notin f_i(\hat{x}_j)$. Since equilibrium strategies satisfy $x_i^* \in f_i(x_j^*)$ and $x_j^* \in f_j(x_i^*)$, they are always feasible (both governments' equilibrium choices are admissible, meaning that they satisfy the household and government budget constraints). When these constraints hold in each country, then the global feasibility constraint also holds in all periods:

$$F(K_t, n_t) + K_t - K_{t+1} - c_t - g_t + F(A_t^*, n_t^*) + A_t^* - A_{t+1}^* - c_t^* - g_t^* = 0. \tag{13}$$

This can be verified by simply adding up the household and government budget constraints in each country.

It is possible that there exist multiple equilibria. However, no matter what the equilibrium outcome is, it will be characterized by zero steady-state capital taxes, as I will show in the next

[11] Of course the equilibrium also depends on the initial conditions, including the capital taxes at time zero.

section. These are optimal independent of all other choices and initial conditions. It is also possible that no pure strategy equilibrium or no steady state exists. I discuss these issues further in section 3 and provide alternative concepts which extend the result to these cases.

2.3 Zero Capital Taxes

In order to show the optimality of zero capital taxes in steady state, three first-order conditions are relevant: those with respect to k_{t+1}, τ_t^n, and τ_t^k.

$$\begin{aligned}
k_{t|t>0}:\ & \psi_t \tau_t^k r_t + \psi_t \tau_t^k K_t \frac{\partial r_t}{\partial k_t} - \psi_t b_t (1-\tau_t^k)\frac{\partial r_t}{\partial k_t} + \psi_t \tau_t^n n_t \frac{\partial w_t}{\partial k_t} + \theta_t[1+(1-\tau_t^k)r_t] + \\
& \theta_t(1-\tau_t^k)k_t \frac{\partial r_t}{\partial k_t} + \theta_t(1-\tau_t^n)n_t \frac{\partial w_t}{\partial k_t} + \theta_t b_t(1-\tau_t^k)\frac{\partial r_t}{\partial k_t} + \theta_t^*/\chi k_t^*(1-\tau_t^k)\frac{\partial r_t}{\partial k_t} \\
& + \mu_t u_c(t)(1-\tau_t^n)\frac{\partial w_t}{\partial k_t} + \zeta_t(1-\tau_t^k)\frac{\partial r_t}{\partial k_t} + \gamma_t^*(1-\tau_t^k)\frac{\partial r_t}{\partial k_t} = \theta_{t-1}/\beta, \qquad (14)
\end{aligned}$$

$$\tau_t^n:\ \psi_t n_t w_t = \mu_t u_c(t) w_t + \theta_t n_t w_t, \qquad (15)$$

$$\tau_{t|t>0}^k:\ \psi_t K_t r_t + \psi_t b_t r_t = \theta_t k_t r_t + \theta_t b_t r_t + \theta_t^*/\chi k_t^* r_t + \zeta_t r_t + \gamma_t^* r_t. \qquad (16)$$

Substituting for ζ and μu_c from equations (15) and (16) in (14), one obtains

$$\psi_t \tau_t^k r_t + \theta_t[1+(1-\tau_t^k)r_t] + \psi_t n_t \frac{\partial w_t}{\partial k_t} + \psi_t K_t \frac{\partial r_t}{\partial k_t} = \theta_{t-1}/\beta. \qquad (17)$$

Since $\partial w/\partial k = F_{nk}$ and $\partial r/\partial k = F_{kk}$ and, due to constant returns to scale, $F_{nk}n = -F_{kk}K$, this simplifies to

$$\psi_t \tau_t^k r_t + \theta_t[1+(1-\tau_t^k)r_t] = \theta_{t-1}/\beta. \qquad (18)$$

This is the same result as in a closed economy, except that the values of the Lagrange multipliers depend on the overall economic conditions, at home and abroad. It implies that the effect of a higher capital stock on prices can be ignored, as long as all factors of production can be taxed separately. From (18) it is straightforward to show that the zero-tax result holds in steady state: Quantities, prices and multipliers are all constant over time, so one can drop time subscripts. The household's Euler Equation (5) then becomes $1/\beta = 1 + r(1-\tau^k)$ and thus

$$\psi\tau^k r + \theta[1 + (1-\tau_t^k)r_t] = \theta[1 + (1-\tau_t^k)r_t] \quad (19)$$

$$\Leftrightarrow \psi r\tau^k = 0. \quad (20)$$

The Lagrange multiplier ψ is always positive, as is the interest rate, therefore $\tau^k = 0$. This can be summarized in the following proposition:

Proposition 1 *With source-based taxation, steady-state capital taxes are optimally zero in any open economy equilibrium, independent of the relative size of the home economy compared to the rest of the world. This includes the limiting points of a closed and a small open economy.*

The assumption of optimal behavior by citizens abroad is important and so is the existence of a steady state, and hence a common discount factor β. All other parameters can be different across countries, from the production function over preferences to government outlays.

I have not explicitly included no-Ponzi conditions, so how can one be sure that governments do not run bigger and bigger deficits? First, the results still hold when I do not allow for government bonds. Second, a rising debt would let the multiplier of the government budget constraint increase, which is not consistent with steady state or the assumption of a constant marginal social value of government funds as in section 3.2.

3 Robustness

The results shown so far rely on steady state, pure strategies, and optimal behavior of all governments. It is not assured that a steady state actually exists, so I present two alternatives to the steady state assumption. Then I discuss the implications of non-optimal behavior by other governments and introduce a different game structure, which will make off-equilibrium behavior feasible. This also permits to analyze mixed strategy equilibria. Next, instead of assuming that government spending is always low enough to have admissible policies, I show next how endogenous government spending guarantees their existence. Ultimately, I briefly lay out how barriers to capital mobility and heterogeneous agents leave the results unaffected; the proofs are relegated to the appendix.

3.1 First-order Approximation

One way to avoid the reliance on steady state is a first-order approximation: I assume here that an additional unit of consumption has negligible effects on the marginal utility of consumption and leisure. This means that a marginal increase in household wealth does not change its labor-leisure or intertemporal decision. A separable utility function (separable over time and between leisure and consumption) that is linear in consumption obviously satisfies these conditions.[12]

To simplify notation, rewrite the equilibrium constraints for domestic capital (with Lagrange multiplier ζ_t for $t > 0$) as

$$(1 + r_t(1 - \tau_t^k))\beta u_c(t) - u_c(t-1) = 0 \tag{21}$$

and for foreign capital (with Lagrange multiplier γ_t for $t > 0$) as

$$r_t^*(1 - \tau_t^{k*}) - r_t(1 - \tau_t^k) = 0. \tag{22}$$

The latter transformation is just adding two binding constraints and thus does not change any results. Then take the derivative of the Lagrangian with respect to c_t

$$\theta_t = u_c(t) + \mu_t[u_{cc}(t)(1 - \tau_t^n)w_t - u_{lc}(t)] + \zeta_t u_{cc}(t)(1 + r_t(1 - \tau_t^k))\beta - \zeta_{t+1}u_{cc}(t)\beta. \tag{23}$$

Under the above-mentioned assumption that the second-order derivatives are zero, it reduces to $\theta_t = u_c(t)$. Substituting this expression for dates t and $t-1$ into equation (18), which I reshow for convenience,

$$\psi_t \tau_t^k r_t + \theta_t[1 + (1 - \tau_t^k)r_t] = \theta_{t-1}/\beta,$$

one can see that

$$\psi_t \tau_t^k r_t + u_c(t)[1 + (1 - \tau_t^k)r_t] = u_c(t-1)/\beta \tag{24}$$

$$\Leftrightarrow \psi_t \tau_t^k r_t = u_c(t-1)/\beta - u_c(t)[1 + (1 - \tau_t^k)r_t] \tag{25}$$

[12]Generally, a quasi-linear utility function does not seem to be a reasonable assumption. However, I do believe that it does provide some interesting insights into the taxation of the initial capital stock. Aiyagari, Marcet, Sargent, and Seppala (2002) also use a quasi-linear utility function. I assume that the Euler equation holds for $t > 0$, i.e. that the initial capital stock is large enough so that the consumer can save or dis-save enough to let it hold in the next period.

From the household's Euler equation (21), the right-hand side is zero:

$$\psi_t \tau_t^k r_t = 0, \tag{26}$$

which implies zero taxes on capital income, since the marginal product of capital r_t and the government's budget constraint multiplier ψ_t are strictly positive.

Since capital taxes have to be announced one period in advance, the solution is time-consistent. In the standard setup with decreasing marginal utility of consumption, capital taxes are initially non-zero as they allow to tax the initial capital stock which does not react to changes in tax rates. Households can partially avoid this tax on k_0 (if τ_0^k is exogenous or bounded enough) by reducing k_1; however, this drives up consumption and reduces marginal utility at time zero. Hence, the ability to avoid a tax on the initial capital stock is limited by how much the marginal utility of consumption decreases as consumption increases. With a utility function linear in consumption, there are no such limits and it is thus impossible for the government to tax the initial capital stock. Capital taxes are optimally set to zero at all times, even if a steady state does not exist, say because g_t does not converge.[13] I summarize it in the following proposition:

Proposition 2 *When the utility function is quasi-linear in consumption and capital taxes are announced one period in advance, then capital taxes τ_{t+1}^k are zero for all $t \geq 0$. Furthermore, the solution is time-consistent.*

3.2 Constant Value of Government Funds

Another alternative to steady state was proposed by Judd (1999), where he shows that a much weaker assumption can be used to obtain a capital tax that is close to zero on average over any long time interval. He assumes "that the marginal social value of government wealth [...] is uniformly bounded below and above over time." I will use a similar idea to show it in this discrete framework. Define the marginal social value of government wealth as $m_t = \psi_t / u_c(t)$. One could also think of it as the marginal rate of substitution between government and private consumption.[14] Assume that

[13] The same is also true for any standard utility function in a small open economy: the government is not able to tax the initial capital stock, as the rate of return is fixed. This has already been shown by Correia (1996a).

[14] Like Judd in his Corollary 9, assume there is a public good $\tilde{g}$, which satisfies $u_{c\tilde{g}} = u_{l\tilde{g}} = 0$ and infinite marginal utility at zero, so that $\tilde{g} > 0$ in all circumstances. The first-order condition would imply $\psi = u_{\tilde{g}}$. $u_{\tilde{g}}/u_c$ is the marginal rate of substitution between government and private consumption. See section 3.5 for further details.

its growth rate is zero on average over a time interval T_1. This can be expressed as

$$\frac{1}{T_1}\sum_{i=t}^{t+T_1} m_i/m_{i-1} = 1. \tag{27}$$

If this is true, then the average distortion, or the ratio of net to gross returns, from t to T_1 is zero. To show this, take the optimal capital accumulation equation (18) and subtract ψ_{t-1}/β from both sides:

$$(\theta_{t-1} - \psi_{t-1})/\beta = \psi_t \tau_t^k r_t + \theta_t[1 + (1 - \tau_t^k)r_t] - \psi_{t-1}/\beta. \tag{28}$$

The first-order condition with respect to government bonds is

$$(\theta_t - \psi_t)[1 + (1 - \tau_t^k)r_t] = (\theta_{t-1} - \psi_{t-1})/\beta. \tag{29}$$

Plugging this into the left-hand side of (28), one obtains

$$(\theta_t - \psi_t)[1 + (1 - \tau_t^k)r_t] = \psi_t \tau_t^k r_t + \theta_t[1 + (1 - \tau_t^k)r_t] - \psi_{t-1}/\beta. \tag{30}$$

This is equivalent to

$$\psi_t(1 + r_t) = \psi_{t-1}/\beta. \tag{31}$$

Substituting for β from the household's Euler equation (21) leads to

$$\psi_t(1 + r_t) = \psi_{t-1}[1 + (1 - \tau_t^k)r_t]u_c(t)/u_c(t-1). \tag{32}$$

Now one can rearrange this to

$$\frac{\psi_t/u_c(t)}{\psi_{t-1}/u_c(t-1)} = \frac{1 + (1 - \tau_t^k)r_t}{1 + r_t} \tag{33}$$

and, using the definition of the marginal social value of government funds, this becomes

$$\frac{m_t}{m_{t-1}} = \frac{1 + r_t(1 - \tau_t^k)}{1 + r_t}. \tag{34}$$

Summing over T_1 time periods and dividing by T_1 yields

$$\frac{1}{T_1}\sum_{i=t}^{t+T_1} m_i/m_{i-1} = 1 = \frac{1}{T_1}\sum_{i=t}^{t+T_1} \frac{1+r_i(1-\tau_i^k)}{1+r_i}. \tag{35}$$

Thus, distortions have to be zero on average. Here I have directly assumed that the marginal social value of government wealth is constant on average over some time interval. If one assumes instead that it is uniformly bounded above and below, then the average growth rate has to be zero in the limit as the time interval grows infinitely long. As long as the value of public compared to private funds stays constant, distortions of the capital-labor ratio are zero on average.These results can be summarized by

Proposition 3 *When the marginal rate of substitution between government and private consumption does not grow on average over some time interval, then average distortions of the rate of return over that time interval are zero. If the marginal social value of government wealth is uniformly bounded below and above over time, then average distortions over a time interval will go to zero as the length of that interval tends to infinity.*

3.3 Non-optimal Capital Taxes Abroad

I would like to emphasize that zero capital taxes are still optimal when the other country sets its taxes non-optimally (i.e. not to zero). The analysis above holds no matter what the beliefs about the other country's actions are. Assume that capital taxes abroad are set to some strictly positive or negative level in steady state, for example due to some political economy constraints or simply because the other government does not behave optimally. Whatever the reason and whatever the capital taxes abroad are, it remains optimal to set steady-state taxes to zero.

It is still necessary that anticipated and actually chosen actions coincide. So far I have assumed (in accordance with standard game theory) that governments can perfectly anticipate actions by the other government and do not make mistakes. That is, they are rational and know that others are rational, that others know that they are rational and so forth. Off-equilibrium outcomes cannot be analyzed in this framework, since when the beliefs about the other player's actions do not coincide with actual actions, then feasibility will be violated.

3.4 Off-equilibrium Behavior and Mixed Strategies

As pointed out above, actions are not feasible off the equilibrium path. In order to ensure that they still are, one has to redefine the game. Taking the other player's actions as given does generally not satisfy the overall feasibility requirement (i.e. the global resource constraint does not hold).

I assume that labor taxes adjust in order to satisfy the government budget constraint (and that expenditures are sufficiently low so that this is always possible). Therefore, a country's choices consist only of the sequences of capital taxes $\{\tau_{t+1}^k\}_{t=0}^\infty$ and bonds $\{b_{t+1}\}_{t=0}^\infty$, while labor taxes adjust automatically so that the government budget constraint holds.

When the government incorporates its budget constraint into its decision problem, then it can choose the labor tax rate, too. In a similar way, foreign taxes on labor income are also determined as a reaction to taxes at home. They react to satisfy the foreign government's budget constraint, thus it is a choice variable for the government at home once the additional constraint is incorporated. The definition of an optimal response function changes accordingly:

Definition 4 (Optimal Adjusted Response Function) *An optimal adjusted response function is a sequence of capital taxes $\{\tau_t^k\}_{t=1}^\infty$ and bond issues $\{b_t\}_{t=1}^\infty$ for each belief of foreign policy $\{\tau_t^{k*}, b_t^*\}_{t=1}^\infty$ maximizing the agent's discounted lifetime utility such that labor tax rates adjust so that the government budget constraints hold every period at home and abroad and that the allocations of consumption, labor supply, and bond and capital holdings at home and abroad satisfy*

1. *agents maximizing utility subject to their budget constraints, taking prices and taxes as given;*

2. *firms maximizing profits, taking prices as given.*

The home government's range of effective choices remains the same, as it can still choose the labor tax rate when it incorporates its own budget constraint into the problem. It may also choose the labor tax rate abroad, but has to satisfy the government budget constraint there, too. It thus does not gain or lose any degrees of freedom. Since any feasible strategy that a foreign government can play involves a balanced budget (allowing for debt and assuming goods cannot be discarded), the assumption that labor taxes adjust is not problematic. Within the set of all feasible strategies, the foreign government is not restricted in its choice.

I let labor taxes adjust since bond issues are generally not a sufficient instrument to satisfy the intertemporal budget constraint. Adjusting capital taxes might not be sufficient for the government budget constraint to hold in every period, but if it does, then results remain unchanged.[15] Any combination of tax rates and bonds would increase the degrees of freedom and thus be subject to the same criticisms as the literature on conjectural variations. The idea behind the concept I propose is not some form of behavioral game theory but to ensure that off-equilibrium choices are still feasible.

The planner's problem remains the same as before, except that the foreign government's budget constraint now has to be included in the constraint set (with multiplier ψ_t^*) and that the foreign labor tax rate τ_t^{n*} is now part of the control variables.[16] It is easily verified that the relevant first-order conditions for the proof do not change, as k_{t+1}, τ_t^k, and τ_t^n or domestic prices r_t and w_t are not part of the foreign government's budget constraint. A similar argument holds when capital taxes adjust.

The implications for optimal long-run capital taxes are hence the same as before. Given any belief of the foreign government's actions, optimal capital taxes are zero. If the other player then turns out to have played something different than expected, the home government would not want to change its capital taxes ex-post, since the best response to *any* foreign policy are zero capital taxes.

Proposition 4 *For a redefined game in which labor taxes adjust in both countries to satisfy the government budget constraint, off-equilibrium outcomes are also feasible. In such a game, capital taxes are optimally zero in steady state, no matter whether the other government plays the equilibrium strategy or not. There is no ex-post incentive to change the steady-state capital tax rate after knowing the other player's strategy.*

The concept of an adjusted response function also allows to consider mixed-strategy equilibria. In what had been previously assumed, feasibility is only satisfied if beliefs and actions taken coincide. Therefore, a non-degenerate mixed strategy would not be feasible. When off-equilibrium behavior is feasible, then a mixed strategy equilibrium is, too. For any mixed strategy, the part of it specifying long-run capital taxes will always be zero, as this is optimal no matter what the other country's

[15]Mendoza and Tesar (2005) let either labor or consumption taxes adjust and obtain different results depending on which one adjusts. This can be explained by the following factors: (i) the initial taxes are not optimal; (ii) taxes are forced to be time-invariant; (iii) taxation of the initial capital stock is driving their results. As I show in the appendix, the availability of consumption taxes is redundant, except for the taxation of the initial capital stock.

[16]If only the foreign government's per-period budget constraint were included, but not τ_t^{n*} as an additional choice variable, then the home government's "optimal response" would generally be a singleton. However, when labor taxes abroad can be chosen, that is no longer true.

policy is.

Proposition 5 *For a redefined game in which labor taxes adjust in both countries to satisfy the government budget constraint, all mixed-strategy equilibria feature zero capital taxes in steady state.*

The proof is in the appendix.

3.5 Endogenous Government Spending

So far I have assumed that the government's set of admissible policies is always non-empty for any belief of the other country's policy, in other words that the exogenous stream of government expenditures and the initial debt are sufficiently low. Alternatively, government expenditures can be endogenous, ensuring that this is true. Assume that initial government debt is zero and that the government can choose spending g. Let the government's per-period objective function be $v(c, l, g)$. g can either be a public good which also enters the household's utility function or it can represent government consumption or some of both. Assume that $\lim_{g\to\infty} v_g = 0$ and that $\lim_{g\to 0} v_g = \infty$. The government will thus adjust its government spending depending on the value of public funds; specifically, it can reduce its spending when the social cost of raising tax revenues is very high. This guarantees that an admissible policy always exists, no matter what the policy in the other country is, while the relevant first-order conditions are not affected. To allow for off-equilibrium behavior as in the last section, government expenditures can then adjust to satisfy the government budget constraint, while the sequence of foreign labor and capital taxes and bonds is taken as given.

3.6 Adjustment Costs

It is a priori not clear whether the assumption of perfect capital mobility is elemental for the result. To explore the robustness of the result in this direction, I introduce adjustment costs to model barriers to capital mobility. Define a symmetric adjustment cost function $Z(x, y)$ such that $Z_{yy} = Z_{xx} = -Z_{xy}$ and $Z_x = Z_y$, where x is capital today and y capital in the next period. A well-known example is the quadratic function $\delta[(k_t - k_{t+1})/k_t]^2$, where δ is some positive, finite parameter.

Proposition 6 *For any symmetric adjustment cost function for the capital stock in any of the countries, optimal steady-state capital income taxes are zero.*

The proof is in the appendix.

3.7 Heterogeneous Agents

To quote from Kocherlakota (2010), the textbook on New Dynamic Public Finance (NDPF), the biggest flaw of Ramsey taxation is: "Its key economic trade-off is that the government would like to make the taxes nonlinear but cannot." Unlike the NDPF, I do not introduce uncertainty (the model is still deterministic) or private information, so I am by no means trying to address the same questions as tackled by the NDPF. But issues of redistribution and wealth heterogeneity are potentially important. It turns out that optimal long-run capital taxes are not affected, though:

Proposition 7 *Under the assumption that agents differ in their initial capital endowments and their time-independent labor productivity, and that the government uses a polynomial of degree* $J > 0$ *in labor income for labor taxes and a linear tax for capital income, optimal steady-state capital taxes are zero. This is independent of the social welfare weights for different agents.*

The proof is in the appendix.

4 Residential Taxes

In a residential tax system, agents pay taxes to their home government for all capital income, no matter where it is invested. In a hybrid system, there are corporate taxes which are paid in the country where the capital is employed, and the remaining capital income is taxed according to the residential system. Capital taxes have to be the same in steady state in all countries (something already explored by Razin and Sadka (1991)). The question is then how to interpret best response functions in steady state. Assuming that countries are symmetric, this ceases to be a problem, since all countries will choose the same tax rates in any case.[17] Long-run capital taxes in a residential system are generally not zero. The sign and magnitude depend on a number of endogenous variables such as the interest rate, the value of government funds, and others. The importance of this result is not whether the sign is positive or negative, but rather that it underscores the importance of a

[17] Mendoza and Tesar (2005) formally define capital taxes as residential, but taxes effectively function as in a territorial system: They assume that all domestic capital is held domestically and that international bonds are not taxed. Correia (1996a) considers a small open economy and concludes that a steady state does not exist if tax rates are not equal.

complete tax system – where every factor of production can be taxed independently – for the zero capital tax. With residential taxation, a government cannot tax domestic capital investment by foreigners. A hybrid system, with both residential and territorial capital taxes, is complete in the sense specified above and effective capital taxes are zero again.

4.1 Pure Residential Taxation

I proceed as before, taking taxes abroad as given, derive a best response function, and then set residential capital taxes in both countries equal to each other to arrive at the equilibrium tax rate. In the symmetric case, all countries would choose the same tax rates, so this is a valid approach, but in the asymmetric case the optimal rates generally do not coincide. I will thus assume that the countries are identical. Klein, Quadrini, and Rios-Rull (2005) derive an equilibrium for asymmetric countries in a framework with imperfect commitment. They find the steady-state foreign net investment that equates the two optimal rates. It is therefore possible to extend the analysis to the case of asymmetric countries. However, this is only useful in a computational implementation, which is beyond the scope of this paper. I leave it for future research.

To simplify the analysis I leave out government bonds, without loss of generality. The planner's Lagrangean is

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} \beta^t \{ & u(c_t, l_t) \qquad (36) \\
& + \psi_t [\tau_t^k r_t k_t + \tau_t^k r_t^* a_t + \tau_t^n w_t n_t - g_t] \\
& + \theta_t [(1 - \tau_t^n) w_t n_t + k_t [1 + r_t (1 - \tau_t^k)] - k_{t+1} + a_t [1 + r_t^* (1 - \tau_t^k)] - a_{t+1} - c_t] \\
& + \mu_t [u_c(t)(1 - \tau_t^n) w_t - u_l(t)] \\
& + \zeta_{t|t>0} [(r_t (1 - \tau_t^k) + 1) - u_c(t-1)/(\beta u_c(t))] \\
& + \gamma_{t|t>0} [(r_t^* (1 - \tau_t^k) + 1) - u_c(t-1)/(\beta u_c(t))] \\
& + \theta_t^* [\cdot] + ... \}.
\end{aligned}
$$

I have omitted showing explicitly the foreign household's budget constraint and first-order conditions, these are subsumed into the shorthand $+\theta_t^*[\cdot] + ...$ I use the same first-order conditions as before to show that optimal steady state capital taxes are generally non-zero. The first-order conditions of the other capital choice variables (for example a or k^*) do not reveal additional information, as the system is over-identified in steady state and outcomes depend on initial capital holdings. Therefore, I do not show these additional conditions here.

$$
\begin{aligned}
k_{t|t>0}:\ & \psi_t[n_t\tau_t^n\frac{\partial w_t}{\partial k_t}+\tau_t^k r_t+\tau_t^k k_t\frac{\partial r_t}{\partial k_t}] && (37)\\
&+\theta_t[n_t(1-\tau_t^n)\frac{\partial w_t}{\partial k_t}+[1+r_t(1-\tau_t^k)]+k_t(1-\tau_t^k)\frac{\partial r_t}{\partial k_t}]+\mu_t u_c(t)(1-\tau_t^n)\frac{\partial w_t}{\partial k_t}\\
&+\zeta_t(1-\tau_t^k)\frac{\partial r_t}{\partial k_t}+\theta_t^* k_t^*(1-\tau_t^{k*})\frac{\partial r_t}{\partial k_t}+\gamma_t^*(1-\tau_t^{k*})\frac{\partial r_t}{\partial k_t}=\theta_{t-1}/\beta\\
\tau_{t|t>0}^k:\ & \psi_t(r_t k_t+r_t^* a_t)=\zeta_t r_t+\gamma_t r_t^*+\theta_t k_t r_t+\theta_t a_t r_t^* && (38)\\
\tau_t^n:\ & \psi_t w_t n_t=\mu_t u_c(t)w_t+\theta_t n_t w_t. && (39)
\end{aligned}
$$

As evident from the two capital accumulation constraints in (36), gross capital returns will always be equal across countries. Hence, net capital returns and thus taxes will be equal in steady state, when gross returns in different countries are equal. Therefore, I convert r^* into r and τ^{k*} into τ^k. Dropping time subscripts, replacing $1/\beta$ with $1+r(1-\tau^k)$ and combining, one obtains

$$\tau^k=\frac{[(\gamma-\gamma^*)+\psi(k^*-a)+(a\theta-k^*\theta^*)]\frac{\partial r_t}{\partial k_t}}{\psi r+[(\gamma-\gamma^*)+(a\theta-k^*\theta^*)-\psi a]\frac{\partial r_t}{\partial k_t}}. \qquad (40)$$

In a symmetric world, $k^*=a$, but the Lagrange multipliers for a domestic and foreign constraint, e.g. γ vs. γ^* and θ vs. θ^*, will generally not be equal. To see this, taking the derivative with respect to c_t and c_t^* shows that both influence the incentive compatibility conditions for working and saving (i.e. the constraints with multipliers μ, ζ, and γ or their equivalents abroad), but only the former affects utility directly. In other words, resources abroad are only valued insofar as they affect prices and hence incentives. Symmetry allows to simplify equation (40) to

$$\tau^k=\frac{[(\gamma-\gamma^*)+(a\theta-k^*\theta^*)]\frac{\partial r_t}{\partial k_t}}{\psi r+[(\gamma-\gamma^*)+(a\theta-k^*\theta^*)-\psi a]\frac{\partial r_t}{\partial k_t}}. \qquad (41)$$

Residential capital taxes are thus generally not zero in steady state. They depend among others on the distribution of assets (how much is invested abroad, a and k^*) and the impact that one country has on the worldwide rate of return ($\partial r_t/\partial k_t$). γ and γ^* are the Lagrange multipliers for the household's Euler equation concerning investment abroad. In the limiting cases of a small open economy or closed economy, taxes are zero again. For a closed economy, it is simply the common Chamley-Judd result; there is no foreign country and no assets abroad to invest in, so in equation (41) $\gamma = \gamma^* = 0$ and $a = k^* = 0$. For a small open economy, optimal capital taxes also have to be zero[18]; it follows from the fact that $\partial r_t / \partial k_t = 0$, as $r = r^*$ from the no-arbitrage condition and r^* is fixed from the small country's perspective. This had been shown by Correia (1996a) in a model where the rest of the world is not explicitly modeled. The results can be summarized in the following proposition:

Proposition 8 *With residential taxation, steady-state capital taxes are generally non-zero in a symmetric large open economy equilibrium. In the limiting cases of a closed and a small open economy, steady-state capital taxes are optimally zero.*

4.2 Hybrid System

Let τ_t^s denote corporate taxes, i.e. taxes that are paid by firms in the country where they produce. τ_t^k is the residential tax, which will be levied on all net capital income of residents, regardless of where it is invested. The planner's problem is thus

$$
\begin{aligned}
L = & \sum_{t=0}^{\infty} \beta^t \{u(c_t, l_t) \qquad (42)\\
& +\psi_t[\tau_t^s r_t K_t + \tau_t^k r_t (1-\tau_t^s) k_t + \tau_t^k r_t^* (1-\tau_t^{s*}) a_t + \tau_t^n w_t n_t - g_t] \\
& +\theta_t[(1-\tau_t^n) w_t n_t + k_t[1 + r_t(1-\tau_t^s)(1-\tau_t^k)] - k_{t+1} + a_t[1 + r_t^*(1-\tau_t^{s*})(1-\tau_t^k)] - a_{t+1} - c_t] \\
& +\mu_t[u_c(t)(1-\tau_t^n) w_t - u_l(t)] \\
& +\zeta_{t|t>0}[(r_t(1-\tau_t^s)(1-\tau_t^k) + 1) - \frac{u_c(t-1)}{\beta u_c(t)}] \\
& +\gamma_{t|t>0}[(r_t^*(1-\tau_t^{s*})(1-\tau_t^k) + 1) - \frac{u_c(t-1)}{\beta u_c(t)}] \\
& +\theta_t^*[\cdot] + ...\}.
\end{aligned}
$$

[18] With two symmetric countries, there is of course no small open economy. However, the model can be easily expanded to any number of countries. For a small open economy, let the number of countries go to infinity. The impact of each country on the world interest rate then is zero.

The tax system is complete in the sense that all domestic factor returns are subject to taxation, so combining the first-order conditions for k_t, τ_t^s and τ_t^n is equivalent to taking the derivative with respect to k_t and taking prices as given:

$$k_{t|t>0}\colon\ \psi_t \tau_t^s r_t + \psi_t \tau_t^k r_t (1-\tau_t^s) + \theta_t [1 + r_t(1-\tau_t^s)(1-\tau_t^k)] = \theta_{t-1}/\beta. \tag{43}$$

In steady state, one can eliminate time subscripts and substitute for β to obtain

$$(\tau^s + \tau^k - \tau^s \tau^k) r \psi = 0. \tag{44}$$

Therefore, effective taxes on domestic capital are zero: $1+r(1-\tau^s)(1-\tau^k) = 1+r$. Each component is not necessarily zero, although $\tau^k = \tau^s = 0$ is of course a solution.

The problem remains that residential taxes around the world have to be equal in steady state, in order to satisfy the no-arbitrage conditions. If that tax rate is non-zero, then the domestic corporate tax will not be zero in steady state, either. If the global residential tax is positive, then it is optimal to introduce a negative corporate tax, and vice versa.

Proposition 9 *With both residential and source-based taxation, the tax distortion on capital returns is zero, i.e.* $(1-\tau^s)(1-\tau^k) = 1$.

5 Conclusion

This paper presents a framework to analyze optimal capital taxation with perfect commitment in large open economies. With a territorial system, in which taxes are paid where the capital is employed, optimal capital taxes are zero in the long run. The result is the same as in a comparable closed economy of a representative infinitely lived dynasty. This is robust to introducing barriers to capital mobility and heterogeneity in agents' initial wealth and labor productivity as well as non-linear labor taxes.

In a residential system, in which capital taxes are paid where the owner resides, capital taxes are generally not zero, except in the limiting cases of a closed and a small open economy. With residential taxes, not all factors of production can be taxed independently, notably domestic capital investment of foreign agents. In a hybrid system with both residential and territorial taxes, all factors are taxable once more and the zero capital-tax result is restored. This underlines the importance of a complete tax system for the Chamley-Judd result.

Some extensions to the presented model come to mind. If taxes were optimally non-zero in a closed economy, say because of firm profits or due to overlapping generations as in Erosa and Gervais (2002), then would a race to the bottom ensue? How does productive infrastructure spending influence tax competition? Although there is a huge literature on this subject already, most models feature an exogenously given capital stock. I analyze tax competition with endogenous capital accumulation in a separate paper, Gross (2012). Furthermore, one could study the implications of aggregate fluctuations on optimal fiscal policy in an open economy, extending the work of Chari, Christiano, and Kehoe (1994) and Farhi (2010). The computational challenges would be considerable, though.

Another venue of research is to look at labor mobility. A somewhat philosophical problem arises here, though, as it is no longer clear whose utility a benevolent government would maximize: the utility of its citizens, no matter whether they migrate or not; the utility of all people living in its jurisdiction; the utility of its citizens as long as they stay in the country etc. Whatever it is, it would be interesting to see whether the zero capital tax result still holds and what optimal taxes on labor would be in such an environment.

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A Appendix

A.1 Consumption Taxes and the Primal Approach

In this appendix, I examine two questions simultaneously: How can one think of the primal approach in a large open economy and what is the impact of consumption taxes on optimal capital tax policy? The primal approach is not as useful here as it is in a closed economy, since many of the prices and taxes cannot be eliminated. However, it still serves the purpose to illustrate that the availability of consumption taxes does not affect results. I maintain the assumption of source-based or territorial capital taxation, but results easily transfer to a system of world-wide or residential capital taxation.

Consider an economy as presented in section two, except that the government can now introduce consumption taxes. The household's per-period budget constraint (2) and the government budget constraint (10) change to

$$c_t(1+\tau_t^c) = (1-\tau_t^n)w_t n_t + [1+(1-\tau_t^k)r_t]k_t - k_{t+1} \tag{45}$$
$$+ [1+(1-\tau_t^{k*})r_t^*]a_t - a_{t+1} + (1+R_t)b_t - b_{t+1}$$
$$g_t + b_t(1+R_t) = \tau_c^t c_t + \tau_t^k r_t K_t + \tau_t^n w_t n_t + b_{t+1}. \tag{46}$$

The household's optimality conditions, equations (4) to (6), along with the optimality condition for government bonds, then become

$$u_l(t) = u_c(t)(1-\tau_t^n)w_t/(1+\tau_t^c) \tag{47}$$
$$u_c(t)/(1+\tau_t^c) = \beta u_c(t+1)[(1-\tau_{t+1}^k)r_{t+1}+1]/(1+\tau_{t+1}^c) \tag{48}$$
$$u_c(t)/(1+\tau_t^c) = \beta u_c(t+1)[(1-\tau_t^{k*})r_{t+1}^*+1]/(1+\tau_{t+1}^c) \tag{49}$$
$$u_c(t)/(1+\tau_t^c) = \beta u_c(t+1)(1+R_{t+1})/(1+\tau_{t+1}^c). \tag{50}$$

First, replace $(1-\tau_{t+1}^k)r_{t+1}$ and $(1-\tau_t^{k*})r_{t+1}^*$ by R_{t+1} in the household's per-period budget constraint. Then one can use equation (45) to isolate b_{t+1} and substitute for it (eliminating b_{t+1} as a choice variable and rendering the household's per-period budget constraint obsolete). This results in an intertemporal budget constraint for the household and a national resource constraint for every period t from the government budget constraint (46):

$$\frac{(b_0+k_0)(1+(1-\tau_0^k)r_0)+a_0(1+(1-\tau_0^{k*})r_0^*)}{1+R_0}=\sum_{t=0}^{\infty}\frac{c_t(1+\tau_t^c)-(1-\tau_t^n)w_tn_t}{\prod_{i=0}^{t}(1+R_i)} \tag{51}$$

$$F_t(K,n)+k_t-k_{t+1}+a_t(1+r_t^*(1-\tau_t^{k*}))-a_{t+1}-k_t^*r_t(1-\tau_t^k)=c_t+g_t. \tag{52}$$

Now one can eliminate R_{t+1} and τ_t^n from the problem by substituting in from equations (50) and (47) (which thereby become obsolete) into (51) to obtain the conventional implementability constraint:

$$\frac{[(b_0+k_0)(1+(1-\tau_0^k)r_0)+a_0(1+(1-\tau_0^{k*})r_0^*)]u_c(0)}{1+\tau_0^c}=\sum_{t=0}^{\infty}\beta^t(u_c(t)c_t+u_n(t)n_t). \tag{53}$$

Finally, use equation (48) to eliminate τ_{t+1}^c from the problem. This leaves the no-arbitrage condition for every period t:

$$(1-\tau_{t+1}^k)r_{t+1}=(1-\tau_t^{k*})r_{t+1}^*. \tag{54}$$

The government's set of constraints now consists of the implementability constraint (53), the national resource constraint (52) and no-arbitrage condition (54) for every period, plus the competitive equilibrium conditions for foreign agents. The choice variables b_{t+1}, τ_t^n, and τ_{t+1}^c were eliminated. Note that none of the variables eliminated are present in the foreign competitive equilibrium conditions. One cannot follow the same approach for the foreign agent, since none of the eliminated variables' foreign counterparts are choice variables.

What is the impact of the availability of consumption taxes on the optimal tax policy? The problem presented here with an implementability constraint clearly shows that consumption taxes do not change the problem except for consumption taxes at time zero. This is the same result as in a closed economy: consumption taxes at time zero represent another opportunity to tax the initial capital stock; apart from that, consumption taxes are irrelevant.

Even though I have only shown it for a territorial tax system, the same is true for a residential tax system. Theoretically, having consumption taxes would allow residential capital taxes to be different across countries in steady state. However, this would require constantly increasing consumption taxes. If one imposes limits on labor income or consumption taxes, then consumption taxes have to be constant in steady state and capital taxes have to equalize across countries.

A.2 Proof: Mixed-strategy Equilibria

Assume that the foreign country is believed to play a mixed strategy consisting of I different elements, played with a probability $\pi_i > 0$, such that $\sum_I \pi_i = 1$. As shown in the main section of the paper, the best response to any single one of the elements of the other country's mixed strategy is to set steady-state capital taxes to zero. I prove now that this is also true in expectation for any foreign mixed strategy.

The government's problem is now to maximize expected lifetime utility of its agents, $\sum_I \pi_i L_i$, where L_i is the Lagrangian as in section 2.2 extended to include the foreign government's budget constraint per period (and with foreign labor taxes as an additional choice variable), as outlined in section 3.4. The set of control variables is then

$$X = \{\{c_{t,i}, c^*_{t,i}, n_{t,i}, n^*_{t,i}, k_{t+1,i}, k^*_{t+1,i}, a_{t+1,i}, a^*_{t+1,i}, b_{t+1}, \tau^k_{t+1}, \tau^n_{t,i}, \tau^{n*}_{t,i}\}_{i=1}^I\}_{t=0}^\infty. \tag{55}$$

The control variables b_{t+1} and τ^k_{t+1} have to be the same for each element of the other country's mixed strategy, but the response of household consumption to domestic policy depends on foreign policy (which is observed when households make their choices). As pointed out before, labor taxes have to adjust, otherwise the government budget constraint would not hold (and thus violate feasibility). The relevant first-order conditions are

$$\begin{aligned} k_{t|t>0,i}\colon\ & \psi_{t,i}\tau^k_t r_{t,i} + \psi_{t,i}\tau^k_t K_{t,i}\frac{\partial r_{t,i}}{\partial k_{t,i}} - \psi_{t,i}b_t(1-\tau^k_t)\frac{\partial r_{t,i}}{\partial k_{t,i}} + \psi_{t,i}\tau^n_{t,i}n_{t,i}\frac{\partial w_{t,i}}{\partial k_{t,i}} + \theta_{t,i}[1+(1-\tau^k_t)r_{t,i}] + \\ & \theta_{t,i}(1-\tau^k_t)k_{t,i}\frac{\partial r_{t,i}}{\partial k_{t,i}} + \theta_{t,i}(1-\tau^n_{t,i})n_{t,i}\frac{\partial w_{t,i}}{\partial k_{t,i}} + \theta_{t,i}b_t(1-\tau^k_t)\frac{\partial r_{t,i}}{\partial k_{t,i}} + \theta^*_{t,i}/\chi k^*_{t,i}(1-\tau^k_t)\frac{\partial r_{t,i}}{\partial k_{t,i}} \\ & + \mu_{t,i}u_c(t,i)(1-\tau^n_{t,i})\frac{\partial w_{t,i}}{\partial k_{t,i}} + \zeta_{t,i}(1-\tau^k_t)\frac{\partial r_{t,i}}{\partial k_{t,i}} + \gamma^*_{t,i}(1-\tau^k_t)\frac{\partial r_{t,i}}{\partial k_{t,i}} = \theta_{t-1,i}/\beta, \end{aligned} \tag{56}$$

$$\tau^n_{t,i}\colon\ \psi_{t,i}n_{t,i}w_{t,i} = \mu_{t,i}u_c(t,i)w_{t,i} + \theta_{t,i}n_{t,i}w_{t,i}, \tag{57}$$

$$\begin{aligned} \tau^k_{t|t>0}\colon\ & \sum_I \pi_i(\psi_{t,i}K_{t,i}r_{t,i} + \psi_{t,i}b_t r_{t,i} \\ & - (\theta_{t,i}k_{t,i}r_{t,i} + \theta_{t,i}b_t r_{t,i} + \theta^*_{t,i}/\chi k^*_{t,i}r_{t,i} + \zeta_{t,i}r_{t,i} + \gamma^*_{t,i}r_{t,i})) = 0. \end{aligned} \tag{58}$$

Considering the steady state, I drop time subscripts. Furthermore, I substitute $1+(1-\tau^k)r_i$ for $1/\beta$, which has to hold in any steady state, into equation (56). Equation (57) can be divided by $w_{t,i}$ and then $\mu_{t,i}u_c(t,i)$ can be substituted into equation (56). Next I multiply that equation by

r_i and π_i, divide it by $\partial r_i/\partial k_i$,[19] and sum over all i. One can then use equation (58), multiplied by $1-\tau^k$, to substitute in for $\sum \pi_i \zeta_i r_i (1-\tau^k)$ and obtain

$$\sum_I \pi_i \psi_i \frac{r_i}{\partial r_i/\partial k_i} \left(\tau^k r_i + n_i \frac{\partial w_i}{\partial k_i} + K_i \frac{\partial r_i}{\partial k_i} \right) = 0 \tag{59}$$

As before, $\partial w/\partial k = F_{nk}$ and $\partial r/\partial k = F_{kk}$. Due to constant returns to scale, $F_{nk}n = -F_{kk}K$, so $\tau^k \sum_I \pi_i \psi_i r_i^2 \partial r_i/\partial k_i = 0$, which implies that $\tau^k = 0$, since $\pi_i, \psi_i, r_i > 0$ and $\partial r_i/\partial k_i < 0$ for all i. Therefore, no matter what the foreign country's strategy is, the home country will always select zero capital taxes in the long run, including mixed strategies.

A.3 Proof: Adjustment Costs

Let the adjustment cost function be twice continuously differentiable and in the home economy be given by $Z(1 - k_{t+1}/k_t)$ and abroad by $Y(1 - a_{t+1}/a_t)$, where each function reaches its respective minimum when $k_{t+1} = k_t$ and $a_{t+1} = a_t$. An agent who seeks to shift capital from one country to another thus faces the cost of reducing the capital stock in one place plus the cost of increasing it in the other place.

The household's budget constraint then becomes

$$\begin{aligned} c_t =& (1-\tau_t^n) w_t n_t + [1 + (1-\tau_t^k) r_t] k_t - k_{t+1} - Z(k_t, k_{t+1}) + \\ & [1 + (1-\tau_t^{k*}) r_t^*] a_t - a_{t+1} - Y(a_t, a_{t+1}). \end{aligned} \tag{60}$$

As apparent from section two, government bonds do not affect the result, so to keep notation as simple as possible, I have removed them. The household's first-order conditions change accordingly

[19] The production function is assumed to be well-behaved, i.e. that $F_{kk} < 0$ for all $K > 0$ and $n > 0$.

and the planner's modified Lagrangean is

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} \beta^t \{ & u(c_t, l_t) \\
& +\psi_t[\tau_t^k r_t K_t + \tau_t^n w_t n_t - g_t] \\
& +\theta_t[(1-\tau_t^n)w_t n_t + [1+(1-\tau_t^k)r_t]k_t - k_{t+1} - Z(k_t, k_{t+1}) + \\
& \quad [1+(1-\tau_t^{k*})r_t^*]a_t - a_{t+1} - Y(a_t, a_{t+1}) - c_t] \\
& +\mu_t[u_c(t)(1-\tau_t^n)w_t - u_l(t)] \\
& +\zeta_{t|t>0}[(1+r_t(1-\tau_t^k) - Z_{k_t}(k_t, k_{t+1}))u_c(t) - (1+Z_{k_t}(k_{t-1}, k_t))u_c(t-1)/\beta] \\
& +\gamma_{t|t>0}[(1+r_t^*(1-\tau_t^{k*}) - Y_{a_t}(a_t, a_{t+1}))u_c(t) - (1+Y_{a_t}(a_{t-1}, a_t))u_c(t-1)/\beta] \\
& +\theta_t^*[\cdot] + ...\}.
\end{aligned} \tag{61}
$$

I have omitted showing explicitly the foreign household's budget constraint and first-order conditions, these are subsumed into the shorthand $+\theta_t^*[\cdot]+...$ As shown before, when taking the first-order conditions with respect to k_{t+1}, τ_t^k and τ_t^n, the terms with second-order derivatives cancel out (the effect of a change in k on w and r), so I will use the first-order condition for k_{t+1}, taking prices as given:

$$
\begin{aligned}
k_{t+1}: \; & -\zeta_t Z_{k_t k_t}(k_t, k_{t+1})u_c(t) - \zeta_t Z_{k_t k_t}(k_{t-1}, k_t)u_c(t-1)/\beta \\
& -\zeta_{t-1} Z_{k_{t-1} k_t}(k_{t-1}, k_t)u_c(t-1)/\beta - \zeta_{t+1} Z_{k_t k_{t+1}}(k_t, k_{t+1})u_c(t) \\
& +\psi_t \tau_t^k r_t + \theta_t r_t(1+(1-\tau_t^k) - Z_{k_t}(k_t, k_{t+1})) = \theta_{t-1}(1+Z_{k_t}(k_{t-1}, k_t))/\beta.
\end{aligned} \tag{62}
$$

In steady state, k is independent of the time dimension, as are the other variables and Lagrange multipliers. The second and cross derivatives of the adjustment cost function with respect to today's (x) and tomorrow's capital (y) satisfy $Z_{yy} = Z_{xx} = -Z_{xy}$. Moreover, $Z_x = Z_y = 0$, since the function reaches its minimum in steady state when $x = y$. Equation (62) then simplifies to

$$
\psi\tau^k r + \theta(1 + r(1-\tau^k)) = \theta/\beta. \tag{63}
$$

In steady state, $1/\beta = 1 + r(1-\tau^k)$, so optimally $\psi\tau^k r = 0$. In fact, any adjustment cost function that satisfies the properties $Z_{yy} = Z_{xx} = -Z_{xy}$ and $Z_x = Z_y$, which I call symmetric, leads to zero capital taxes. Even when the first derivative is not zero in steady state, then equation (63) becomes $\psi\tau^k r + \theta(1+(1-\tau^k)r - Z_k) = \theta(1+Z_k)/\beta$. In this case $1/\beta$ is equal to $(1+r(1-\tau^k)-Z_k)/(1-Z_k)$, so as before $\psi r\tau^k = 0$.

A.4 Proof: Heterogeneous Agents and Non-linear Taxes

There are I different types of agents of unit mass each in every country,[20] with time-independent, finite and strictly positive labor productivity $x^i \in [\underline{x}, \overline{x}]$ per hour worked, and initial capital holdings k_0^i. I assume an interior solution, i.e. that all household's labor decisions are in the interval $(0, 1)$. This can be guaranteed by choosing an appropriate utility function or parameter values.[21]

A household of type i has the following budget constraint

$$\begin{aligned} c_t^i = {} & k_t^i(1 + r_t(1 - \tau_t^k)) - k_{t+1}^i + a_t^i(1 + r_t^*(1 - \tau_t^{k*})) - a_{t+1}^i \\ & + b_t^i(1 + R_t) - b_{t+1}^i + n_t^i x^i w_t - \sum_J \tau_{t,j}^n (n_t^i x^i w_t)^j. \end{aligned} \tag{64}$$

The household's first-order conditions change accordingly. The government's budget constraint is

$$\tau_t^k r_t K_t + B_{t+1} - B_t(1 + R_t) - g_t + \sum_I \sum_J \tau_{t,j}^n (n_t^i x^i w_t)^j = 0, \tag{65}$$

where K is the total capital stock employed in the home country, i.e. the sum of domestic capital holdings over all agents at home and abroad, similarly for total government debt B. Implicit in the formulation above is the assumption that wages are paid per efficiency unit of labor.

Let α^i denote the planner's utility function (increasing and concave) when individuals of type i obtain utility $u(c_t^i, n_t^i)$. As above, I do not show explicitly the foreign household's budget constraint

[20]It is easy to incorporate different sizes for each type, but it clogs up notation and does not yield any additional insights.

[21]A caveat worth mentioning is the case of log-utility. As Lansing (1999) has shown, a "capitalist" class that does not work (complemented by a working class that does not save) and has log-utility can be taxed at will. Since income and substitution effects cancel each other out, their savings decision is independent of the tax on capital. In this paper, everybody is allowed to work and save, though, so Lansing's limiting case does not apply here.

and first-order conditions. The planner's problem is

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} & \beta^t \{ \sum_I \alpha^i (u(c_t^i, n_t^i)) \\
&+ \psi_t \left[\tau_t^k r_t K_t + \sum_I \sum_J \tau_{t,j}^n (n_t^i x_t^i w_t)^j - B_t(1 + r_t(1 - \tau_t^k)) + B_{t+1} - g_t \right] \\
&+ \sum_I \theta_t^i [k_t^i (1 + r_t(1 - \tau_t^k)) - k_{t+1}^i + a_t^i (1 + r_t(1 - \tau_t^{k*})) - a_{t+1}^i + \\
&\quad b_t^i (1 + r_t(1 - \tau_t^k)) - b_{t+1}^i - c_t^i + n_t^i x^i w_t - \sum_J \tau_{t,j}^n (n_t^i x^i w_t)^j] \\
&+ \sum_I \mu_t^i \left[u_c^i(t)(w_t x_t^i n_t^i - \sum_J \tau_{t,j}^n j (w_t x_t^i n_t^i)^j) + u_n^i(t) n_t^i \right] \\
&+ \sum_I \zeta_{t|t>0}^i \left[(r_t(1 - \tau_t^k) + 1) - \frac{u_c^i(t-1)}{\beta u_c^i(t)} \right] \\
&+ \sum_I \gamma_{t|t>0}^i \left[(r_t^*(1 - \tau_t^{k*}) + 1) - \frac{u_c^i(t-1)}{\beta u_c^i(t)} \right] \\
&+ \sum_I^* \theta_t^{i*} [\cdot] + ... \}.
\end{aligned}
\tag{66}
$$

In the following, I will drop time subscripts in order to keep the notation as simple as possible. Since I am only concerned with steady state solutions, it comes without loss of generality. The relevant first-order conditions are

$$
\tau_j^n: \quad \sum_I \theta^i (n^i x^i w)^j + \sum_I \mu_i u_c^i j (n^i x^i w)^j = \psi \sum_I (n^i x^i w)^j \tag{67}
$$

$$
\tau^k: \quad \sum_I \theta^i (k^i + b^i) r + \sum_{I^*} \theta^{*i} k^{*i} r + \sum_I \zeta^i r + \sum_{I^*} \gamma^{i*} r = r(K + B)\psi \tag{68}
$$

$$
\begin{aligned}
k_i: \quad & \sum_I \theta^i \left((k^i + b^i)(1 - \tau^k) \frac{\partial r}{\partial k} + n^i x^i \frac{\partial w}{\partial k} - \sum_J \tau_j^n j (n^i x^i w)^j \frac{\partial w}{\partial k} / w \right) \\
&+ \psi \left(\tau^k r + \tau^k K \frac{\partial r}{\partial k} - B(1 - \tau^k) \frac{\partial r}{\partial k} + \sum_I \sum_J \tau_j^n j (n^i x^i w)^j \frac{\partial w}{\partial k} / w \right) \\
&+ \sum_I \mu^i u_c^i \frac{\partial w}{\partial k} / w \left(n^i x^i w - \sum_J \tau_j^n j^2 (n^i x^i w)^j \right) + \sum_I \zeta^i (1 - \tau^k) \frac{\partial r}{\partial k} \\
&+ \sum_{I^*} \theta^{i*} k^{i*} (1 - \tau^k) \frac{\partial r}{\partial k} + \sum_{I^*} \gamma^{i*} (1 - \tau^k) \frac{\partial r}{\partial k} + \theta^i (1 + r(1 - \tau^k)) = \theta^i / \beta.
\end{aligned}
\tag{69}
$$

Conditions (67) and (69) hold for all j and i respectively. Combining equations (68) and (69) and substituting for $1/\beta = (1 + r(1 - \tau^k))$ yields:

$$\begin{aligned} &\sum_I \theta^i \left(n^i x^i \frac{\partial w}{\partial k} - \sum_J \tau_j^n j (n^i x^i w)^j \frac{\partial w}{\partial k} / w \right) \\ &+ \psi \left(\tau^k r + K \frac{\partial r}{\partial k} + \sum_I \sum_J \tau_j^n j (n^i x^i w)^j \frac{\partial w}{\partial k} / w \right) \\ &+ \sum_I \mu^i u_c^i \frac{\partial w}{\partial k} / w \left(n^i x^i w - \sum_J \tau_j^n j^2 (n^i x^i w)^j \right) = 0. \end{aligned} \tag{70}$$

Taking condition (67) for $j = 1$ and using the fact that $\sum_I n^i x^i \frac{\partial w}{\partial k} = -K \frac{\partial r}{\partial k}$ by constant returns to scale, (70) simplifies to:

$$\begin{aligned} &\psi \tau^k r + \psi \sum_I \sum_J \tau_j^n j (n^i x^i w)^j \frac{\partial w}{\partial k} / w \\ &= \sum_I \sum_J \theta^i \tau_j^n j (n^i x^i w)^j \frac{\partial w}{\partial k} / w + \sum_I \sum_J \mu^i u_c^i \tau_j^n j^2 (n^i x^i w)^j \frac{\partial w}{\partial k} / w. \end{aligned} \tag{71}$$

Now take condition (67) and multiply by $j\tau_j^n$ and then sum over j. Substituting this into (71) eliminates all terms but one:

$$\psi \tau^k r = 0. \tag{72}$$

Hence, capital income taxes in steady state are zero, even when allowing for heterogeneity among agents and non-linear taxes on labor income.