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Animal Spirits as an Engine of Boom-Busts and Throttle of Productivity Growth

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ANIMAL SPIRITS AS AN ENGINE OF BOOM-BUSTS AND THROTTLE OF PRODUCTIVITY GROWTH

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Abstract

The news-shock literature interprets empirical news-shock identifications as signals about future productivity. Under this view, changes in productivity cause changes in expectations. I investigate an alternative interpretation whereby changes in expectations cause changes in productivity. I present a model where firms adopt the technology of a deterministic frontier, and where self-fulfilling expectational-shocks unleash a frenzy of adoption through which firms increase productivity. Consistent with the news evidence, stock prices and aggregate activity boom, yet TFP increases with a lag. Simulations using i.i.d. expectational-shocks yield moments consistent with the data, and qualitatively capture both high-frequency boom-busts as well as lower-frequency fluctuations. Finally, estimating a Beaudry-Portier style VECM on the simulated model output to identify a "news shock" recovers impulse response functions largely consistent with the Beaudry and Portier (2006) results.

Keywords: expectations-driven business cycle, technological adoption, sunspot, multiple equilibria, indeterminacy, animal spirits, technology, news shock, intangible capital, embodied, productivity, technological adoption
JEL Classification: C68,E00,E2,E3,O3,O4

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1 Introduction

Much has been made about the role of "over-exuberance" and "optimism" in the great stock market run-ups of the 20th century[1]. Moreover the fact that these events didn't exist in isolation in asset markets alone - the real economy boomed during these periods - has lent credence to some about the possibility of an independent role for the "animal spirits" of consumers and firms in driving the economy independent of fundamentals such as productivity. Yet with the advantage of hindsight, one could argue that this exuberance was not completely unfounded: the boom of the 1990's eventually led to a period of growth in total factor productivity (TFP) not seen in 20 years; the 1920's yielded similar growth in TFP.

Indeed this casual observation about a connection between stock market booms and growth in fundamentals has empirical support. Using a VAR-based empirical strategy including stock prices and total factor productivity (TFP), Beaudry and Portier (2006) investigate the connection between changes in expectations, aggregate economic activity, and long-run economic fundamentals (TFP). They find that: (i) a shock to expectations produces a boom in stock prices and aggregate economic activity; (ii) such booms can account for a material fraction of the variance of economic activity at business cycle frequencies; and (iii) these booms precede eventual growth in TFP.

Nearly universally, the theoretical literature has interpreted these finding in terms of "news shocks", whereby economic agents receive advanced information about changes in future TFP, to which their response induces an economic boom[2]. Yet from the empirical analysis alone, it is not clear which direction the causation runs: do exogenous changes in (future) fundamentals cause the change in expectations, or could exogenous changes in expectations cause the changes in fundamentals?[3]. In this paper I investigate this alternative interpretation whereby exogenous changes in sentiment can produce not just business cycle booms and busts, but also shape productivity growth itself. I develop a theoretical model where firms endogenously increase productivity by adopting new technology into production through a costly-adoption process. The frontier of technology evolves without shocks, yet the rate that firms implement new technology is a function of their self-fulfilling beliefs. When agents are suddenly optimistic, a frenzy of adoption ensues that leads to a boom in stock prices and aggregate quantities followed by eventual productivity growth, consistent with the empirical news-shock evidence. Moreover, consistent with the literature on intertemporal shocks, the relative price of capital falls. Yet in contrast with the traditional stochastic view of technology, the rapid growth in technology is a consequence of the increase in optimism, the same mechanism responsible for the business cycle boom. I also show that simulated business cycle moments in response to only these belief shocks are consistent with various unconditional moments in the data. Moreover, I demonstrate that in simulations over 100-year spans, i.i.d. expectational shocks

[1]See Shiller (2009).

[2]See Beaudry and Portier (2004), Jaimovich and Rebelo (2009), Christiano et al. (2008), Gunn and Johri (2011).

[3]In a recent empirical news-shock paper that largely corroborates the results of Beaudry and Portier (2006), Beaudry et al. (2011) write, "we find that there is a very close link between [identified mood shocks and subsequent developments in fundamentals], suggesting that agents' feelings of optimism and pessimism are at least partially rational as total factor productivity (TFP) is observed to rise 8-10 quarters after an initial bout of optimism. While this later finding is consistent with some previous findings in the news shock literature, we cannot rule out that such episodes reflect self-fulfilling beliefs."

can qualitatively account for both high-frequency boom-busts in asset prices and macro variables, as well as also occasional medium-frequency highly persistent secular "bull" and "bear" markets that seem to have characterized the 20th century U.S. data. Finally, I estimate a Beaudry-Portier style VECM on both data and on the simulated sunspot model output to identify a "news shock", and show that the procedure recovers impulse response functions from the sunspot model largely consistent with those estimated on the data.

In illustrating this alternative view, I provide not just a different way to think about the connections between expectations and technology over business cycles in general, but also way to think about several key macroeconomic episodes of concentrated technological change such as the 1920's and 1990's and their connection to the business cycle and asset markets. In particular, my model is consistent with the view that the concentrated growth in productivity that we observed during these periods was not necessarily a unique outcome of technological change. For example, during the technology boom of the 1990's in the United States we observe a dramatic rise in stock prices accompanied by an overall boom in economic activity, and then towards the latter part of the decade, a broad-based increase in productivity growth. A strict interpretation of this phenomena through the traditional stochastic view of technology would suggest that stock prices, economic activity and productivity rose uniquely given a change in the state of technology. In contrast, from the perspective of the model that I present in this paper, firms' independent enthusiasm about the newly emerging technologies increased their rates of adoption, which as a consequence increased economic activity and subsequent productivity growth.

While many existing models of sunspots, such as Benhabib and Farmer (1994), allow for theories of self-fulfilling cycles whereby exogenous changes in agents' beliefs trigger business cycle responses, many such models typically rely on contemporaneous externalities or point-in-time increasing returns that imply a "recovered" productivity that appears to rise contemporaneously. For example, Farmer and Guo (1994) show that one can generate a simulated series of business cycle and "TFP" from the model of Benhabib and Farmer (1994) to produce an observationally equivalent series to that of a standard real business cycle model with contemporaneous productivity shocks. Yet contemporaneous TFP shocks are inconsistent with the types of business cycles identified by the empirical news literature where the benefits of technology appear to follow the boom through the growth in technology.

Instead, my modeling approach draws upon the long literature on technological adoption and diffusion and incorporates two critical features emphasized by this literature: the inherently time-consuming nature of technological adoption and the role of physical capital in embodying new technology[4]. Thinking about the frontier of technology as "technological ideas", I model technological ideas as embodied in new capital, in the sense that firms must first purchase new capital in order to implement these ideas. Thus in the process of increasing their stock of physical capital through new investment, firms also effectively grow a stock of "technology capital" that represents potential productivity increases to the firm. Firms then undergo a process of costly adoption through which

[4]See Greenwood and Yorukoglu (1997), Aghion and Howitt (1998), David (1990), Helpman and Trajtenberg (1994), Jovanovic and Rousseau (2005), Carlaw and Lipsey (2002), Atkeson and Kehoe (2007), Basu and Fernald (2008), Bresnahan et al. (2002), Oliner et al. (2007) and Comin and Hobijn (2007).

they transfer resources out of goods production in order to convert this potential productivity into a from of intangible capital which serves as an additional input into goods production, thereby increasing their productivity of goods production for given levels of labour and physical capital.

The presence of these modeling features opens up interesting mechanisms for generating the indeterminacies necessary to support self-fulfilling sunspot shocks. In all simulations in the model I maintain constant returns to scale to all factors in production, yet the model can still generate indeterminacies through the interaction of physical and intangible capital. I define the term "knowledge intensity" to represent the ratio of embodied intangible capital to physical capital in production, and illustrate how the impact of investment on this ratio can enable indeterminacies. As agents invest, they simultaneously acquire both a stock of physical capital and embodied intangible capital, however due to the nature of intangible capital accumulation, these stocks accrue at different rates, and thus knowledge intensity varies over time. In the simplest form of the model, an increase in investment drives up future knowledge intensity, leading to an increase in the return to capital.

The assumption of intangible capital as an additional input into production is supported by a large body of research which highlights the role of unmeasured intangible capital in understanding changes in measured productivity and technology. Corrado et al. (2009) provide evidence for a significant role for intangible capital in growth in general, and Hall (2000) and McGrattan and Prescott (2009) argue that intangible capital grew rapidly during the technological boom of the 1990s. Both the critical role for physical capital as well as the delayed realization of productivity benefits is motivated by a host of studies following the boom of the 1990's that link information technology capital with eventual delayed productivity gains. For example, Basu and Fernald (2008) study a data-set of 40 industries in the U.S. over the period of 1986 to 2004 and find that TFP gains after mid-1990's were broad-based across industries, located primarily in information and communications technology (ICT) capital-*using* rather than *producing* industries, and that industry TFP accelerations in 2000's were positively correlated with industry ICT capital growth in mid-1990's. An overarching theme of researchers studying the role of information technology is that that realizing the benefits of IT capital requires firms to make complementary organizational investments in intangible assets such as workplace organization and business processes, highlighting a critical link between physical and intangible capital [5].

To give some concrete substance to the above discussion of technological ideas, embodiment and adoption, it is helpful to look at a specific example. One such candidate is the concept of supply chain management which was popularized in the 1990's during the IT revolution and involved the process by which firms plan and manage the flow of goods. As the late 1980's approached, the experiences of the early technological revolution gave way to widely-known transformational ideas about how to reorganize processes within firms, between firms, and between firms and customers using the new hardware and software technologies. Yet in order to implement these ideas, firms first had to purchase the necessary hardware and software infrastructure to interact with their existing capital stock. Moreover, these benefits were not immediate upon purchasing new capital; firms in general had to go through periods of implementation and learning such that the productivity

[5] See Bresnahan et al. (2002), Corrado et al. (2009), Oliner et al. (2007), Brynjolfsson et al. (2002).

benefits came well after initial investments in new capital.

In attempting to make a link between endogenous adoption and aggregate fluctuations, I am proceeding in the spirit of Comin et al. (2009), who model an endogenous adoption process in response to stochastic shifts in the technology frontier. Unlike Comin et al however, I attempt to investigate the link between endogenous adoption and aggregate fluctuations in the absence of any sudden shocks to the technology frontier, and in an environment where the only form of uncertainty is changes in beliefs about the value of adoption. From a modeling standpoint, the adoption aspects of my model share features similar to those of Comin and Hobijn (2007) who study implementation and innovation in the context of a growth model. By considering the relation between adoption and beliefs, I draw on ideas similar to those in implementation models such as Shleifer (1986), Francois and Lloyd-Ellis (2003) and Francois and Lloyd-Ellis (2008), whereby firms must choose to implement a new technology to realize its productivity benefits. These papers show how clustering of firm-level implementations can lead to non-stochastic, endogenous cycles in an environment where profits related to the innovation are short-lived. In contrast, I explore how belief-driven adoption can lead to booms followed by a delayed realization of productivity. Moreover, whereas Shleifer (1986) illustrates an environment that yields cyclical equilibria, I develop a model of stationary sunspot equilibria whereby fluctuations are driven by exogenous expectational sunspot shocks.

It is important to note that the self-fulfilling interpretation that I provide does not necessarily supplant the news-shock interpretation. In fact, the two interpretations in tandem can provide a richer view of technological booms. Presupposing the existence of news-shocks, the analysis of this paper can help us understand how the overall response of both the immediate boom and eventual productivity growth may vary from the unique response suggested by a news-shock if we allow firm's "sentiment" to vary independently of the news-shock signal. Moreover, the interpretation that I provide does not account for a potential role for forecast errors in business cycles due to "noisy news" originally illustrated by Beaudry and Portier (2004).

The remainder of the paper is structured as follows. In Section 2 I present a stripped-down simple form of the model and then in Section 3 I investigate the response of this simple model to a single sunspot shock. While the model illustrates the core mechanism of indeterminacy, on its own does not produce results qualitatively consistent with empirically-identified news shocks. As such, in Section 4 I develop the *Full Model* to address these shortcomings, examining simulations of a calibrated version of the *Full Model* economy driven only by sunspot shocks in Section 5. Section 6 concludes.

2 A simple model of investment-driven knowledge

I begin with the simplest form of the model that incorporates into an otherwise standard RBC model with imperfect competition a mechanism through which the evolution of knowledge depends on investment in physical capital. While this minimal departure from the standard model does not produce results that are qualitatively consistent with empirically-identified news shocks when the model is subject only to sunspot shocks, its simple form makes clear the role that the knowledge mechanism plays in generating indeterminacy.

The economy consists of an infinitely-lived representative household, a single final goods firm that nonetheless acts competitively, and a continuum of monopolistically competitive intermediate goods firms on a unit measure, each ith firm producing a differentiated good. Intermediate goods firms accumulate stocks of both physical and intangible capital, financing their expenditures through shares sold to households. This latter stock of intangible capital represents a firm's "technological knowledge" that I will discuss in more detail below. Upper case variables denote aggregate quantities. Lowercase variables represent individual quantities for the ith firm, except where confusion may arise where I explicitly include the index i.

2.1 Final goods firm

The final goods producer purchases intermediate goods $y_t(i)$ from intermediate goods firms and combines these goods into a single final good Y_t according to the technology

$$Y_t = \left(\int_0^1 y_t(i)^\nu di \right)^{\frac{1}{\nu}}, \tag{1}$$

where $\nu \in (0,1)$ determines the elasticity of substitution between the intermediate goods. The producer then sells the final good into the final goods market to be used as consumption for households or investment for intermediate goods firms. Each period the producer chooses its demand for each intermediate good $y_t(i)$ by maximizing its profits given by

$$Y_t - \int_0^1 P_t(i) y_t(i) di, \tag{2}$$

where $P_t(i)$ is the relative price of the ith intermediate good $y_t(i)$ in terms of the final good y_t. The resulting optimality condition then yields a demand function for the ith good as

$$y_t(i) = P_t(i)^{\frac{1}{\nu-1}} Y_t. \tag{3}$$

2.2 Intermediate goods firms and technological knowledge

Each ith intermediate goods firm produces differentiated output y_t according to the production function

$$y_t = (A_t n_t)^\alpha k_t^\theta j_t^{1-\alpha-\theta}, \tag{4}$$

where A_t is exogenous labour-augmenting technical change, n_t is labour, k_t is physical capital, and j_t is a form of intangible capital which I will call *technology capital*. Note that this specific parameterization implies constant returns to all factors in production, n, k, and j.

As a firm invests in new physical capital, its stock of physical capital evolves according to

$$k_{t+1} = [1-\delta] k_t + i_t, \tag{5}$$

where i_t is investment in units of the final good. Additionally, as it acquires this new capital, the firm also gains the ability to implement the technological ideas that the new capital embodies. These technological ideas are transformational in that they allow a firm to raise the effectiveness of its existing factors of production, in the sense of the production blueprint-altering potential of new capital stressed in work such as Bresnahan et al. (2002), Corrado et al. (2009), Oliner et al. (2007) and Brynjolfsson et al. (2002)[6]. To capture this blueprint-altering effect in a simple way, I model the technological ideas as accumulating in the form of the technology capital j_t as a by-product of investing, distinct from the stock of physical capital k_t. The accumulated stock of technology capital j_t then enters production directly as an additional input as shown in (4), such that changes in the stock of technology capital j_t alter the marginal products of labour and capital. The stock of technology capital accumulates according to

$$j_{t+1} = [1 - \delta_j] X_t j_t + z_t i_t, \tag{6}$$

which is symmetrical to that of physical capital accumulation (5) except for the presence of the terms z_t and X_t. The term $X_t = X(\Psi_t, S_t)$ is a factor external to a given firm that imposes an upper-bound of Ψ_t on the accumulation of j_t, where S_t is a vector of endogenous state variables and $X(\cdot)$ is a function that I will define below. While firms can increase their technological capabilities in this economy by acquiring new capital, they cannot do so without bound. Instead, the exogenous factor Ψ_t represents the technological frontier of embodied ideas that bounds the level of a firm's technology capital j_t. In this sense, Ψ_t represents the potential level of technological capital available once a sufficient stock of physical capital has been installed in the economy. In general, X_t imposes a *negative* externality on individual firms as the economy-wide average level of potential productivity approaches that of the frontier Ψ_t.

The term z_t is defined as $z_t = \kappa \frac{J_t}{k_t}$, where κ is a constant and J_t is the average economy-wide technology in the economy. The presence of J_t reflects a positive network externality such that at the margin, the marginal contribution of a unit of new investment to a firm's own j_t is proportional to the existing state of economy-wide technology capital. The k_t in the denominator is a scale factor ensuring the amount that a firm needs to invest to grow its technology capital is proportional to the size of the firm, and moreover, to ensure that the growth rate of j_t on the balanced growth path is independent of the scale of the economy.

I define the factor X_t by $X_t[1 - \delta_j] = 1 - \delta_j - \kappa \frac{J_t}{\Psi_t} \frac{I_t}{K_t}$, such that in a symmetric equilibrium, the aggregate law of motion for J_t follows a discrete logistic-type equation of the form,

$$J_{t+1} = [1 - \delta_j] J_t + \mathcal{Z}_t I_t, \tag{7}$$

[6] As an example of the ability of new capital to catalyze a blueprint-altering effect in production that does not necessarily imply scrapping existing capital, consider the 1990's supply chain example from earlier. Many firms were able to enhance the productivity of their existing "bricks and mortar" capital stock by using the new technologies to reorganize production of goods and services. Examples of this range from transportation and distribution companies that used IT systems to optimize scheduling and deployment, to companies such as Wal-Mart who used the technology from new capital goods to drive inefficiencies out of their supply chain selling traditional products.

where $\mathcal{Z} = \kappa \frac{J_t}{K_t}[1 - \frac{J_t}{\Psi_t}]$, which is similar to the technological adoption specifications of Nelson and Phelps (1966) and Greenwood and Yorukoglu (1997). For the parameterizations that I consider, (7) is bounded above by the frontier Ψ_t, and in general the rate of change of J_t will be a function of the relative distance of the current state of technology capita J_t to the frontier Ψ_t. To further illuminate this equation, using the aggregate equation $K_{t+1} = (1 - \delta_k) + I_t$ to substitute out I_t, we can re-write (7) as

$$\%\Delta J^*_{t+1} = \kappa \left(1 - \frac{J_t}{\Psi_t}\right) \%\Delta K^*_{t+1}, \tag{8}$$

where $\%\Delta J^*_{t+1} = \frac{J_{t+1} - J_t}{J_t} + \delta_j$ and $\%\Delta K^*_{t+1} = \frac{K_{t+1} - K_t}{K_t} + \delta_k$, which is similar to Arrow (1962) in that the rate of change of technological knowledge (net of depreciation) is proportional to the rate of change of physical capital (net of depreciation)[7]. Unlike Arrow however, the proportionality term here is the endogenous "technological gap" term $(1 - \frac{J_t}{\Psi_t})$, which reduces the rate of growth of technological knowledge as the economy approaches the frontier.

If in reality embodied technological ideas vary over time, then the potential of new capital to influence the productivity of other factors should change over time also. Variation in Ψ_t over time captures this effect. If the frontier Ψ_t is growing, there is a positive gap between J_t and Ψ_t and new capital investment increases the stock of technology capital J_t as well as the stock of physical capital K_t. In contrast, if there is no growth in technological ideas and Ψ_t stagnates, J_t will converge to its bound and stagnate also. In this case new capital investment will continue to lead to growth in the stock of physical capital K_t, yet there will be no by-product effect and technology capital J_t will remain fixed.

In equilibrium the evolution of both the firm-level and economy-wide technology capital will be governed by (7), however the behaviour of the economy will be driven by which aspects of this process individual firms internalize. In particular, firms are aware that the technological benefit is bounded by Ψ_t, and that there is a complementarity with their own technology capital and that of the overall economy, however, they do not internalize the impact of their own investment in altering the gap between the current state of technological capabilities J_t and the bound Ψ_t.

Before moving on to the typical firm's problem, there are some key features of this model economy that require emphasis. Firstly, the notion of embodiment here is different from that in the investment-specific technological change literature. In that literature, growth in embodied technology simply raises the efficiency of the marginal investment unit in capital accumulation, and thus only impacts the technology of goods production indirectly through the effect that it has on the quantity of physical capital accumulated. In contrast in this model, growth in embodied technology impacts the technology of goods production more directly through its effect on an additional input into goods production, j_t. Thus the notion of embodiment in this model stresses the impact of technological ideas in the *use of* new investment goods in the sense that using new capital allows firms to transform and reorganize their production process, a point emphasized explicitly by Oliner et al. (2007) and Basu and Fernald (2008). This lies in contrast to the notion of embodiment in the *production of* new investment goods emphasized in the investment-specific technological change

[7] I define equilibrium aggregate quantities below.

literature. Secondly, since technology capital j_t enters directly into the production function, I am implicitly assuming that there is no implementation time in the *Simple Model*: once a firm acquires new capital, the firm instantaneously implements the embodied ideas into production[8]. Finally, firms are not innovators in this model. Rather, through purchasing capital, they simple gain the ability to adopt and implement technological ideas Ψ_t that are already in the public domain. I leave the origin of these technological ideas unmodeled and external to the model.

2.3 Intermediate goods firm's problem

A typical intermediate goods firm's problem is to maximize $E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} d_{t+s}$ subject to (5) and (6), where $d_t = P_t(i)y_t - w_t n_t - i_t = Y_t^{1-\nu} y_t^{\nu} - w_t n_t - i_t$ is the firm's dividend, $\beta \frac{\lambda_{t+1}}{\lambda_t}$ is the household owner's stochastic discount factor, and y_t is given by (4).

The firm's n_t first-order condition is standard. Letting q_{kt} and q_{jt} be the Lagrange multipliers on (5) and (6), the firm's i_t, k_{t+1} and j_{t+1} first-order conditions are

$$1 = q_{kt} + q_{jt}\kappa \frac{J_t}{k_t} \tag{9}$$

$$q_{kt} = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left(\nu\theta P_t(i) \frac{y_{t+1}}{k_{t+1}} + q_{kt+1}\left[1-\delta_k\right] - q_{jt+1}\kappa \frac{j_{t+1}}{k_{t+1}} \frac{i_{t+1}}{k_{t+1}} \right) \right\} \tag{10}$$

$$q_{jt} = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left(\nu(1-\alpha-\theta) P_t(i) \frac{y_{t+1}}{j_{t+1}} + q_{jt+1}[1-\delta_j] X_{t+1} \right) \right\}, \tag{11}$$

where $X_t = 1 - \frac{\kappa}{1-\delta_j} \frac{J_t}{\Psi_t} \frac{I_t}{K_t}$.

The firm's investment first-order condition (9) shows how the firm internalizes the by-product effect of investment, considering both its benefit in adding to the physical capital stock as well as its contribution in developing technology capital in equating the marginal benefit of investment to its (unitary) cost. The firm values each of these two effects according to their respective shadow prices.

The firm's physical capital first-order condition (10) is standard with the exception of the final term that reflects the firm's recognition that that marginal contribution of each unit of investment to technology capital depends on the size of the capital stock. As such, when increasing the size of the capital stock next period this term puts downward pressure on the value of new capital since it decreases the relative contribution of investment to the new technology capital.

Equation (11) shows the firm's optimal choice of next period's technology capital. The first term inside the inner brackets on the right hand side is the marginal product of technology capital in goods production. The second term is the value of the firm's remaining stock of technology capital next period, which includes the factor X_t. Recalling that X_t is decreasing in J_t, this term capture the dynamic *dis*-economy of scale a firm faces as the economy-wide average level of potential productivity increases. Since this dynamic dis-economy of scale operates as a negative externality, no individual firm considers the impact of its own increase in i or j' in moving the overall economy

[8] In the full-model I allow for implementation/adoption lags.

towards the frontier. Nevertheless, the impairing effect this dis-economy of scale has on the value of technology capital means that new investment becomes less and less attractive time, putting a gradually increasing brake on advancing booms.

2.4 Household

The household side of the model is standard so I discuss it briefly. The representative household has preferences defined over sequences of consumption C_t and hours-worked N_t with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t), \tag{12}$$

where β is the household's subjective discount factor, $0 < N_t < 1$. The period utility function $u(C_t, N_t) = \frac{1}{1-\sigma}\{[C_t v(N_t)]^{1-\sigma} - 1\}$ is of the class of preferences described in King et al. (1988), where $\sigma > 0$ and $v(\cdot)$ is a non-negative, strictly decreasing concave function. The household's budget constraint is given by

$$C_t + \int_0^1 v_t(i) B_{t+1}(i) di = w_t N_t + \int_0^1 [d_t(i) + v_t(i)] B_t(i) di, \tag{13}$$

where v_t is the price of firm i's share, $B_t(i)$ the household's holdings of shares of firm i and w_t is the real wage. The household's problem is to maximize (12) subject to (13). Letting λ_t by the Lagrange multiplier on (13), the household's first-order conditions are

$$u_c(C_t, N_t) = \lambda_t \tag{14}$$

$$-u_l(C_t, N_t) = \lambda_t w_t \tag{15}$$

$$v_t(i) = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} [d_{t+1}(i) + v_{t+1}(i)] \right\} \quad \forall i. \tag{16}$$

2.5 Exogenous technology

The exogenous technological frontier Ψ_t and labour-augmenting technology A_t evolve as

$$\Psi_t = \Psi_{t-1} g_\Psi, \tag{17}$$

$$A_t = A_{t-1} g_A, \tag{18}$$

where g_Ψ and g_A are the deterministic growth rates of Ψ and A respectively. Note that in the context of the model, $g_\Psi > 1$ implies positive growth in technological ideas over time, and $g_\Psi = 1$ implies stagnation of technological ideas [9].

[9] The results of the model remain unchanged if I allow for a stochastic trend in either Ψ_t or A_t. Since in the paper I focus on the role of expectations in the absence of technology shocks, for simplicity I assume deterministic growth rates only.

2.6 Equilibrium and balanced growth path

I define a symmetric rational expectations equilibrium and balanced growth path of this *Simple Model* economy in the appendix. To simplify the notation, I redefine the equilibrium shadow prices in terms of household utility as $\mu_t = q_{kt}\lambda_t$ and $\zeta_t = q_{jt}\lambda_t$. On the balanced growth path, the non-stationary variables inherit trends as some function of the trend in A_t and Ψ_t, with J_t growing at the same rate as the frontier Ψ_t. Finally, the model contains a unique balanced growth path and steady state.

For reference in the discussion, I define a "naive" observed TFP as

$$T\hat{F}P_t = \hat{Y}_t - \alpha\hat{N} - (1-\alpha)\hat{K}_t, \tag{19}$$

where "hat's" denote deviations from steady state, and the $1-\alpha$ coefficient on K_t assumes constant returns to scale when one assumes (naively) that labour and physical capital are the only factors of production. Thus we can see that with this measure of TFP - which is consistent with common methods of calculating TFP, such as those in Fernald (2012) - variation in productivity is a result of mis-measurement of production-inputs due to the exclusion of intangible capital from the TFP calculation.

3 Examining the role of self-fulfilling beliefs: *Simple Model*

In this section I explore the properties of the model under a parameterization that yields indeterminacy such that a sunspot expectational shocks can produce fluctuations in the absence of any fundamental shocks such as technology shocks. Later when illustrating the *Full Model* I perform a considered calibration, however for the purpose of illustrating this *Simple Model* I simply present an illustrative parameterization with three variations variations - referred to as *Parameterizations 1*, *2*, and *3* - to show a range of behaviour, leaving discussion of the parameterization to the *Full Model*. Many of the parameters are common to the literature and I adopt a standard parameterization for these. Over the three parameterizations I then vary preferences and the technology capital parameters. With preferences given by the general form $u(C_t, N_t) = \frac{1}{1-\sigma}\{[C_t v(N_t)]^{1-\sigma} - 1\}$, in all parameterizations I let the slope of the consumption-constant labour supply curve $\gamma = N\frac{h'(N)}{h(N)} = 0$, where $h(N) = -\frac{v'(N)}{v(N)}$, but in *Parameterization 1*, $\sigma = 1$, and in *Parameterizations 2* and *3*, $\sigma = 0.25$. Additionally, in *Parameterization 1* and *2* $\delta_j = 0.022$ and $\kappa = 13.5$, whereas in *Parameterization 3*, $\delta_j = 0.0$ and $\kappa = 1$. Table 1 summarizes the calibration for the *Simple Model*.

To solve the model I linearize the non-linear system around the non-stochastic state state, and then reduce the system to a first-order linear system of the form

$$\tilde{\mathcal{Q}}_{t+1} = \tilde{\mathcal{A}}\tilde{\mathcal{Q}}_t + \tilde{\mathcal{B}}\tilde{\varepsilon}_t, \tag{20}$$

where $\tilde{\mathcal{Q}}_t = [\hat{k}_t, \hat{j}_t, \hat{i}_t]'$ and $\tilde{\varepsilon}_t = [0, 0, e^i_t]'$ and where e^i_t is an i.i.d. sunspot shock to investment, which one can interpret as "animal spirits" driving investment. Note that all the roots of $\tilde{\mathcal{A}}$ are inside

Parameters common to all variants							
β	ν	δ_k	α	θ	ϵ	g_ψ	g_A
0.995	0.9	0.022	0.68	0.16	0.16	1.0123	1.0038
Parameters specific to variant							
	σ	δ_j	κ				
Parameterization 1	1	0.022	13.5				
Parameterization 1	0.25	0.022	13.5				
Parameterization 1	0.25	0	1				

Table 1: *Simple Model* parameterization.

the unit circle, and the system is a Markovian stable process such that any value of e_t^i will set the system on a stable path that eventually returns to steady state. I elect to include investment i_t as one of the non-predetermined co-states to aid in the intuitive explanations in that we can then think about animal spirits as shocks to investment. Appendix 7.1.3 provides the details of the solution method.

3.1 Response to i.i.d. sunspot shock

Figure 1 shows the stationary impulse response functions relative to the stationary de-trended steady-state for both parameterizations of the model to a 10% i.i.d. sunspot shock to investment. For all parameterizations, in the initial period investment rises by the amount of the sunspot shock, resulting in a rise in hours-worked and output and fall in consumption. Moreover, the by-product effect of the rise in investment in the initial period leads to an increase in the stock of *both* physical K and technology capital J in the subsequent period. The behaviour of TFP, stock prices and the relative prices of K and J differs by parameterization, and I will discuss these below. As is clear from the figure, none of the parameterizations are able to fully capture a news-like boom featuring an initial co-moving surge in aggregate variables and stock prices followed by an eventual rise in TFP.

We can understand the common behaviour of hours-worked, output and consumption in response to the shock to investment over all the parameterizations by thinking about the interaction of the rise in investment with the labour market. In the initial period, the stocks of K and J are fixed in the firm's production function, and with no shift in productivity, there is no shift in labour demand. Thus in this *Simple Model*, the movement in hours-worked can only be a result of a shift in labour supply[10]. The firm's demand curve is standard and downward-sloping, and for all parameterizations, the labour supply curve is horizontal, implying that in the absence of shifts to labour demand, increases in hours-worked and therefore output are associated with a drop in consumption. Thus the initial rise in investment results in a drop in consumption and associated downward shift in labour supply, yielding the rise in hours-worked and output, and drop in the real wage shown in the figure for all parameterizations. Note that this interaction of the sunspot

[10] This lack of a shift in labour-demand in the initial period is a feature of the *Simple Model* only. In the *Full Model*, a shift in labour demand in the initial period plays and important role also.

shock with consumption and the labour market shares similarities with "investment shock" models that affect the marginal efficiency of investment, including those describing investment-specific technical change, credit and capital installation shocks, such as in Greenwood et al. (1988), Fisher (2006), Primiceri et al. (2006) and Justiniano et al. (2010). Such shocks typically imply negative co-movement between consumption and investment without additional modeling mechanisms that promote co-movement, and the drop in consumption in response to the rise in investment in the figure is consistent with this response.

Despite these similarities, the above investment-shock models are saddle-path stable, with the investment shocks be represented as a shock to fundamentals, yet the present *Simple Model* has an indeterminate equilibrium and the investment shock is a non-fundamental sunspot shock. How does this *Simple Model* allow for indeterminacy? To address this question, it is helpful to first consider why a standard RBC-style model does *not* exhibit indeterminacy in response to such an investment shock, and then highlight what aspects of this *Simple Model* alter this. As such, consider a standard RBC model with log-consumption and linear-leisure preferences as in *Parameterization 1.* Starting on some arbitrary equilibrium path, we can then conjecture a sentiment-driven rise in investment, and then determine if the system can support this level of investment as a new rational expectations equilibrium. In the standard RBC model, a rise in investment leads to a rise in physical capital K_{t+1} next period. Looking at the capital first-order condition for such a standard model,

$$1 = E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\theta\frac{Y_{t+1}}{K_{t+1}} + [1-\delta_k]\right)\right\}, \tag{21}$$

we can see that with constant returns to scale in production, this rise in K_{t+1} will lead to a fall in the marginal product of capital next period. Noting that $E_t\beta\frac{\lambda_{t+1}}{\lambda_t}$ is the inverse of the risk-free rate, under certainty equivalence we can see that this fall in next period's marginal product of capital thus leads to a fall in the risk-free rate. For this to be a new equilibrium, the fall in the risk-free rate must be consistent with the resulting behaviour of consumption and hours-worked in the economy. For the household with standard preferences separable in consumption and leisure, the fall in the risk-free rate leads to a rise in consumption, and associated leftward shift in its labour supply curve, increasing the real wage in the labour market and decreasing hours-worked and therefore decreasing output. Clearly this cannot be an equilibrium, since both consumption and investment cannot rise when output falls which would violate the equilibrium resource constraint.

To illustrate how the *Simple Model* model provides for indeterminacies relative to this standard RBC model, we can first re-write the constant returns production function for the model as

$$Y_t = (A_tN_t)^{\alpha}K_t^{\theta}J_t^{1-\alpha-\theta} = (A_tN_t)^{\alpha}K_t^{1-\alpha}\left(\frac{J_t}{K_t}\right)^{1-\alpha-\theta} = Q_t\left(\frac{J_t}{K_t}\right)^{\theta}, \tag{22}$$

using the fact that in the parameterization J_t is capital-augmenting such that $1-\alpha-\theta=\theta$, and where $Q_t = (A_tN_t^{\alpha})K_t^{1-\alpha}$. The term J_t/K_t, which I refer to as *knowledge intensity*, will have the effect of shifting what would otherwise be considered the "standard" production function $Q_t = (A_tN_t)^{\alpha}K_t^{1-\alpha}$. As firms invest in new capital, they acquire *both* a stock of embodied ideas in addition to physical capital itself. When the rate of growth of these two stock differs, next period's knowledge intensity will vary, such that a unit of investment today has an asymmetric impact on next period's marginal productions of K and J in production. To see the effect of this, we can re-write the firm's K and J first-order conditions as

$$1 = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\nu\theta \frac{y_{t+1}}{k_{t+1}} + q_{kt+1}\left[1-\delta_k\right] - q_{jt+1}\kappa \frac{j_{t+1}}{k_{t+1}} \frac{i_{t+1}}{k_{t+1}}}{q_{kt}} \right) \right\} \tag{23}$$

$$1 = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\nu\theta \frac{y_{t+1}}{j_{t+1}} + q_{jt+1}[1-\delta_j] - q_{jt+1}\kappa \frac{J_{t+1}}{\Psi_{t+1}} \frac{I_{t+1}}{K_{t+1}}}{q_{jt}} \right) \right\}, \tag{24}$$

using $P_t(i) = P_t = 1$ in equilibrium, $1-\alpha-\theta = \theta$ from the calibration, and $X_t[1-\delta_j] = 1-\delta_j-\kappa\frac{J_t}{\Psi_t}\frac{I_t}{K_t}$ from the definition of X_t.

With a high κ as in *Parameterization 1*, investment today is associated with a rise in knowledge intensity, putting upward pressure on the marginal product of K relative to the marginal product of J, which all else equal would raise the return on K relative to J[11]. As such, to equalize the overall return in equilibrium between K and J, the price q_{kt} must rise relative to q_{jt} such that there is negative co-movement between these two prices. Yet as we recall from the firm's investment first-order condition (9), with the stock of J and K fixed in the initial period, these two prices must negatively co-move in equilibrium, so the negative co-movement requirement above is consistent with equilibrium. As discussed earlier, for the separable preference specification of *Parameterization 1*, consumption must drop relative to investment for labour market equilibrium in response to the investment shock. This implies that the real interest rate must rise in the initial period, which we can see in the figure with the downward-sloping path of the marginal utility of consumption, λ. Equations (23) and (24) are consistent with this rise in the real rate of return as long as q_{kt} does not rise too far, and as long as q_{jt} falls enough[12] .

As a result, unlike in the discussion of the standard RBC model above whereby investment today leads to a fall in the rate of return, in *Parameterization 1*, investment today can lead to a rise in the rate of return through upward pressure on the marginal product of capital combined with relative price effects, providing the consistency with labour market equilibrium necessary for the indeterminate equilibrium.

In *Parameterization 2*, which is the same as *Parameterization 1* except that $\sigma = 0.25$ instead of 1 in *Parameterization 1*, the nonseparability in preferences means that the fall in consumption can now be associated with a *fall* in the marginal utility of consumption, which we can see in the figure, and thus a fall in the real interest rate[13]. As such, while q_{kt} and q_{jt} still negatively co-move, q_{kt}

[11] There is a subtle additional asymmetric impact of investment on (23) and (24) involving the right-most terms in the numerators of these two equations that works in the same direction as the impact of investment on the marginal products in each respective equation, reinforcing the effect.

[12] Note that the relation between q_{kt} and q_{jt} in (9) is driven in part by the fact that firms internalize that their own investment leads to growth in their stock stock of technology capital, but do not internalize their role in this investment in pushing the economy closer to the frontier Ψ_t.

[13] We can see this impact of preferences on λ with the linearized form of the consumption first-order condition, $\hat{\lambda}_t = -\sigma\hat{C}_t + (\sigma-1)\psi\hat{N}_t$, where for $\sigma \neq 1$, λ responds to both C *and* N. For $\sigma = 0.25$, the upwards pressure of a

now falls and q_{jt} now rises, consistent with an initial fall in the real rate of return. Since for these preferences this fall in the real interest rate still implies a fall in consumption, the labour market dynamics are still consistent with equilibrium.

In *Parameterization 3*, which is the same as *Parameterization 2* except that κ is now low, next period's knowledge intensity now falls since J grows slower than K initially, reversing the asymmetric effect between (23) and (24). As such, q_{kt} must full less and q_{jt} must rise more, which we can see in the figure.

Having addressed how the model supports indeterminacy, we can now consider the behaviour of the remaining variables. Firstly, the behaviour of *knowledge intensity* J_t/K_t also governs the path of the naive productivity calculation in the *Simple Model.* Combining our definition of TFP given in (19) with the production function gives

$$TFP = A_t^{\alpha} \left(\frac{J_t}{K_t} \right)^{\theta}. \tag{25}$$

Adjusting for the effect of labour-augmenting technology A_t which is consistent with the stationary IRFs, the %-change in TFP will be proportional to the %-change in knowledge intensity. Thus for higher values of κ where J grows faster than K, we would expect current investment to lead to a rise in TFP next period, which is consistent with the response of *Parameterizations 1* and *2* that use a higher κ. Convesely, for lower values of κ where J grows slower than K, we would expect a current investment to lead to a fall in TFP next period, which is consistent with the response of *Parameterization 3* in the figure that uses a lower κ.

We turn now to the behaviour of stock prices. In the model the time-path behaviour of stock prices varies relatively significantly depending on the particular parameterization, especially with regards to the parameter κ. In general however, the stock price is influenced significantly by relative price of J with respect to that of K, and we can see in the figure that the stock prices rises for Parameterization 2 and 3 where q_j rises and q_k falls, and falls in Parameterization 1 where q_j falls and q_k rises.

Finally, it is clear form the figure that the persistence of the IRFs differs substantially between the parameterizations. We can understand this by again considering the effect of the external factor X_t. The increase in investment also leads to a fall in the external effect X that captures the un-internalized effect of investment in pushing J closer to the frontier Ψ, the process of which makes investment less and less productive in growing J. Recalling that the rate that additional investment pushes J towards the frontier Ψ_t is proportional to the parameter κ, we can see from the figure that the larger value of κ in *Parameterization 1* and *2* is associated with a much more rapid drop in X than with the small value of κ in *Parameterization 3.* As a result, the impairing effect of this term on the relative price of J in the J first-order condition plays a significant role in the dynamics over time in *Parameterization 1* and *2*, dramatically reducing the persistence of the response of investment, consumption and hours-worked as firms' incentive to continue investing drops rapidly, relative to the much more gradual and persistent response of *Parameterization 3.*

drop in C on the marginal utilization of consumption is offset by the downwards pressure from the rise in N.

3.1.1 Dependence of indeterminancy on production parameterization

If we relax the assumption of constant returns to N, K and J, we can investigate how the potential for indeterminacy varies with the parameterization of the production function and overall returns to scale, re-writing the production function in the form

$$Y_t = (A_t N_t)^\alpha K_t^\theta J_t^\epsilon \tag{26}$$

where ϵ is no longer restricted to equal $1 - \alpha - \theta$. I use the baseline *Parameterization 1* where preferences are separable, with the exception of θ and ϵ which I leave variable. I then solve the model for each combination of θ and ϵ on a 150x150 grid ranging from 0 to 0.8 for each of these parameters, determining the stability properties of the system for each combination. Figure 2 shows the results of this exercise. Also shown on the figure is a line of constant returns (CRS line) such that $\alpha + \theta + \epsilon = 1$, and the line of equal contribution to capital share, $\theta = \epsilon$. For reference, note that *Parmeterization 1* lies at the intersection of these two lines. As the figure shows, the model exhibits indeterminacies for a very wide range of degrees to scale, ranging from severe decreasing returns to scale to large increasing returns to scale. In general the returns to scale is not important for indeterminacies; rather, the figure shows that the relative size of the elasticities of K and H in production is important. As ϵ gets small relative to the θ, the influence of the knowledge intensity term J/K diminishes and the model moves towards saddle-path stability.

4 Full Model

To address the shortcomings of the *Simple Model*, in this section I add a costly-adoption element as well as variable capacity utilization, referring to the resulting model as the *Full Model.* Relative to the *Simple Model*, only the structure of the intermediate goods technology is different.

4.1 Intermediate goods firms

One salient feature of technological implementation/adoption is that it takes *time* to implement a technology, and the literature on technological diffusion and adoption typically describes the adoption process as a period through which firms learn, experiment and refine the technological ideas into their production process. Moreover this process is typically described as being costly to the firm in the sense that it must devote resources to implementation and adoption that would otherwise be engaged in current profit-enhancing activities.

To model such a technology adoption process in this environment with technology capital j_t, I introduce an additional intangible capital variable h_t called *firm-specific productivity.* As in the *Simple Model*, j_t continues to represent the stock of technological ideas that the firm can implement into production given its capital acquisitions. Unlike the *Simple Model* however, this stock of technological capital does not immediately impact production. Rather, the firm must devote resources to implement technology capital into production, after which it takes the form of h_t, which acts as a stock of technological capability operative in the firm's production process. Thus in a nutshell, the firm undergoes costly adoption to convert its technological capital j_t into *actual* firm-specific productivity h_t.

Since h_t represents knowledge operative in production, h_t now enters directly into the production process instead of j_t, such that a typical intermediate goods firm produces differentiated output y_t according to the production function

$$y_t = (A_t n_{yt})^{\alpha}(u_t k_t^{\frac{\theta}{1-\alpha}} h_t^{\frac{1-\alpha-\theta}{1-\alpha}})^{1-\alpha}, \tag{27}$$

where n_{yt} is labour engaged in goods production and u_t is the utilization rate of the "effective capital" composite $k_t^{\theta} h_t^{1-\alpha-\theta}$. Note again that as in the *Simple Model*, this parameterization of the production function implies constant returns to all factors n_y, k, and h.

The firm bears the cost of increased capacity utilization in terms of increased depreciation of physical capital, such that capital accumulation now follows

$$k_{t+1} = [1 - \delta(u_t)]k_t + i_t, \tag{28}$$

where the function $\delta(\cdot)$ is a standard time-varying cost of utilization as a convex function of the utilization rate, with properties $\delta'(\cdot) > 0$, $\delta''(\cdot) > 0$.

As in the *Simple Model*, as firms invest in physical capital, they accumulate technology capital j_t as a by-product according to (6). To model the adoption process through which firms implement this technology capital j_t into production, I follow an approach similar to that of Howitt and Mayer-Foulkes (2005) and Comin and Gertler (2006) and assume that firms implement the technology of their technology capital j_t into goods production with a stochastic success rate, where the firm's success rate is a function of the labour resources that the firm directs towards adoption, denoted as n_{ht}. A firm that attempts to increase its productivity from the its current state h_t to the potential j_{t+1} through implementation is successful with probability ω_t. As such, the firm's expected productivity next period is $E_t h_{t+1} = \omega_t j_{t+1} + [1-\omega_t]h_t$. The probability ω_t that the firm is successful is an increasing function of the labour resources that the firm directs towards adoption, such that $\omega_t = \Phi(n_{ht}) = n_{ht}^{\eta}$, where $0 < \eta \leq 1$, and n_{ht} is labour that the firm directs to the adoption process. Substituting this definition of ω_t into the expression for expected productivity next period and re-arranging yields a partial-adjustment equation of the form,

$$E_t h_{t+1} = h_t + \Big[j_{t+1} - h_t \Big] \Phi(n_{ht}). \tag{29}$$

Equation (29) has the property that if the realizations of $\Phi(n_{ht}) < 1$, h_t is bounded above by j_{t+1}, imposing a type of "limit to learning" on the firm such that the firm cannot increase its firm-specific productivity h_{t+1} beyond its technology capital j_{t+1}. In contrast to similar adoption specifications in Greenwood and Yorukoglu (1997), Comin and Hobijn (2007) and Nelson and Phelps (1966) where the bound of adoption is tied to the overall technological frontier outside of the control of the firm, here the bound j_{t+1} is endogenously controlled by the firm as a function of its history of investment in physical capital, acting as an “effective frontier” facing the firm.

Finally, firms purchase total labour n_t at wage w_t, and allocate it between goods production and adoption according to

$$n_{y_t} + n_{ht} = n_t. \tag{30}$$

4.1.1 Intermediate goods firm's problem

The typical intermediate goods firm's problem is the same as in the *Simple Model*, with the addition of the constraints (29) and (30), and with the production function and capital accumulation equations now given by (27) and (28) respectively. Defining q_{ht} as the Lagrange multiplier on (29) in addition to those already defined in the Simple Model, the firm's first-order conditions are now,

$$w_t = \nu\alpha \frac{P_t(i)y_t}{n_{yt}} \tag{31}$$

$$w_t = q_{ht}\Big[j_{t+1} - h_t\Big]\Phi'(n_{ht}) \tag{32}$$

$$q_{kt}\delta'(u_t)k_t = \nu(1-\alpha)\frac{P_t(i)y_t}{u_t} \tag{33}$$

$$1 = q_{kt} + q_{jt}\kappa\frac{J_t}{k_t} \tag{34}$$

$$q_{kt} = E_t\Big\{\beta\frac{\lambda_{t+1}}{\lambda_t}\Big(\nu\theta\frac{P_{t+1}(i)y_{t+1}}{k_{t+1}} + q_{kt+1}\left[1-\delta(u_{t+1})\right] - q_{jt+1}\kappa\frac{J_{t+1}}{k_{t+1}}\frac{i_{t+1}}{k_{t+1}}\Big)\Big\} \tag{35}$$

$$q_{jt} = q_{ht}\Phi(n_{ht}) + E_t\Big\{\beta\frac{\lambda_{t+1}}{\lambda_t}q_{jt+1}[1-\delta_j]X_{t+1}\Big\} \tag{36}$$

$$q_{ht} = E_t\Big\{\beta\frac{\lambda_{t+1}}{\lambda_t}\Big(\nu\epsilon\frac{P_{t+1}(i)y_{t+1}}{h_{t+1}} + q_{ht+1}\Big[1-\Phi(n_{ht+1})\Big]\Big)\Big\}. \tag{37}$$

Equations (31) and (32) are the firm's y-hours and h-hours first-order conditions respectively, and show that the firm allocates labour between goods production and adoption to equalize the value of the marginal products of labour in each use. Note in (32) that the gap term $[j_{t+1} - h_t]$ increases the technical effectiveness of hours in adoption, whereas the shadow value q_{ht} expresses the marginal value of adoption in terms of the firm's output. Inspection of (31) and (32) reveals that all else equal, an increase in either q_{ht} or $[j_{t+1} - h_t]$ will shift the firm's total labour demand n_t rightward in wage-hours space. Thus unlike the *Simple Model*, shifts in labour demand due to adoption will play a more prominent role in this *Full Model*.

The firm's utilization first-order condition (33) is standard and shows simply that in choosing u, the firm equates the marginal product of u in goods production to the marginal cost of adjusting utilization, where the marginal cost reflects both physical depreciation of physical capital, and the relative value of physical capital in terms of the consumption good.

The firm's investment first-order condition (34) and physical capital first-order condition (35) are identical to that in the Simple Model, with the exception that (35) now reflects the impact of variable capacity utilization. The firm's technology capital first-order condition (36) is similar to that of the *Simple Model*, with that difference that the first term on the right-hand side now shows that the dependence of the value of technology capital on the value of the marginal impact of technology capital in the technology adoption process.

Finally, (37) describes the firm's optimal choice of firm-specific productivity next period, h_{t+1}. The first term on the right-hand side is the discounted value of the marginal product of h in goods production, and the second term is the discounted value of the remaining h next period. The $\Phi(n_{ht+1})$ term reflects the firm's recognition that increasing h next period reduces the growth rate of h next period since it will push it closer to its effective frontier j.

4.2 Equilibrium and balanced growth path

I define a rational-expectations equilibrium and balanced growth path of this *Full Model* economy in the appendix. Note now that the equilibrium shadow prices in terms of household utility are given as $\mu_t = q_{kt}\lambda_t$ and $\zeta_t = q_{jt}\lambda_t$. As in the *Simple Model*, on the balanced growth path, the non-stationary variables inherit trends as some function of the trend in A_t and Ψ_t, with both H_t and J_t now growing at the same rate as the frontier Ψ_t. As with the *Simple Model*, the *Full Model* contains a unique steady state.

I define adjusted equilibrium observed adjusted total factor productivity (TFP) as

$$T\hat{F}P_t = \hat{Y}_t - \alpha\hat{N}_{yt} - (1-\alpha)\hat{u}_t - (1-\alpha)\hat{K}_t \tag{38}$$

which now controls for capacity utilization, and where N_{yt} captures total labour adjusted for "effort". Note again that the $1-\alpha$ coefficient on K_t and u_t assumes constant returns to scale when one assumes (naively) that labour and physical capital are the only factors.

5 Examining the role of self-fulfilling beliefs: *Full Model*

In this section I explore the properties of a calibrated version of the *Full Model*. Solving and reducing the model as for the *Simple Model* now gives $\tilde{\mathcal{Q}}_t = [\hat{k}_t, \hat{h}_t, \hat{j}_t, \hat{\imath}_t]'$, and $\tilde{\varepsilon}_t = [0, 0, 0, e^i_t]'$, where e^i_t is again an i.i.d. sunspot shock to investment. Appendix 7.2.3 provides the details of the solution method.

To calibrate the *Full Model*, I assign values to parameters using either values typical to the literature, or determined from restrictions on the model steady-state. Beginning with the standard parameters, I set the household's subjective discount factor $\beta = 0.995$, implying an annualized risk-free of 2%, and the curvature on the final goods aggregator ν to 0.9, implying a markup of approximately 11%. For the convex cost of capacity utilization, my solution method requires that I need only specify the elasticity of marginal depreciation to utilization, $\epsilon_u = \frac{\delta''(u)}{\delta'(u)}u$, which I set to 1.4 in accordance with Greenwood et al. (1988), and the steady state value of depreciation $\delta(u_{ss}) = \delta_k$, which I set to 0.022.

To promote comovement, I use preferences not separable in consumption and leisure. Within the general class of preferences $u(C_t, N_t) = \frac{1}{1-\sigma}\{[C_t\upsilon(N_t)]^{1-\sigma} - 1\}$ defined in King et al. (1988), I use specific the nonseparable indivisible labour preferences used in King and Rebelo (2000) that involve specifying the $\upsilon(N_t)$ function as $\upsilon(N_t) = \left[\left(\frac{N_t}{\mathcal{H}}\right)\upsilon_1^{\frac{1-\sigma}{\sigma}} + \left(1 - \frac{N_t}{\mathcal{H}}\right)\upsilon_2^{\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}}$, where $\mathcal{H}$ is the fixed shift length, and υ_1 and υ_2 are constants representing the leisure component of utility of the underlying employed group (who work $\mathcal{H}$ hours) and unemployed group (who work zero hours) respectively. Basu and Kimball (2002) empirically investigate the general class of King et al. (1988) preferences not additively separable in consumption and leisure and find estimates of the *labour held constant elasticity of intertemporal substitution in consumption* of 0.5-0.67 during the sample period 1982 to 1999, larger than the near-zero values of the intertemporal elasticity of consumption estimated by Hall (1988) that assumed no non-separabilities in consumption and leisure. During the sample period 1949 to 1982 they estimated this quantity to be not significantly different from zero, in line with the results of Hall (1988). I choose $\sigma = 4.5$, which yields a value of the labour held constant elasticity of intertemporal substitution in consumption of 0.22 to represent the entire sample period. I then set the average household's share of time allocated to market work N_{ss} to 0.3.

For $\sigma > 1$ these preferences are not separable in consumption and leisure, and for $\sigma = 1$ they reduce down to standard separable indivisible labour preferences with log-consumption and linear leisure. In the results section, I use the preferences non-separable in consumption and leisure with $\sigma = 4.5$ as part of the *Full Model* calibration, but also discuss the results for the more standard log-consumption and linear leisure associated with $\sigma = 1$, referring to these as *separable preferences*.

Many of the remaining parameters depend upon the presence of intangible capital in the production function. The concept of intangible capital that I present in this paper has a very specific interpretation, and thus should only be considered one particular sub-category of the wider array of intangible capital studied in the literature. The work of Corrado et al. (2009) (henceforth CHS) is especially helpful in this regard, since they are able to quantify the contribution to growth of a wide array of sub-classes of intangible capital based on observed expenditures on "investment" into each class of intangible capital. The intangible capital that I explore most closely aligns with the "Economic competencies - Firm-specific resources (FSR)" category of CHS, which they say includes "the costs of employer-provided worker training and an estimate of management time devoted to enhancing the productivity of the firm".

I also assume that the firm-specific productivity H acts as a capital-augmenting growth factor, implying that $1 - \alpha - \theta = \theta$. This means that intangible capital contributes 50% to the share of total capital in production (where the total capital share is the sum of the physical capital share and intangible capital share), which is in the range of the estimate from CHS who find that all forms of intangible capital together contribute about 40% of the total capital share in production. This notion of capital augmentation also seems reasonable given my modeling assumptions about the dependence between intangible and physical capital, and as well is consistent with my specification of technology capital J_t (which is converted into intangible capital H_t) accumulation whereby J_t accumulates relative to the size of the capital stock K_t. Thus under this capital augmenting calibration, we can think of firm-specific productivity H_t as an index which grosses up the impact of the stock of capital K_t in goods production, depending on the economy's knowledge of that capital stock.

Calculating α and θ based on this information requires determining the share of labour engaged in goods production. The overall labour share in output S_n is a function of labour in both the Y and H uses, such that $S_n = S_{ny} + S_{nh}$, where $S_{ny} = \nu\alpha$ and S_{nh} are the Y and H labour shares in output respectively. I set the share of total labour S_n to 0.64, determine S_{nh} using results from the literature on intangible capital, and then determine S_{ny} as the difference. CHS report spending data by firms on this FSR form of intangible capital back to the 1950's. In my model this corresponds to the term wN_h, which is the firm's costly-component of adoption. I convert CHS's data into shares of income, analogous to S_{nh} in my model, and calculate that the average of this share from the from 1950's to end of 1990's gives 2.4%. As such, I use $S_{nh} = 2.5\%$. This and the information above then gives $S_{ny} = 0.64 - 0.025 = 0.615$, and then $\alpha = 0.68$ and $\theta = 0.16$.

The parameters of technological adoption that require calibration are the depreciation rate of potential productivity, δ_j, the deterministic growth rate of the exogenous frontier, g_ψ, the curvature on labour engaged in H-accumulation, η, and the constant term κ in J accumulation. Given the capital-augmenting form of H in the production function, and the conceptual link of embodied knowledge being related to the stock of capital, I assume a symmetrical depreciation rate for J such that $\delta_j = \delta_k = 0.022$. To calibrate g_Ψ, my strategy involves assigning the sources of growth in the model between that due to growth in the embodied technological frontier Ψ_t and that due to "all other factors", which I lump into labour-augmenting technological progress A_t. I begin with an observed annual growth rate of output of 2.2%, determine Ψ_t's role in this based on implications of the model, and then treat the growth in A_t as a residual. While we don't observe Ψ_t directly in the data, the model implies that the growth rate of firm-specific productivity H_t is equal to that of the technological frontier Ψ_t on the balanced growth path. Since H_t is a form of intangible capital, by determining a realistic growth rate of this form of intangible capital, we can infer a growth rate for Ψ_t. Corrado et al. (2009) estimate growth of the FSR stock as 5.3% from 1973-95, and 6.2% from 1995-2003. I use 5%, implying that H_t, and therefore Ψ_t, grow at 5% annually, implying quarterly growth rate $g_\psi = 1.0123$. This then yields a quarterly growth rate for "all other factors" of $g_A = 1.0038$.

We can determine η by implicitly solving the expression for the ratio of labour shares from the steady state of the model, $\frac{S_{nh}}{S_{ny}} = \frac{N_h}{N_y} = \frac{(1-\alpha-\theta)(\eta/\alpha)(g_\Psi-1)}{g_\Psi/(\beta g_y^{1-\sigma})-(1-N_h^\eta)}$, where N_h is is determined by the labour shares and $N_{ss} = 0.3$. Solving this expression yields $\eta = 0.85$.

To calculate the remaining parameter, κ, I choose it in order to produce a moment condition consistent with the data. In the model, κ parameterizes the rate at which J_t grows outside of steady state. To illustrate the impact of κ, we can can compare the symmetry of K (ignoring utilization) and J accumulation equations, given by

$$\begin{aligned}
\hat{k}_{t+1} &= \frac{(1-\delta_k)}{g_y}\hat{k}_t + \frac{i}{k}\hat{i}_t \\
\hat{j}_{t+1} &= \frac{(1-\delta_j)}{g_\Psi}\hat{j}_t + \tilde{z}\frac{i}{j}[\hat{i}_t + \hat{z}_t],
\end{aligned} \tag{39}$$

Parameters common to all variants											
	β	ν	δ_k	α	θ	ϵ	g_ψ	g_A	η	δ_j	κ
	0.995	0.9	0.022	0.68	0.16	0.16	1.0123	1.0038	0.85	0.022	13.5
Parameters specific to variant											
	σ	ϵ_u									
Full	4.5	1.44									
Sep, no util	1	∞									
Non-sep, no util	4.5	∞									

Table 2: *Full Model* parameterization.

where $\hat{z}_t$ is the linearization of the stationary "gap" term $\tilde{z}_t = \kappa \frac{g_y}{g_\Psi}\frac{j_t}{k_t}(1 - \frac{j_t}{g_\Psi})$. Note that since $\hat{z}_t$ is a function of endogenous states, it is predetermined in period t, and thus the contemporaneous percentage change in j_{t+1} will be $\tilde{z}\frac{i}{j} = \left[\kappa \frac{g_y}{g_\Psi}(1 - \frac{j}{g_\Psi})\right]\frac{i}{k} = \kappa z^* \frac{i}{k}$, for a given percentage change in i_t, where $z^* = \frac{g_y}{g_\Psi}(1 - \frac{j}{g_\Psi})$. This compares with the contemporaneous percentage change in k_{t+1} of simply $\frac{i}{k}$ for a given percentage change in i_t. Thus for a given change in investment in period t, k_{t+1} will increase by $\frac{i}{k}$ percent, whereas j_{t+1} will increase by $\kappa z^* \frac{i}{k}$ percent. In the subsequent periods after period t, the growth rate of j will also be influenced by the evolution of the gap term $\hat{z}_t$, but from inspection we can see that $\hat{z}_t$ will eventually decrease as the economy moves towards the frontier, which will put downward pressure on the impact of new investment. For the above calibration, $z^* = 0.069$, which means that the impact of investment on j_{t+1} is very small unless κ is considerably larger than unity. The faster that J_t accumulates, the more immediate is the return for the firm to transferring labour out of goods production into adoption. As such, κ has a large impact on the properties of hours-worked. I set the parameter such that the model produces a cross-correlation of hours with output of 0.88, as in the data, giving a value of $\kappa = 13.5$, and $\kappa z^* = 1.2399$. This means that in the initial period of the shock, j will growth slightly faster than k for given investment, but this will slow considerably over time as the decreasing returns of the gap term set in. Table 2 summarizes the calibrations for the *Full Model*.

5.1 Response to i.i.d. sunspot shocks

Figure 3 shows impulse response functions relative to trend of the model economy to a 10% i.i.d. sunspot shock on investment. As in the *Simple Model*, in period 1 I_t rises by the amount of the shock, reflecting the change in value from the belief shock, leading to both a rise in total hours-worked and output. Unlike the *Simple Model*, consumption now rises, accompanied by a rise in stock prices and drop in the relative price of capital, as well as a small initial rise in the real wage. Additionally, the boom in investment leads to an increase in the rate of technological adoption, shown by the rise in adoption hours-worked dedicated to adoption, N_h, in addition to a rise in hours-worked dedicated to production, N_y. As a consequence, firm-specific productivity H and TFP increase gradually with a lag. Moreover, in contrast to the *Simple Model*, both the increase in the rate of costly-adoption and the accompanied rise in capacity utilization now lead to a rise in labour demand. Additionally, from the figure we can see that many of the variables such as hours-worked and output respond in a hump-shaped manner, a feature that exists strongly in the data. Overall, the model behaviour is qualitatively consistent with the new-shock interpretation: an initial boom in stock prices and aggregate activity, followed by eventual growth in TFP.

5.2 Understanding the *Full Model* relative to *Simple Model*

I now explore the impact of the presence of costly-adoption, non-separability and capacity utilization in the *Full Model* relative to the *Simple Model*, which contains none of these elements. To do so, I first "remove" non-separability and capacity utilization from the *Full Model*, and then add each back sequentially.

5.2.1 Impact of costly-adoption

Firstly, I set $\sigma = 1$, which gives the *separable preferences* parameterization equivalent to the log-consumption and linear leisure preference specification of *Parameterization 1* of the *Simple Model*. Additionally, I set an an extremely high cost of capacity utilization such that utilizations remains constant. As such, compared to *Parameterization 1* of the *Simple Model*, this exercise differs only by the addition of the adoption process for firm-specific productivity H.

Figure 4 shows the impulse response of this modification of the *Full Model* to the same investment sunspot shock. Similar to the *Simple Model*, investment and hours rise and consumption falls. Note however that compared to *Parameterization 1* of the *Simple Model*, q_{kt} now falls, and the overall response of all variables is considerably more persistent. Both of these effects are primarily due to the presence of firm-specific productivity H. Since firms must now undergo costly-adoption to implement technology capital J into production and convert it into H, firm-specific productivity is slow to grow. In fact, as firms acquire new capital in response to the shock, K grows faster than H, and thus knowledge intensity, which I now define for this model economy as H_{t+1}/K_{t+1}, *falls* initially, and then grows gradually over time. Thus unlike *Parameterization 1* of the *Simple Model* (and similar to *Parmaterization 3*), there is no amplifying effect on the marginal product of capital next period, and thus q_{kt} falls such that the increase in investment is still associated with a rise in the real rate of return, consistent with the separable preferences. The rise in the risk-free rate shifts labour supply rightward, but now costly-adoption shifts labour demand shifts rightwards also, and both of these work to push up hours and therefore output. Note also that this fall in and then eventual sustained rise in knowledge intensity is a significant driver of the dynamics of TFP in the *Full Model* from the second period onwards.

Note from Figure 4 that q_{jt} rises initially. In the *Simple Model* where J entered directly into production, we could think about this in terms of the asymmetry in the relative marginal products of K and J in production. Now however, J is an input into costly-adoption only. We can understand the behaviour of this price by first looking at the equilibrium firm N_{ht} first-order condition,

$$w_t = q_{ht}\Big[J_{t+1} - H_t\Big]\Phi'\Big(N_{ht}\Big). \tag{40}$$

As firms invest in new capital, it leads to an increase next period's potential productivity J_{t+1}, effectively pushing out the effective frontier that the firm faces, and shifting its demand for N_{ht} labour. Firms respond by increasing N_{ht}, which drives up the marginal impact of J_{t+1} in H accumulation, since it is non-diminishing in H. Looking at the equilibrium first-order condition for J_{t+1},

$$q_{jt} = q_{ht}\Phi(N_{ht}) + E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}q_{jt+1}[1-\delta_j]X_{t+1}\right\}, \tag{41}$$

we can see that all else equal this rise in the marginal impact of J_{t+1} in H accumulation, shown in the first term on the right-hand side, drives up the relative price q_{jt}. Thus, as in the *Simple Model*, there is an asymmetry in how investment impacts the initial marginal contribution of K and J, thereby producing the negative co-movement in q_{kt} and q_{jt} consistent with that implied by the firm's investment first-order condition (34).

I now show how the potential for indeterminacy varies with the parameterization of the production function as in the *Simple Model*. Again relaxing the assumption of constant returns to N, K and H, I re-write the production function in the form

$$Y_t = (A_tN_t)^{\alpha}K_t^{\theta}H_t^{\epsilon} \tag{42}$$

where ϵ is no longer restricted to equal $1-\alpha-\theta$, and then solve the model for each combination of θ and ϵ on a 150x150 grid as in the *Simple Model*. Figure 5 shows the results of this exercise. In general the region of indeterminacy is similar to that in the *Simple Model*, however now the presence of costly-adoption produces a larger unstable region for high degrees of overall returns to scale.

5.2.2 Impact of costly-adoption and non-separable preferences

Relative to the previous exercise, we can now add back the effect of non-separable preferences by setting $\sigma = 4.5$. Figure 6 shows how the stability of this permutation of the model depends on the parameterization of the production function, again repeating the approach described earlier. Note now that the region of indeterminacy is shifted towards the origin so that indeterminacies exist only for overall decreasing returns to scale. Moreover, at the intersection of the CRS line and $\theta = \epsilon$ line, the model is saddle-path stable. As such, I do not show an impulse response for this permutation.

The reduction in the indeterminate space for this parameterization of $\sigma = 4.5$ is a result of the indeterminacy-suppressing impact of the low intertemporal elasticity of substitution (IES) associated with $\sigma = 4.5$, consistent with the result of Bennett and Farmer (2000) who in a different model find that with nonseparable preferences, a high IES makes it easier to generate an indeterminate equilibrium, whereas a lower IES makes it harder. In the context of my model, we can see the effect by examining the linearized equilibrium household consumption and hours-worked first-order conditions using the general non-separable preference specification,

$$\hat{\lambda}_t = -\sigma\hat{C}_t + (\sigma-1)\psi\hat{N}_t \tag{43}$$

$$\hat{w}_t = \hat{C}_t + \gamma \hat{N}_t \tag{44}$$

where "hat's" denote %-deviations from steady state, λ_t is the marginal utility of consumption, ψ is a constant pinned down by the steady state of the model, and γ is constant determined from preference parameters. Equation (43) clearly shows that for $\sigma > 1$, the marginal utility of consumption is increasing in hours-worked. Equation (44) is the *consumption-constant* labour supply curve, with slope γ. In the previous case with separable preferences, the drop in consumption and associated rise in current marginal utility of consumption λ_t was associated with a rightward shift in the labour supply curve, which along with the shift in labour demand, increased output. In this case, since the marginal utility of consumption is increasing in hours-worked, the household will meet the rise in investment with a rise in hours-worked and less of a drop in consumption, or a small *rise* in consumption. As a consequence, labour supply now either shifts rightward less, or leftward (if consumption rises), putting downward pressure on output. Thus the rightward shift in labour demand is left with a larger role in expanding output enough to fund both an increase in consumption and investment in equilibrium. In general, this impact on the labour supply curve is larger for higher returns to scale in production, since the marginal product of capital rises more, and thus the interest rate rises more, impacting the marginal utility of consumption more. Thus as the figure shows, for the baseline parameterization, the region of indeterminacy is limited to values of θ and ϵ associated with decreasing returns to scale in production. Note that the magnitude of the increase in labour demand can be amplified by either increasing ϵ relative to θ, which can be seen from the figure, or increasing κ, which is not shown [14].

5.2.3 Impact of costly-adoption, non-separable preferences and capacity utilization

Finally, we return to the Full Model. Figure 7 shows the response of the model economy described in the calibration to the same investment sunspot shock. This figure is the same case as Figure 3 except with panels consistent with the previous exercises. Note that again q_k falls and q_j rises, however, compared to the previous exercises, q_h now rises and then falls.

The presence of capacity utilization in the model has an interesting impact through the interaction of the falling relative price of capital q_k. Similar to the mechanism described in Greenwood et al. (1988) and as well in Jaimovich and Rebelo (2009), when q_{kt} falls, the cost of utilization falls, thereby increasing the optimal choice of utilization, and shifting labour demand rightwards. Additionally, this effect is amplified by the now larger increase in labour demand for costly adoption due to the rise in both q_{ht} and j_{t+1}. The combined effect of these two labour-demand shifting mechanisms can be seen through the small rise in the real wage in the figure.

Figure 8 shows how the stability of the *Full Model* depends on the parameterization of the production function, again repeating the approach described earlier. Note that the presence of capacity utilization expands the region of indeterminacy significantly through its impact on labour demand.

[14] Increasing κ enough can expand the region of indeterminacy away from the origin enough such that intersection of the CRS and $\theta = \epsilon$ lines lie within the region of indeterminacy

	Std. dev'n relative to Y			
	C	I	N	w
Data	0.74	2.93	0.99	0.56
Model	0.75	3.71	1.09	0.55

	Cross corr'n with Y			
	C	I	N	w
Data	0.88	0.8	0.88	0.12
Model	0.95	0.87	0.88	0.32

Table 3: Select data and model moments.

5.3 Simulation conditional on stochastic sunspot

Thus far we have examined the response of the model economy to a single sunspot shock. I now perform various simulations on the *Full Model* to explore if the model economy can provide a theory of cycles in general that is consistent with the data, assuming the model economy is driven only by an i.i.d. sunspot shock process. To determine the variance of the shock process, I set its standard deviation such that the resulting sample standard deviation of HP-filtered output in the simulated series is consistent with that measured in the data. Accordingly, I set the standard deviation of the sunspot shock process to 3.2, yielding a standard deviation of HP filtered output of 1.81 as reported by King and Rebelo (2000).

5.4 Sample moments

Table 3 shows the sample moments for the *Full Model* driven by the sunspot process using a sample period of 10000 quarters, with an additional initial period of 1000 quarters that is discarded to minimize the influence of boundary conditions. The counterpart data moments are those reported in King and Rebelo (2000).

Firstly, recall that κ was calibrated to yield a cross-correlation of labour and output equal to 0.88, as in the data. As the table shows, the model subject to a sunspot shock only does a relatively good job of matching moments with the data, especially with the relative standard deviations of consumption and the real wage, and the cross correlation of investment. Both investment and hours worked are more volatile than in the data, and consumption and the real wage are more highly correlated with output than in the data. With regards to the relative volatility of hours-worked, note that that total hours in the model economy is the sum of hours in Y and H production, whereas only Y-hours are engaged in Y-production, such that the link between hours and output is potentially not as tight as in a standard RBC model.

Figures 9 and 10, corresponding to the data and the model simulation respectively, show auto-correlations of output *growth*, and dynamic cross-correlograms of stock prices and real GDP, stock prices and adjusted TFP, and real GDP and adjusted TFP[15]. For the autocorrelation panel in

[15]In Figure 9, output is the log of the seasonally adjusted quarterly Real Gross Domestic Product, GDPC96, in

both the data and model, output growth is differenced log non-stationary real GDP. For the dynamic cross-correlograms in both the data and model, all series are HP-filters of the non-stationary log-levels.

The hump-shaped response of output (and other variables) shown earlier in the impulse response functions suggests an autocorrelation of output growth, a feature that is generally found to be present in the data, but is typically difficult to re-produce in RBC-style models, as studied in depth in Cogley and Nason (1995). Cogley and Nason find that in the data GNP growth displays positive autocorrelation over short horizons (up to 3-4 quarters) and weak and possibly statistically insignificant negative autocorrelation at longer horizons. In contrast, typical RBC-style models produce almost no autocorrelation in output growth at short or long horizons. The first panel of Figure 9 for the data shows output growth is positively autocorrelated at short horizons, consistent with Cogley and Nason, with a maximum autocorrelation coefficient of just under 0.4. At longer horizons the autocorrelations are for the most part not statistically significant. Consistent with the above, the first panel of Figure 10 for the *Full Model* shows positive autocorrelation at short lags, with a maximum autocorrelation coefficient of just over 0.2, and then negative autocorrelation at longer lags.

In general, "news-shock" theory suggests a pattern of business cycles whereby forward-looking variables such as stock prices and measures of economic activity such as output respond in advance of a change in fundamentals, such as adjusted TFP. Aside from conditional identifications of such expectational shocks, to the extent that this theory bears out in the data, one might expect this pattern to show up in unconditional correlations in the data[16]. The dynamic cross-corellograms in panels 2, 3 and 4 of Figures 9 and 10 explore this idea. Panel 2 of Figure 9 for the data shows the well-know result that stock prices tend to lead output at short horizons of a several quarters, and the corresponding panel 2 of Figure 10 shows that the model does a very good job of reproducing this feature of the data. Interestingly, the model also does a very good job of matching the overall pattern of cross-correlations found in the data, re-producing the statistically significant negative cross-correlation of stock prices and output at leads of about 15 quarters and lags of about 5 quarters.

Panel 3 of Figures 9 and 10 shows the dynamic correlogram for stock prices and TFP, of special interest for news-shock theory given the conditional empirical identifications that suggest stock prices rise in advance of future TFP growth. From the figure we can see a statistically significant negative correlation in stock prices and output a short lead horizons about 2 quarters, but no statistically significant rise in stock prices in advance of TFP. There are of course many reasons why a pattern that emerges in conditional empirical identifications does not bear out significantly in unconditional correlations; one possibility of the influence of other shocks on the overall statistical significant. Indeed, the overall pattern of the correlogram does suggest a nearly significant rise in

the FRED database. Stock prices are the log of the quarterly average of the monthly real S&P 500 index obtainable from the online data of Robert Shiller, http://www.econ.yale.edu/ shiller/data.htm. TFP is the adjusted-TFP series using the updated series of TFP adjusted for utilization from Fernald (2012), converted into log-levels from growth rates.

[16] I thank an anonymous referee for suggesting this exercise.

stock prices at leads of about 10 quarters in advance of TFP. Panel 3 of 10 for the model shows results largely consistent with this interpretation: stock prices lead TFP positively at horizons of about 2 years, but but lead negatively at short horizons of around 2 quarters.

Panel 4 of Figures 9 and 10 show the dynamic correlaogram for real GDP and TFP. The panel in Figure 9 for the data shows a statistically significant rise in GDP at leads about about 8 quarters (as well as lages of about 8 quarters), but essentially no significant correlations inside of this horizon. Panel 4 of Figure 10 shows the model does a good job or reproducing the positive lead at about 6-8 quarters. The lag pattern however is much more delayed in the model than in the data, with peak positive lag correlation at around 20 quarters.

Overall, the dynamic correlations above - both in the data and the model - suggest a pattern largely consistent with news-shock business cycle theory: "fast-moving" stock prices react before slower moving inertial real variables such as output, and both tend to positively precede future adjusted TFP.

5.4.1 Example simulations

I now examine several example simulations over different time frames to illustrate the qualitative character of fluctuations driven only by the i.i.d sunspot shock. Figure 11 shows a sample realization over a 25 year span to illustrate the higher-frequency fluctuations produced by the sunspot shock. The bottom panel of the figure shows the shock itself - ie the i.i.d investment sunspot - the top panel shows the response of the stationary percent deviations of stock prices, output, TFP (adjusted for utilization) and the relative price of capital, and the middle panel shows the associated non-stationary movements as log-levels (assuming an initial condition at steady state) [17]. In the figure we can see distinct business cycle frequency behaviour present with cycle amplitude in the range of 5 years. As in the data, expansions are characterized by rising stock prices and output, and falling relative price of capital, with stock prices seeming to anticipate the booms and busts in the real economy. Also, consistent with the empirical news-literature, adjusted TFP rises with a lag. Recessions are then characterized by the reverse behaviour. Remarkably, the strong propagation mechanisms of the model transmit the i.i.d. sunspot shocks - which have zero persistence and little resemblance to business cycle behaviour - into booms and busts resembling business cycles. From inspection we can see that these booms result from a cluster of periods where the stock price realizations happened to lie above the mean - ie exuberance over a moderate duration. Recessions, by symmetry, are pessimism over a moderation duration.

I now examine the qualitative ability of the model to propagate the i.i.d. sunspot disturbances to frequencies lower than business cycles in attempt to see if the model can account for the lower frequency secular trends that we tended to observe in the twentieth century, such as the long run-up in stock prices, real activity and TFP during the 1920's and 1990's, and the lull in stock prices and TFP during the 1970's and early 1980's. Figure 12 shows a different sample realization, this time over a 100 year span. As in the previous figure, we can see fluctuations that appear to look

[17]The relative price of capital is a non-stationary variable in this model so I exclude it from the non-stationary plot.

like business cycles, but now with the advantage of the longer sample period additional behaviour becomes apparent: less frequent periods of lower-frequency fluctuations in stock prices, output, adjusted TFP and the relative price of capital. During such high-growth periods, output and stock prices grow over longer periods, followed by an extended period of highly-persistent TFP growth. Low-growth periods are the reverse. These lower frequency, secular trends occur less frequently than business cycles, since given the i.i.d sunspot distribution, the probability of there arising such a necessary succession of realizations to produce these lower frequency movements is less than the probability of there arising a shorter success of realizations required for business cycles.

From the perspective of the model then, business cycle booms emerge as the result of elevated sentiment over moderate durations of time; lower-frequency secular trends that produce bull-markets in stock prices and sustained growth in productivity emerge as the result of elevated sentiment over longer periods of time. In symmetry, recessions emerge as the result of depressed levels of sentiment over moderate periods, and lower-frequency bear markets associated with low productivity growth emerge as the result of depressed levels of sentiment over longer periods.

5.5 Implication of Beaudry-Portier style VECM news-shock identification

Thus far we have seen that the model impulse response functions and stochastic stimulations yield results that are largely qualitatively and quantitatively consistent with the news-shock view of business cycles, as well as with key unconditional business cycle properties more generally. The natural next question to ask then is whether the empirical news-shock identifications in the style of Beaudry and Portier (2006) would hold up if indeed self-fulfilling effects of sunspots exist in reality in the data. In this section I explore this question, effectively treating the model as the data-generating mechanism, and applying the empirical news identifications to the model output.

Adding a stationary (but near unit-root) neutral technology shock to the *Full Model*, I simulate the model subject to stochastic variation of *both* the non-fundamental sunspot shock and the fundamental neutral technology shock[18]. Recovering the non-stationary data from this simulation in accordance with the model, I then run a bi-variate vector error-correction model (VECM) on the simulated model output of the log of stock prices and the log of TFP as in the core procedure in Beaudry and Portier (2006), identifying the "news shock" with a no-impact restriction that orthagonalizes the innovation in stock prices with respect to contemporaneous TFP. The model simulation sample size is 2000 quarters. I estimate the regression using 5 lags as in Beaudry and Portier (2006).

To serve as the data benchmark for comparison, I run the same procedure on the data, using the log-level of stock prices from the Shiller data-set, and the log of utilization-adjusted TFP from the Fernald data, effectively replicating a Beaudry-Portier style news shock identification using this data sample. The data sample consists of quarterly U.S. time series from Q1 1948 to Q4 2014.

To determine the standard deviation of the technology shock relative to the sunspot shock, I

[18]To add the neutral technology shock to the model, I redefine the production function as $y_t = \Xi_t(A_t n_{yt})^\alpha u_t^{1-\alpha} k_t^\theta h_t^{1-\alpha-\theta}$, where Ξ_t is the stochastic neutral technology that follows $\log \Xi_t = \rho_\Xi \log \Xi_{t-1} + \epsilon_t^\Xi$, and were I set $\rho_\Xi = 0.999$.

normalize the standard deviation of the sunspot shock to 1, and then calibrate the standard deviation of the technology shock so that the contribution of the forecast error variance decomposition of TFP at a horizon of 20 quarters accounted for by the stock price innovation is the same in the VECM on the model simulation as in the VECM on the data, which is 7%. Following this approach leads to a standard deviation of 0.06 for the technology shock[19].

Figure 13 shows the results of the VECM for the data, displaying the response of TFP and stock prices to a unit standard-deviation identified "news shock". As in the results of Beaudry and Portier (2006), stock prices rise immediately in response to the "news shock". By construction TFP does not respond in the initial period, but then eventually begins a sustained period of statistically significant growth over time[20]. After the initial rise in stock prices, there is a small additional rise in stock prices in the next few periods, followed by a relatively constant level of stock prices (there does not appear to be a statistically significant continued growth in stock prices as with TFP).

Figure 14 shows the results the VECM for the model simulation, displaying the response of TFP and stock prices to a unit standard-deviation identified "news shock". As the figure shows, the results are largely consistent with the same procedure applied to the data: stock prices rise immediately, TFP is unaffected in the initial period (by construction). Following this, TFP grows and continues a statistically significant growth rate. Stock prices eventually reach a relatively constant (or slightly increasing) level. The primary difference between this VECM and that applied to the data in the previous figure is the behaviour of stock prices in the approximately 2 quarters after the intial period. In the data VECM there is a further rise above the initial time 0 rise, although the error-bands suggest this feature may not be that prominent. Nevertheless, in the model VECM, stock prices are at their peak in period 0[21]. In any case, the VECM applied to the model captures the general pattern of that applied to the data: stock prices rise in the present in advance of future growth in productivity. The strict "news shock" view interprets this identified pattern as the result of expectations responding in the present to expected exogenous changes in TFP in the future; in contrast, I show that a "self-fulfilling" view is also consistent with this identified pattern, whereby exogenous changes in expectations in the present drive endogenous growth in TFP in the future.

6 Conclusion

In the majority of macroeconomic business cycle theory, the notions of "technology" and "productivity" are used interchangeably. Sudden shifts in the production frontier are "technology shocks", conceived of as shocks to some productivity measure in the production function. Under this view,

[19]Note that this procedure of normalizing the sunspot standard deviation to 1 and then determining that of the technology shock relative to this will not produce an overall standard deviation of TFP or stock prices (attributable to both shocks) that lines up with the data. This will just affect the "size" of the recovered shocks however, and not the identification itself.

[20]The figure suggests a slight fall in TFP after the initial period, however, this effect is not statistically significant.

[21]This fall after the initial rise in stock prices is similar to the behaviour of stock prices in the impulse response functions shown earlier, where stock prices were at their peak in the initial period. This however is not a necessary feature of the model; indeed for certain variants of the calibration, stock prices due indeed rise in a hump-shaped manner.

there is no distinction made between agents' "visions of technology" and the eventual reality of technology in production. Yet as I show in this paper, when we separate these concepts, interesting ideas emerge about how business cycles and productivity growth may interact in the absence of shocks to technology or productivity. If technological ideas exist in the economy, and agents must implement these in production in order to realize their productivity benefits, then the possibility exists that the variation in productivity growth that we observe is connected to variations in the whims of the decision-makers in the economy that can also trigger boom-busts in asset markets and real quantities. To generate the indeterminacies that give rise to such a self-fulfilling episodes, I include modeling assumptions reflecting important features of technological adoption noted in the literature on technological growth and diffusion, namely, the assumption that many technological ideas are embodied in physical capital, and that firms must embark on costly learning in adoption to implement these ideas into production.

More than just providing a new take on possible connections between productivity growth and business cycles, I illustrate that the model is qualitatively consistent with the results of the important recent empirical news shock literature that finds a significant role for changes in expectations in business cycles. While the theoretical outgrowth of this literature has interpreted the empirical evidence as imperfect signals about future changes in productivity that trigger business cycle behaviour, my model provides an alternative interpretation whereby exogenous changes in expectations trigger both business cycle behaviour as well as variation in productivity growth itself. Additionally, beyond just addressing the conditional response to this one type of empirically-identified shock, I show that simulations of the model yield business cycle moments that are quite consistent with the unconditional moments in the data. Moreover, I use the model to demonstrate how both business cycle frequency and lower frequency "secular bull and bear markets" can emerge even when the expectational shocks are i.i.d., when a succession of stochastic realizations arise on one side of the distribution of beliefs. Finally, I estimate a Beaudry-Portier style VECM on both data and on the simulated sunspot model output to identify a "news shock", and show that the procedure recovers impulse response functions from the sunspot model largely consistent with those on the data estimated on data. While this procedure does not of course *p*rove that self-fulfilling effects contribute to the Beaudry and Portier (2006) results, it does validate an alternative interpretation of the results beyond the traditional "manna from heaven" saddle-path stable view.

The model has a number of interesting implications. Firstly, to the extent that the empirical news shock literature helps us to understand an important expectational source of business cycle fluctuations, this model offers a mechanism through which an expectational shock can produce this behaviour. Secondly, while not the first to make a connection between business cycles and lower frequency movements in productivity, I offer a different view of this connection, and thus an additional challenge to the traditional macroeconomic approach of separating the trend from the cycle. Finally, by demonstrating a connection between belief-driven asset price behaviour and productivity growth, the model illustrates an additional mechanism for understanding the nature of productivity growth itself, as well as highlighting a possible trade-off of productivity growth for policy actions that seek to smooth or shape asset price behaviour.

References

Aghion, Philippe and Peter Howitt, "On the Macroeconomic Effects of Major Technological Change," *Annales d'Economie et de Statistique*, 1998, (49-50), 53–75.

Arrow, K.J., "The Economic Implications of Learning by Doing," *Review of Economic Studies*, 1962, *3* (29), 155–173.

Atkeson, Andrew and Patrick J. Kehoe, "Modeling the Transition to a New Economy: Lessons from Two Technological Revolutions," *American Economic Review*, March 2007, *97* (1), 64–88.

Basu, S and M.S. Kimball, "Long-Run Labor Supply and the Elasticity of Intertemporal Substitution for Consumption," Technical Report, University of Michigan 2002.

Basu, Susanto and John G. Fernald, "Information and communications technology as a general purpose technology: evidence from U.S. industry data," *Economic Review*, 2008, pp. 1–15.

Beaudry, Paul and Franck Portier, "An exploration into Pigou's theory of cycles," *Journal of Monetary Economics*, September 2004, *51* (6), 1183–1216.

_ and _ , "Stock Prices, News, and Economic Fluctuations," *American Economic Review*, September 2006, *96* (4), 1293–1307.

_ , Deokwoo Nam, and Jian Wang, "Do mood swings drive business cycles and is it rational?," Technical Report 2011.

Benhabib, J. and R.E.A. Farmer, "Indeterminacy and Increasing Returns," *Journal of Economic Theory*, June 1994, *63* (1), 19–41.

Bennett, Rosalind L. and Roger E. A. Farmer, "Indeterminacy with Non-separable Utility," *Journal of Economic Theory*, July 2000, *93* (1), 118–143.

Bresnahan, Timothy F., Erik Brynjolfsson, and Lorin M. Hitt, "Information Technology, Workplace Organization, And The Demand For Skilled Labor: Firm-Level Evidence," *The Quarterly Journal of Economics*, February 2002, *117* (1), 339–376.

Brynjolfsson, Erik, Loren Hitt, and Shinkyu Yang, "Intangible Assets: How the Interaction of Computers and Organizational Structure Affects Stock Market Valuations," *Brookings Papers on Economic Activity*, 2002, *33* (1), 137–198.

Carlaw, Kenneth I. and Richard G. Lipsey, "Externalities, technological complementarities and sustained economic growth," *Research Policy*, December 2002, *31* (8-9), 1305–1315.

Christiano, Lawrence, Cosmin Ilut, Roberto Motto, and Massimo Rostagno, "Monetary policy and stock market boom-bust cycles," Working Paper Series 955, European Central Bank October 2008.

Cogley, Timothy and James M Nason, "Output Dynamics in Real-Business-Cycle Models," *American Economic Review*, June 1995, *85* (3), 492–511.

Comin, Diego A., Mark Gertler, and Ana Maria Santacreu, "Technology Innovation and Diffusion as Sources of Output and Asset Price Fluctuations," NBER Working Papers 15029, National Bureau of Economic Research, Inc June 2009.

Comin, Diego and Bart Hobijn, "Implementing Technology," NBER Working Papers 12886, National Bureau of Economic Research, Inc February 2007.

_ and Mark Gertler, "Medium-Term Business Cycles," *American Economic Review*, June 2006, *96* (3), 523–551.

Corrado, Carol, Charles Hulten, and Daniel Sichel, "Intangible Capital And U.S. Economic Growth," *Review of Income and Wealth*, 09 2009, *55* (3), 661–685.

David, Paul A, "The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox," *American Economic Review*, May 1990, *80* (2), 355–61.

Farmer, Roger E. A., *Macroeconomics of Self-fulfilling Prophecies, 2nd Edition*, Vol. 1 of *MIT Press Books*, The MIT Press, 1999.

_ and Jang-Ting Guo, "Real Business Cycles and the Animal Spirits Hypothesis," *Journal of Economic Theory*, June 1994, *63* (1), 42–72.

Fernald, John, "A quarterly, utilization-adjusted series on total factor productivity," Technical Report 2012.

Fisher, Jonas D. M., "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks," *Journal of Political Economy*, June 2006, *114* (3), 413–451.

Francois, Patrick and Huw Lloyd-Ellis, "Animal Spirits Through Creative Destruction," *American Economic Review*, June 2003, *93* (3), 530–550.

_ and _ , "Implementation Cycles, Investment, And Growth," *International Economic Review*, 08 2008, *49* (3), 901–942.

Greenwood, Jeremy and Mehmet Yorukoglu, "1974," *Carnegie-Rochester Conference Series on Public Policy*, June 1997, *46* (1), 49–95.

_ , Zvi Hercowitz, and Gregory W Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, June 1988, *78* (3), 402–17.

Gunn, Christopher and Alok Johri, "News and knowledge capital," *Review of Economic Dynamics*, January 2011, *14* (1), 92–101.

Hall, Robert E, "Intertemporal Substitution in Consumption," *Journal of Political Economy*, April 1988, *96* (2), 339–57.

Hall, Robert E., "E-Capital: The Link between the Stock Market and the Labor Market in the 1990s," *Brookings Papers on Economic Activity*, 2000, *31* (2), 73–118.

Helpman, Elhanan and Manuel Trajtenberg, "A Time to Sow and a Time to Reap: Growth Based on General Purpose Technologies," NBER Working Papers 4854, National Bureau of Economic Research, Inc September 1994.

Howitt, Peter and David Mayer-Foulkes, "R&D, Implementation, and Stagnation: A Schumpeterian Theory of Convergence Clubs," *Journal of Money, Credit and Banking*, February 2005, *37* (1), 147–77.

Jaimovich, Nir and Sergio Rebelo, "Can News about the Future Drive the Business Cycle?," *American Economic Review*, September 2009, *99* (4), 1097–1118.

Jovanovic, Boyan and Peter L. Rousseau, "General Purpose Technologies," in Philippe Aghion and Steven Durlauf, eds., *Handbook of Economic Growth*, Vol. 1 of *Handbook of Economic Growth*, Elsevier, 2005, chapter 18, pp. 1181–1224.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti, "Investment shocks and business cycles," *Journal of Monetary Economics*, March 2010, *57* (2), 132–145.

King, Robert G. and Sergio T. Rebelo, "Resuscitating Real Business Cycles," NBER Working Papers 7534, National Bureau of Economic Research, Inc February 2000.

_ , Charles I. Plosser, and Sergio T. Rebelo, "Production, growth and business cycles : I. The basic neoclassical model," *Journal of Monetary Economics*, 1988, *21* (2-3), 195–232.

McGrattan, Ellen R. and Edward C. Prescott, "Unmeasured investment and the puzzling U.S. boom in the 1990s (technical appendix)," Technical Report 2009.

Nelson, R.R. and E.S. Phelps, "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economic Review*, 1966, *1/2* (56), 69–75.

Oliner, Stephen D., Daniel E. Sichel, and Kevin J. Stiroh, "Explaining a Productive Decade," *Brookings Papers on Economic Activity*, 2007, *38* (1), 81–152.

Primiceri, G.E., E. Schaumburg, and A. Tambalotti, "Intertemporal Disturbances," Technical Report, NBER working paper 12243 2006.

Shiller, R.J., *Irrational Exuberance: (Second Edition)*, Princeton University Press, 2009.

Shleifer, Andrei, "Implementation Cycles," *Journal of Political Economy*, December 1986, *94* (6), 1163–90.

7 Appendix

This appendix describes details of the models omitted from the main text.

7.1 Simple Model

7.1.1 Equilibrium

The market clearing conditions in the Simple Model economy for the labour market, the goods market and the stock market are $N_t = \int_0^1 n_t(i)di$, $Y_t = C_t + \int_0^1 i_t(i)di$, and $\int_0^1 b_t(i)di = 1 = B_t$, where the left and right-hand sides of each of the equalities are the supply and demand respectively.

A rational expectations equilibrium for this economy is then a collection of policies for households $B' = B(\mathcal{S}_t)$, $N = N(\mathcal{S}_t, a_t)$, policies for intermediate goods firms $k'(i) = k(\mathcal{S}_t, s_t(i)$, $j'(i) = j(\mathcal{S}_t, s_t(i))$, $n(i) = n(\mathcal{S}_t, s_t(i))$, $i(i) = i(\mathcal{S}_t, s_t(i))$ $\forall i$, policies for the final goods producer $Y = Y(\mathcal{S}_t)$, $y(i) = y(i)(\mathcal{S}_t)$ $\forall i$, price systems $w(\mathcal{S}_t)$, $v_i(\mathcal{S}_t)$, and aggregate laws of motion $K = K(S)$ and $J = J(S)$, such that: (i) the representative household solves its problem (ii) intermediate goods producers solve their problems; (iii) the final goods producer solves its problem; (iv) the labour, goods and stock markets clear, and; (v) the policy rules confirm the aggregate laws of motion.

I consider a symmetric equilibrium where $p_t(i) = p_t$, $y_t(i) = y_t$, $n_t(i) = n_t$, $k_t(i) = k_t$, $j_t(i) = j_t$, and $i_t(i) = i_t$ $\forall i$. Aggregate quantities of variables associated with intermediate goods producers variables are given by $K_t = \int_0^1 k_t(i)di$, $J_t = j_t$, $N_t = \int_0^1 n_t(i)di$ and $I_t = \int_0^1 i_t(i)di$. Note that since j_t can interpreted as indexes of productivity based on how I have defined j_t as independent of the scale of a firm (where scale is measured by k_t), in a symmetrical equilibrium the aggregate value of J is equivalent to the individual values. The external factor X_t is defined as $X_t[1-\delta_j] = 1 - \delta_j - \kappa \frac{J_t}{\Psi_t}\frac{I_t}{K_t}$, and the aggregate laws of motion are given by

$$K_{t+1} = [1 - \delta(u_t)]K_t + I_t \tag{45}$$

and

$$J_{t+1} = [1 - \delta_j]J_t + \mathcal{Z}_t I_t, \tag{46}$$

where $\mathcal{Z} = \kappa \frac{J_t}{K_t}[1 - \frac{J_t}{\Psi_t}]$.

Substituting $y_t(i) = y_t$ into the final goods aggregate technology (1) yields the condition $y_t = Y_t$. Recognizing that under perfect competition the final goods firm's profits will be zero then implies that $p_t(i) = p_t = 1$. Finally, substituting $y_t = Y_t$, $n_y = N_t$ and $k_t = K_t$ into the ith intermediate goods firm's production function (4) yields the aggregate production function

$$Y_t = A_t^{\alpha} N_t^{\alpha} K_t^{\theta} J_t^{1-\alpha-\theta}. \tag{47}$$

In a symmetrical equilibrium, all firms' shadow prices will be equivalent, such that $q_{kt}(i) = q_{kt}$ and $q_{jt}(i) = q_{jt}$ $\forall i$. To represent this in equilibrium system in the reduced solution form I then redefine these internal prices in aggregate in terms of household utility as $\mu_t = q_{kt}\lambda_t$ and $\zeta_t = q_{jt}\lambda_t$.

7.1.2 Balanced growth path

I define a balanced growth path for the simple economy whereby N_t is constant, and the other endogenous variables inherit trends as some function of the trend in A_t and Ψ_t. The equilibrium system implies that C, I, Y, D, w, v and K contain trend $X_t^Y = \Psi_t^{\frac{\theta}{1-\theta}} A_t^{\frac{\alpha}{1-\theta}}$, λ_t contains trend $1/X_t^{Y\sigma}$ and J_t contains trend $X_t^J = \Psi_t$. On the balanced growth path, the growth rates are then $g^y = \frac{X_{t+1}^Y}{X_t^Y}$, $g^\Psi = \frac{\Psi_{t+1}}{\Psi_t}$ and $g^A = \frac{A_{t+1}}{A_t}$ $\forall t$.

The following transformations then yield stationary variables on the balanced growth path, denoted with a tilde: $\tilde{C}_t = \frac{C_t}{X_t^Y}$, $\tilde{I}_t = \frac{I_t}{X_t^Y}$, $\tilde{Y}_t = \frac{Y_t}{X_t^Y}$, ...etc., $\tilde{K}_t = \frac{K_t}{X_{t-1}^K}$, $\tilde{J}_t = \frac{J_t}{X_{t-1}^J}$, $\tilde{\lambda}_t = \lambda_t X_t^{Y\sigma}$, $\tilde{\mu}_t = \mu_t \frac{X_t^K}{X_t^{Y\,1-\sigma}}$ and $\tilde{\zeta}_t = \zeta_t \frac{X_t^J}{X_t^{Y\,1-\sigma}}$. Finally, the resulting stationary system contains a unique non-stochastic steady state.

7.1.3 Solution method

To solve the model I linearize the non-linear system around the non-stochastic state state, resulting in a first-order linear system of the form

$$E_t \mathcal{Q}_{t+1} = A\mathcal{Q}_t + B\epsilon_t, \tag{48}$$

where $\mathcal{Q}_t = [\hat{k}_t, \hat{j}_t, i_t, \hat{\zeta}_t,]'$, and $\epsilon_t = [0,0,0,0]'$ such that there is no external source of uncertainty. Hats above variables denote %-deviations from steady state.

The linear system (48) contains two predetermined endogenous states (k,j) and two forward-looking non-predetermined co-states (i,ζ). Note that I elect to include investment i as one of the non-predetermined co-states to aid in the intuitive explanations in that we can then think about animal spirits as shocks to investment. The system will exhibit saddle-path stability if the number of eigenvalues of the matrix A outside of the unit circle is equal to the number of forward-looking non-predetermined variables, and will display indeterminacy if the number of eigenvalues of A lying outside the unit circle is less than the number of forward-looking non-predetermined variables.

To analyze the response of the system to intrinsic uncertainty, I follow the approach of Farmer (1999), and replace the expectations of a variable with the variable less the expectational error, so that now (48) re-writes as,

$$\mathcal{Q}_{t+1} = A\mathcal{Q}_t + B\varepsilon_t, \tag{49}$$

where ε_t is defined as $\varepsilon_t = [0,0,w_t^i,w_t^\zeta]'$, where $w_t^i = E_t i_{t+1} - i_{t+1}$ and $w_t^\zeta = E_t\zeta_{t+1} - \zeta_{t+1}$ are the one-step ahead forecast errors on the forward-looking variables. Note also that by definition the expectational error of a predetermined variable is zero yielding the two zeros in ε_t.

For the parameterizations that I consider that yield indeterminacy, the matrix A has one less root outside the unit-circle than forward-looking variables, leaving one forward-looking variable with an unstable root. Thus under indeterminacy we can interpret one of the expectational errors as an i.i.d. sunspot shock. Diagonalizing the system and iterating out the remaining unstable root as in a saddle-path solution yields a restriction on (49) that relates the unstable forward-looking variable to the stable variables.

After solving out the unstable root, the system reduces down to

$$\tilde{\mathcal{Q}}_{t+1} = \tilde{\mathcal{A}}\tilde{\mathcal{Q}}_t + \tilde{\mathcal{B}}\tilde{\varepsilon}_t \tag{50}$$

where now $\tilde{\mathcal{Q}}_t = [\hat{k}_t, \hat{j}_t, \hat{i}_t]'$, and $\tilde{\varepsilon}_t = [0, 0, e^i_t]'$ and where e^i_t is an i.i.d. sunspot shock to investment. Note now that all the roots of $\tilde{\mathcal{A}}$ are inside the unit circle, and the system is a Markovian stable process such that any value of e^i_t will set the system on a stable path that eventually returns to steady state.

7.2 Full Model

7.2.1 Equilibrium

The market clearing conditions in the Full Model economy for the labour market, the goods market and the stock market are $N_t = \int_0^1 n_t(i)di$, $Y_t = C_t + \int_0^1 i_t(i)di$, and $\int_0^1 b_t(i)di = 1 = B_t$, where the left and right-hand sides of each of the equalities are the supply and demand respectively.

A rational expectations equilibrium for this economy is then a collection of policies for households $B' = B(\mathcal{S}_t)$, $N = N(\mathcal{S}_t, a_t)$, policies for intermediate goods firms $k'(i) = k(\mathcal{S}_t, s_t(i)$, $j'(i) = j(\mathcal{S}_t, s_t(i))$, $h'(i) = h(\mathcal{S}_t, s_t(i))$, $n_y(i) = n_y(\mathcal{S}_t, s_t(i))$, $n_h(i) = n_h(\mathcal{S}_t, s_t(i))$, $i(i) = i(\mathcal{S}_t, s_t(i))$ $\forall i$, policies for the final goods producer $Y = Y(\mathcal{S}_t)$, $y(i) = y(i)(\mathcal{S}_t)$ $\forall i$, price systems $w(\mathcal{S}_t)$, $v_i(\mathcal{S}_t)$, and aggregate laws of motion $K = K(S)$, $J = J(S)$, and $H = H(S)$, such that: (i) the representative household solves its problem (ii) intermediate goods producers solve their problems; (iii) the final goods producer solves its problem; (iv) the labour, goods and stock markets clear, and; (v) the policy rules confirm the aggregate laws of motion.

As in the Simple Model I consider a symmetric equilibrium, now defined by $p_t(i) = p_t$, $y_t(i) = y_t$, $n_{yt}(i) = n_{yt}$, $n_{ht}(i) = n_{ht}$, $n_t(i) = n_t$, $k_t(i) = k_t$, $j_t(i) = j_t$, $h_t(i) = h_t$ and $i_t(i) = i_t$ $\forall i$. Aggregate quantities of variables associated with intermediate goods producers variables are given by $K_t = \int_0^1 k_t(i)di$, $J_t = j_t$, $H_t = h_t$, $N_{yt} = \int_0^1 n_{yt}(i)di$, $N_{ht} = \int_0^1 n_{yt}(i)di$, and $I_t = \int_0^1 i_t(i)di$. The external factor X_t is defined as in the Simple Model. The aggregate laws of motion for K_t and J_t are as in the Simple Model. In addition, the aggregate law of motion of H_t is given by

$$H_{t+1} = H_t + [J_{t+1} - H_t]N^{\eta}_{ht}. \tag{51}$$

Following the approach for the Simple Model above, the aggregate production function is now

$$Y_t = A^{\alpha}_t N^{\alpha}_{yt} u^{1-\alpha}_t K^{\theta}_t H^{1-\alpha-\theta}_t. \tag{52}$$

Similarly, the shadow prices in terms of household utility are $\mu_t = q_{kt}\lambda_t$, $\Upsilon_t = q_{ht}\lambda_t$ and $\zeta_t = q_{jt}\lambda_t$.

7.2.2 Balanced growth path and steady state

I define a balanced growth path for the Full Model economy whereby N_t, N_{yt}, N_{ht} and u_t are constant, and the other endogenous variables inherit trends as some function of the trend in A_t and Ψ_t. The trend and growth-rate definitions are the same as for the Simple Model, with the addition of the trend in H_t given by $X_t^H = \Psi_t$. The stationary transformations are the same as in the Simple Model, with the addition of $\tilde{H}_t = \frac{H_t}{X_{t-1}^H}$ and $\tilde{\Upsilon}_t = \Upsilon_t \frac{X_t^H}{X_t^{Y^{1-\sigma}}}$.

7.2.3 Solution method

Following the same solution method as in the Simple Model, the linearized system results in the form

$$E_t \mathcal{Q}_{t+1} = A\mathcal{Q}_t + B\epsilon_t, \tag{53}$$

where now $\mathcal{Q}_t = [\hat{k}_t, \hat{h}_t, \hat{j}_t, i_t, \hat{\Upsilon}_t, \hat{\zeta}_t,]'$, and $\epsilon_t = [0, 0, 0, 0, 0, 0]'$.

The linear system (53) contains two predetermined endogenous states (k,h,j) and three forward-looking non-predetermined co-states (i,Υ,ζ).

Replacing the expectations iwth expectational errors gives (53) re-writes as,

$$\mathcal{Q}_{t+1} = A\mathcal{Q}_t + B\varepsilon_t, \tag{54}$$

where ε_t is now defined as $\varepsilon_t = [0, 0, 0, w_t^i, w_t^\Upsilon, w_t^\zeta]'$, where $w_t^i = E_t i_{t+1} - i_{t+1}$, $w_t^\Upsilon = E_t \Upsilon_{t+1} - \Upsilon_{t+1}$ and $w_t^\zeta = E_t \zeta_{t+1} - \zeta_{t+1}$.

Under indeterminacy where the matrix A has one less root outside the unit-circle than forward-looking variables, there are two forward-looking variables with unstable roots. Diagonalizing the system and iterating out the two remaining unstable roots yields a restriction that relates the two unstable forward-looking variables to the stable variables.

After solving out the unstable roots, the system reduces down to

$$\tilde{\mathcal{Q}}_{t+1} = \tilde{\mathcal{A}}\tilde{\mathcal{Q}}_t + \tilde{\mathcal{B}}\tilde{\varepsilon}_t \tag{55}$$

where now $\tilde{\mathcal{Q}}_t = [\hat{k}_t, \hat{h}_t, \hat{j}_t, \hat{i}_t]'$, and $\tilde{\varepsilon}_t = [0, 0, 0, e_t^i]'$ and where e_t^i is again an i.i.d. sunspot shock to investment.

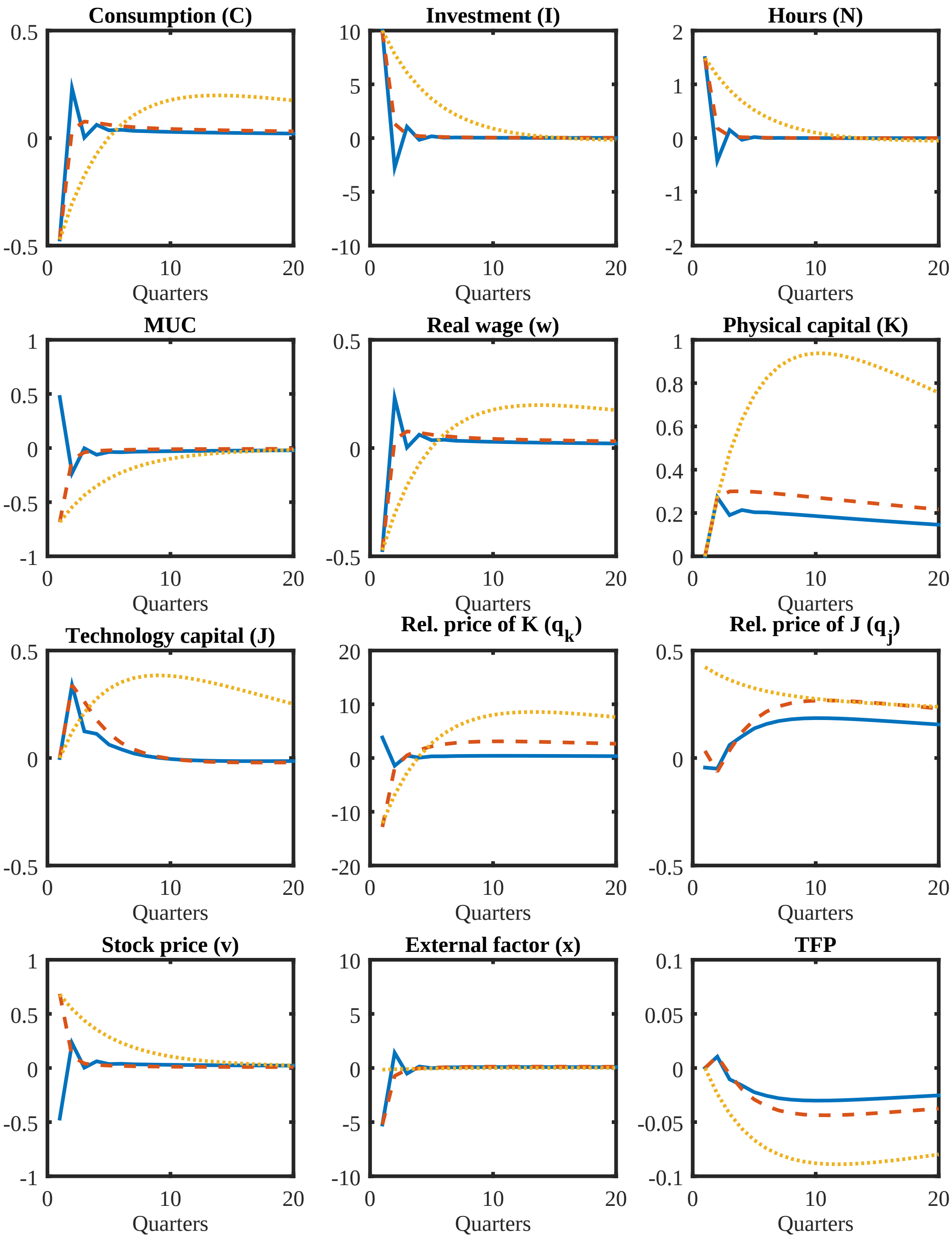

Figure 1: Response to i.i.d. investment sunspot shock: *Simple Model, Parameterization 1 (solid line), Parameterization 2 (broken line) and Parameterization 3 (dotted line).* All IRF's are in stationary form, expressed as %-deviations from stationary (de-trended) steady state.

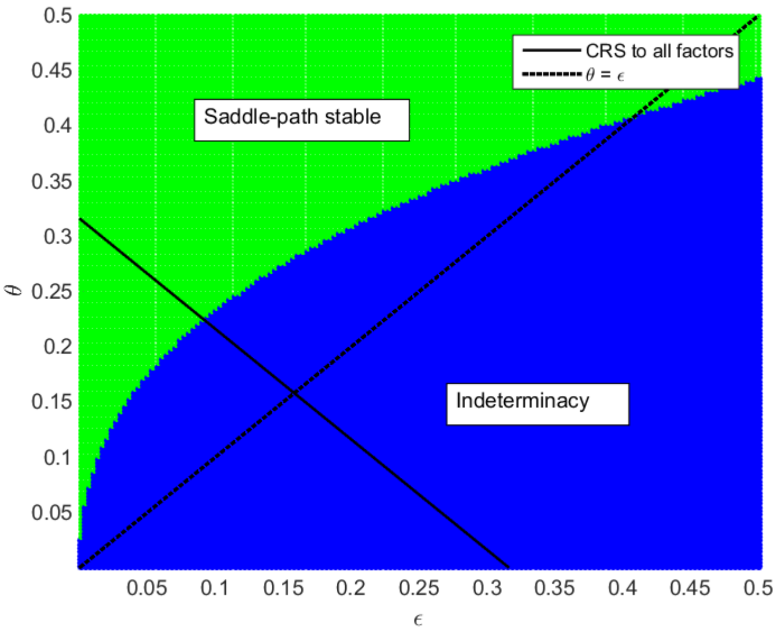

Figure 2: Region of indeterminacy: *Simple Model, Parameterization 1*

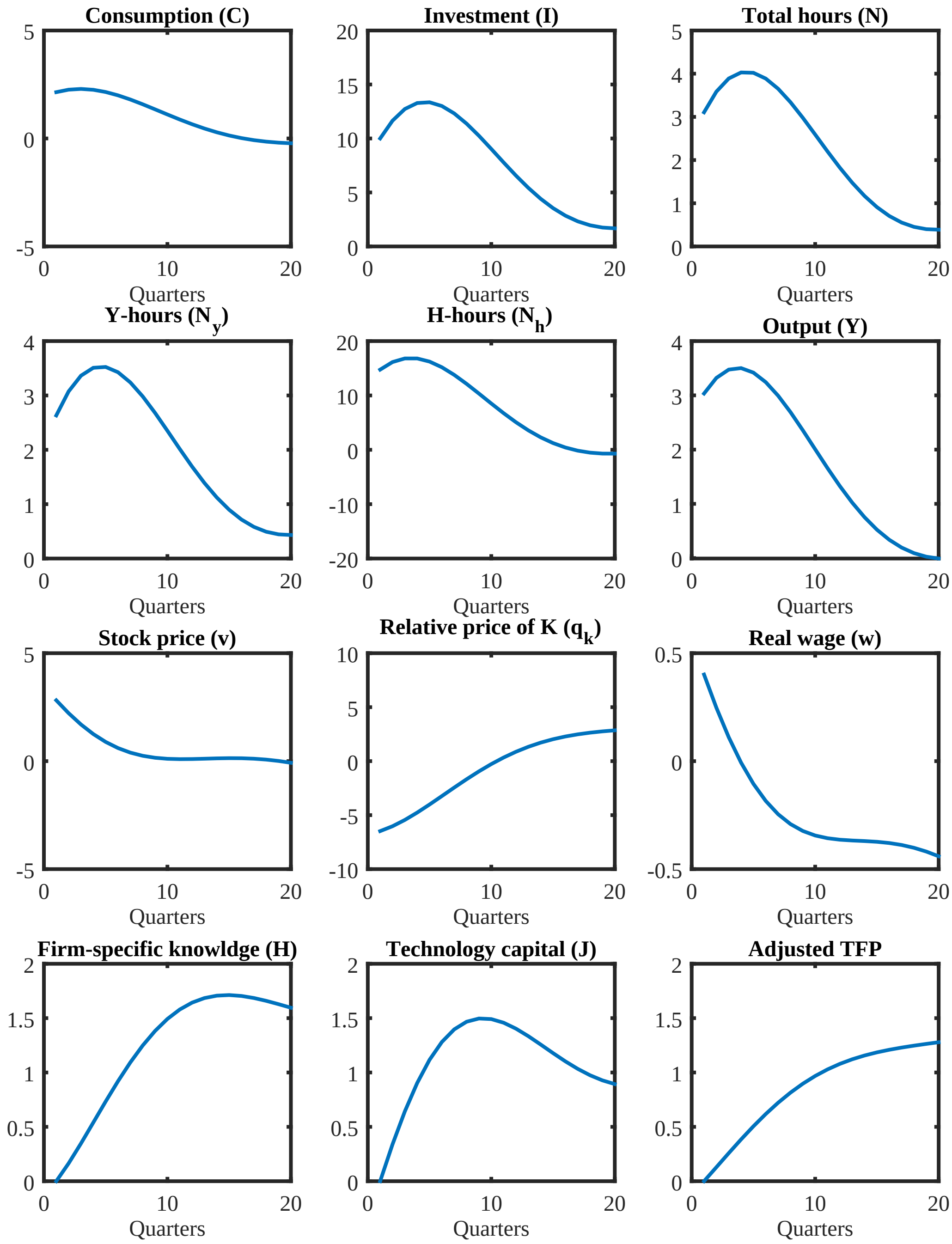

Figure 3: Response to i.i.d. investment sunspot shock: *Full Model*. All IRF's are in stationary form, expressed as %-deviations from stationary (de-trended) steady state.

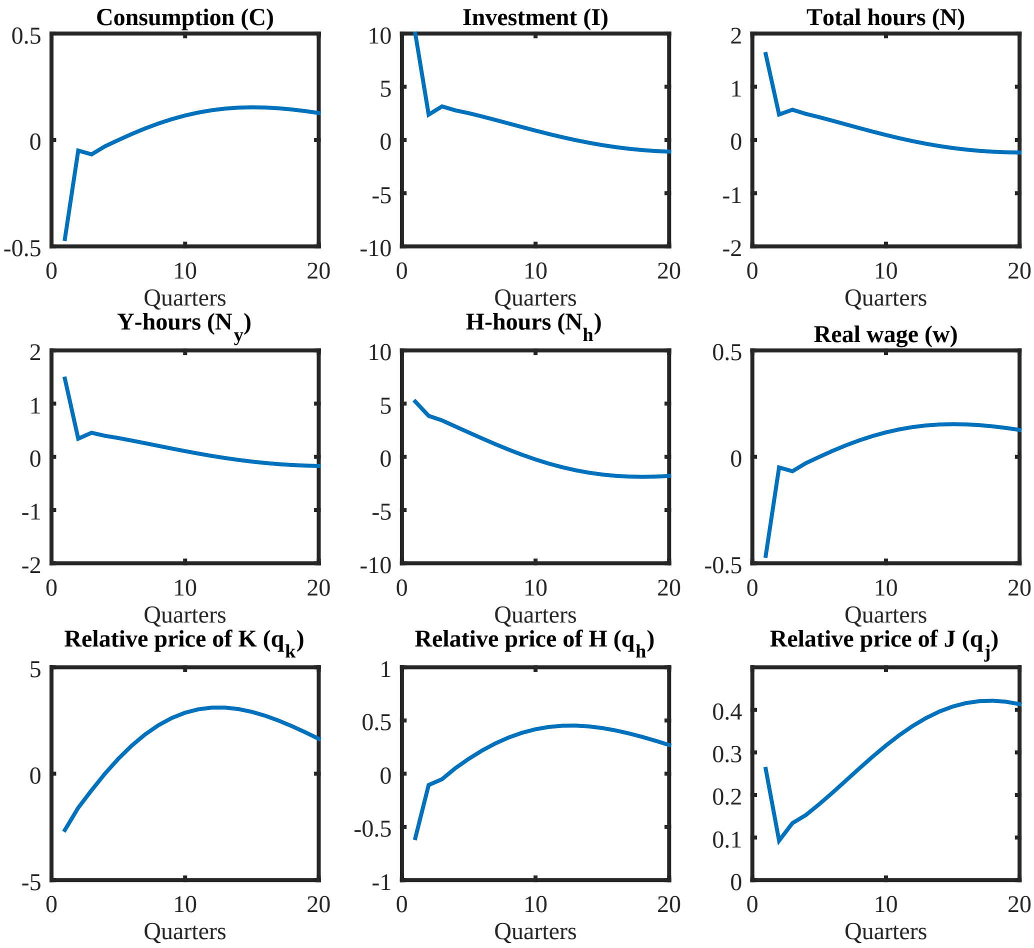

Figure 4: Response to i.i.d. investment sunspot shock: *Full Model, separable preferences, no utilization. All IRF's are in stationary form, expressed as %-deviations from stationary (de-trended) steady state.*

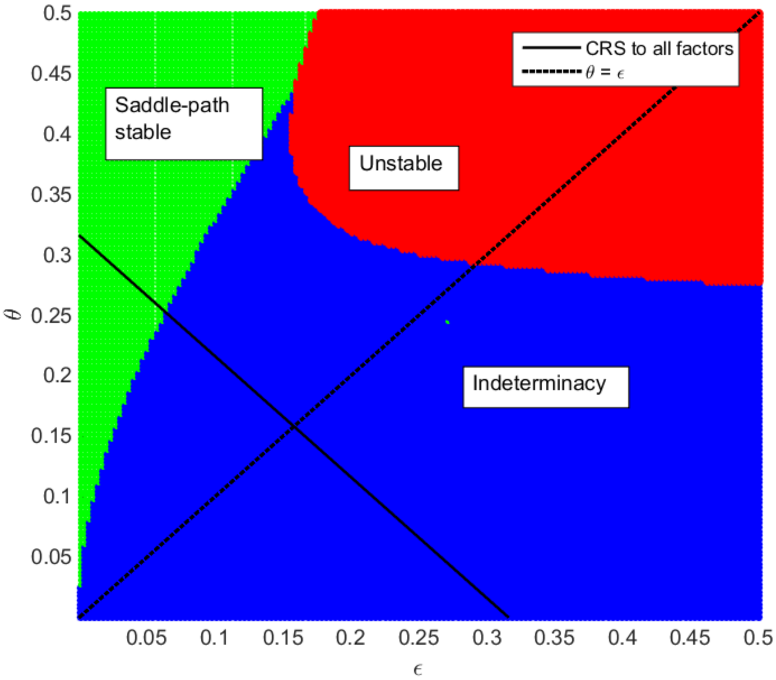

Figure 5: Region of indeterminacy: *Full Model, separable preferences, no utilization*

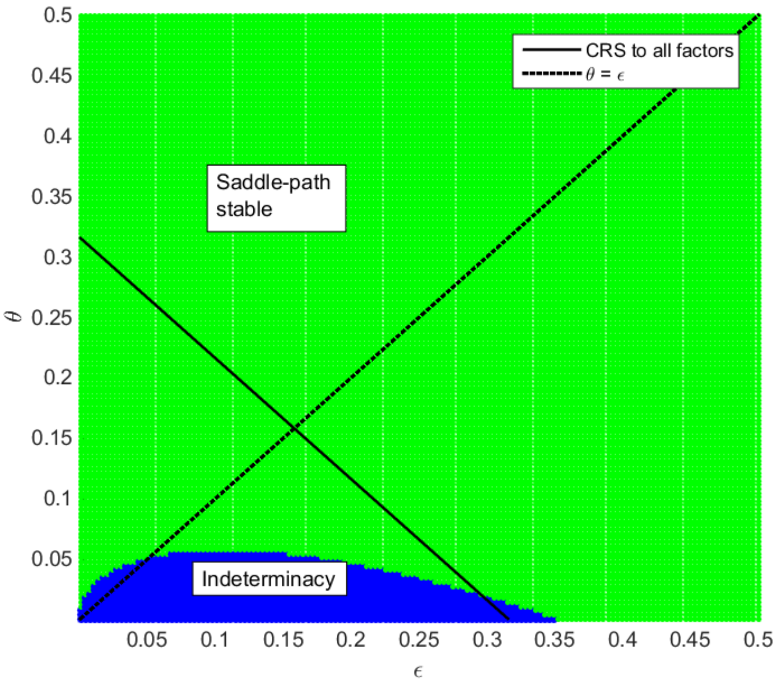

Figure 6: Region of indeterminacy: *Full Model, non-separable preferences, no utilization*

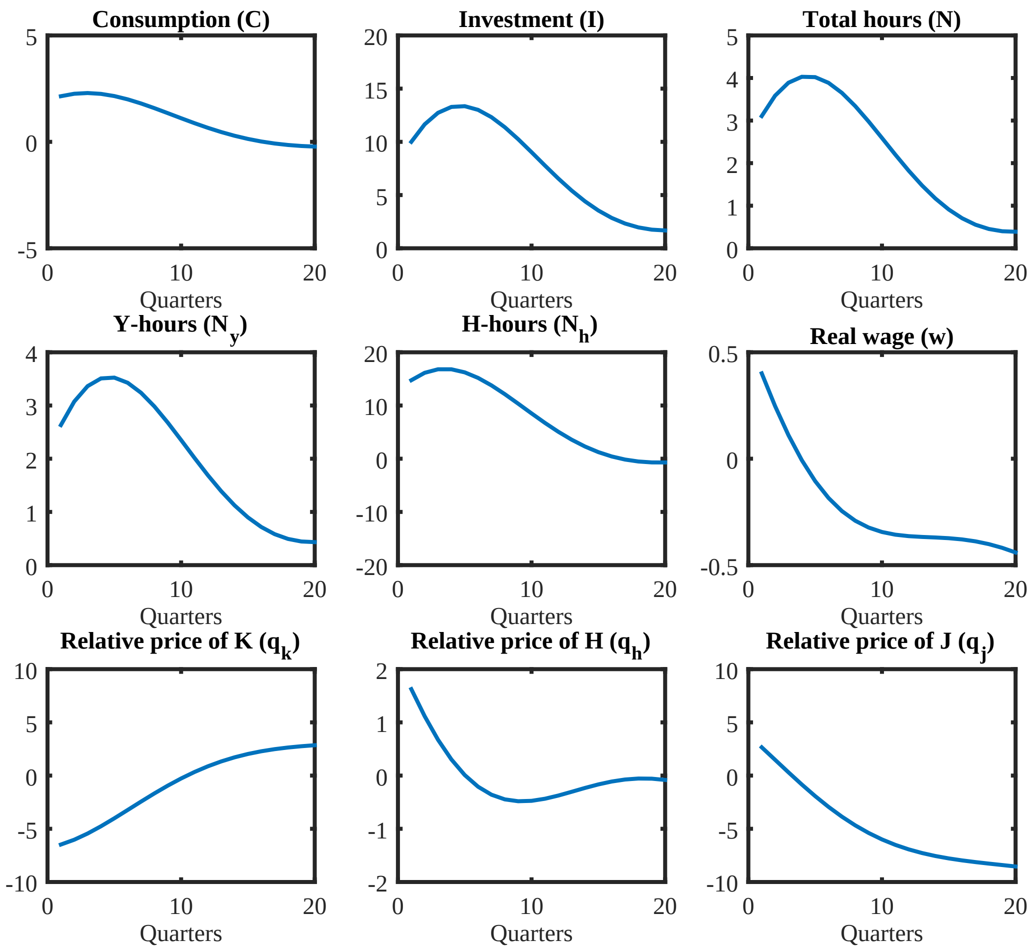

Figure 7: Response to i.i.d. investment sunspot shock: *Full Model*. All IRF's are in stationary form, expressed as %-deviations from stationary (de-trended) steady state.

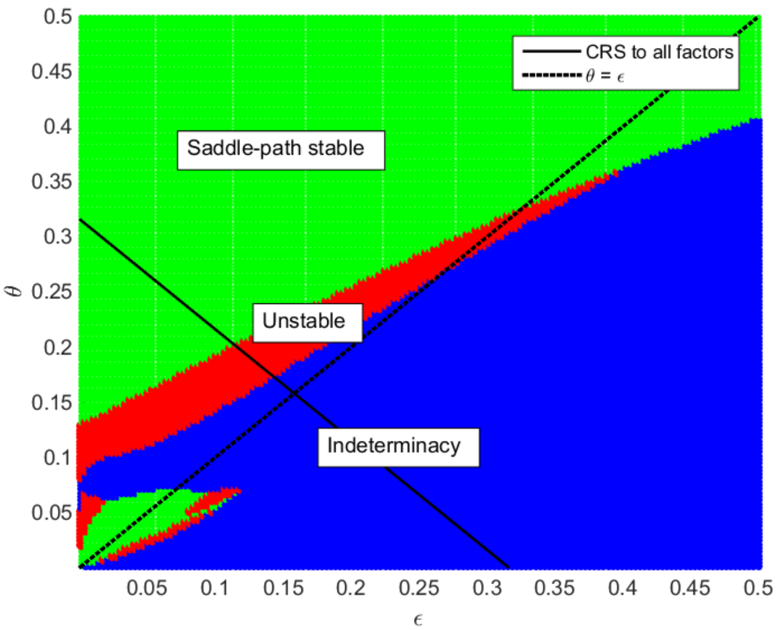

Figure 8: Region of indeterminacy: *Full Model*

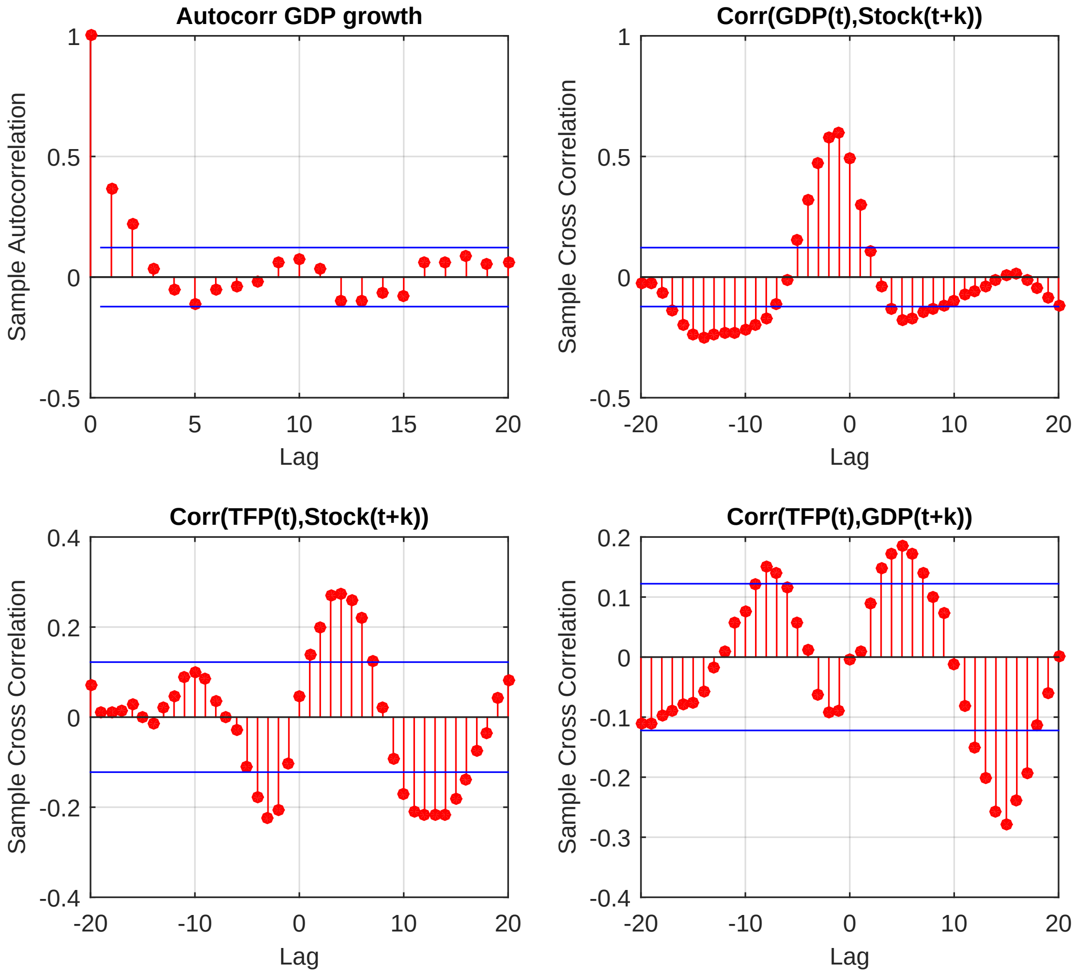

Figure 9: Dynamic correlations in **data**: quarterly observations, 1948 - 2014. *TFP series is adjusted-TFP (per Fernald). All sample data are HP-filtered. 95% error bands shown in blue.*

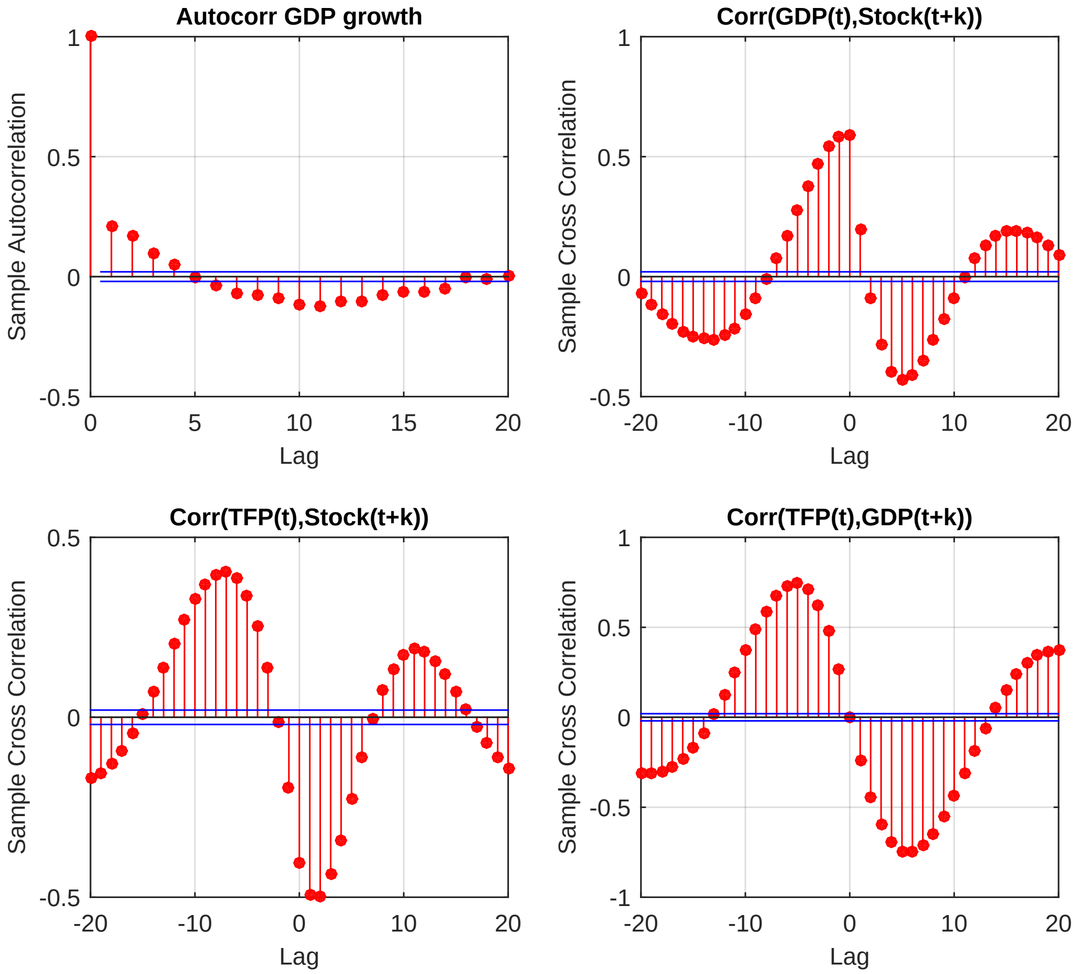

Figure 10: Dynamic correlations in **model simulation, sunspot shock only**: Sample size 11000, initial truncation 1000. *All sample data are HP-filtered. 95% error bands shown in blue.*

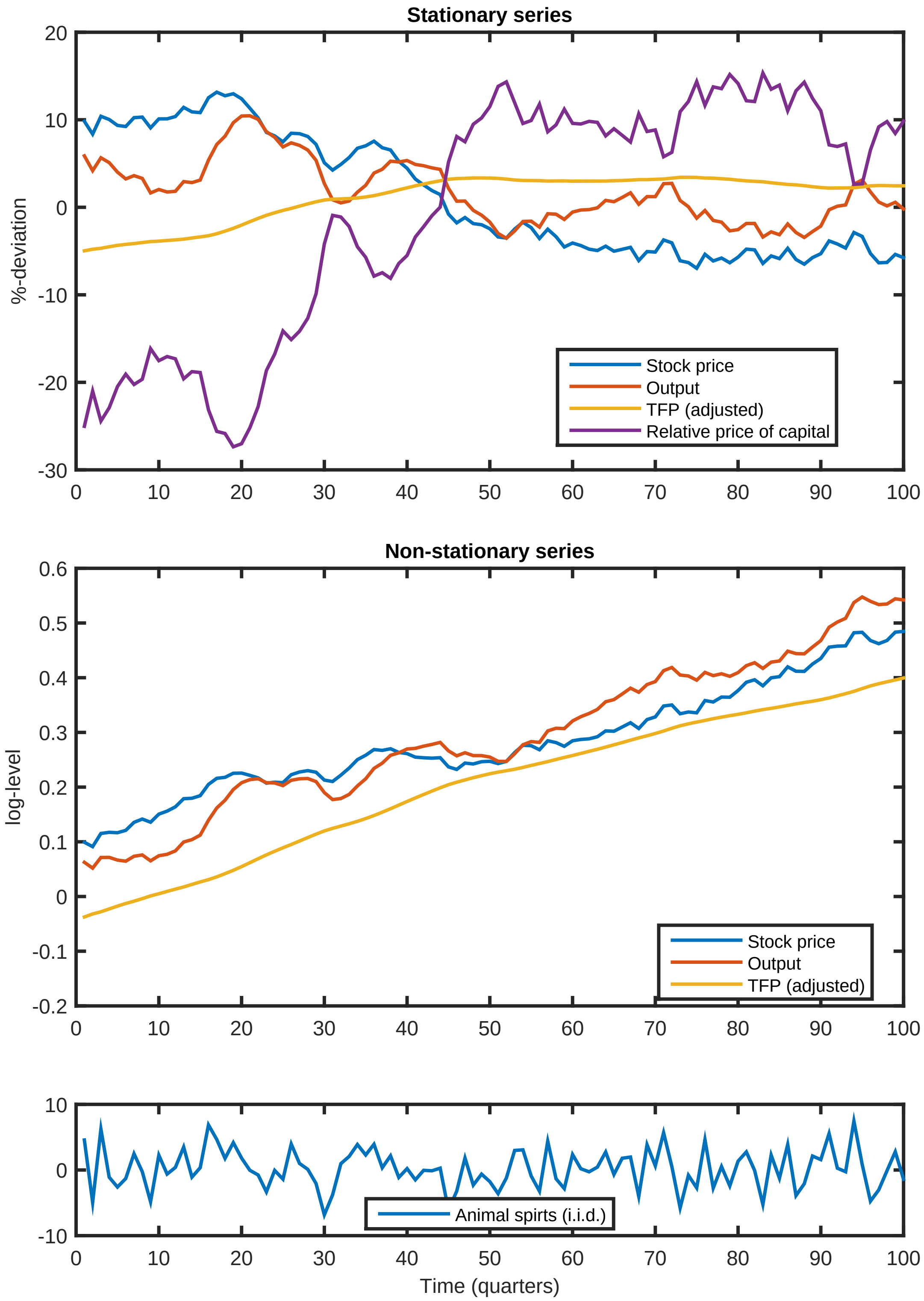

Figure 11: 25-year simulation (stationary series)

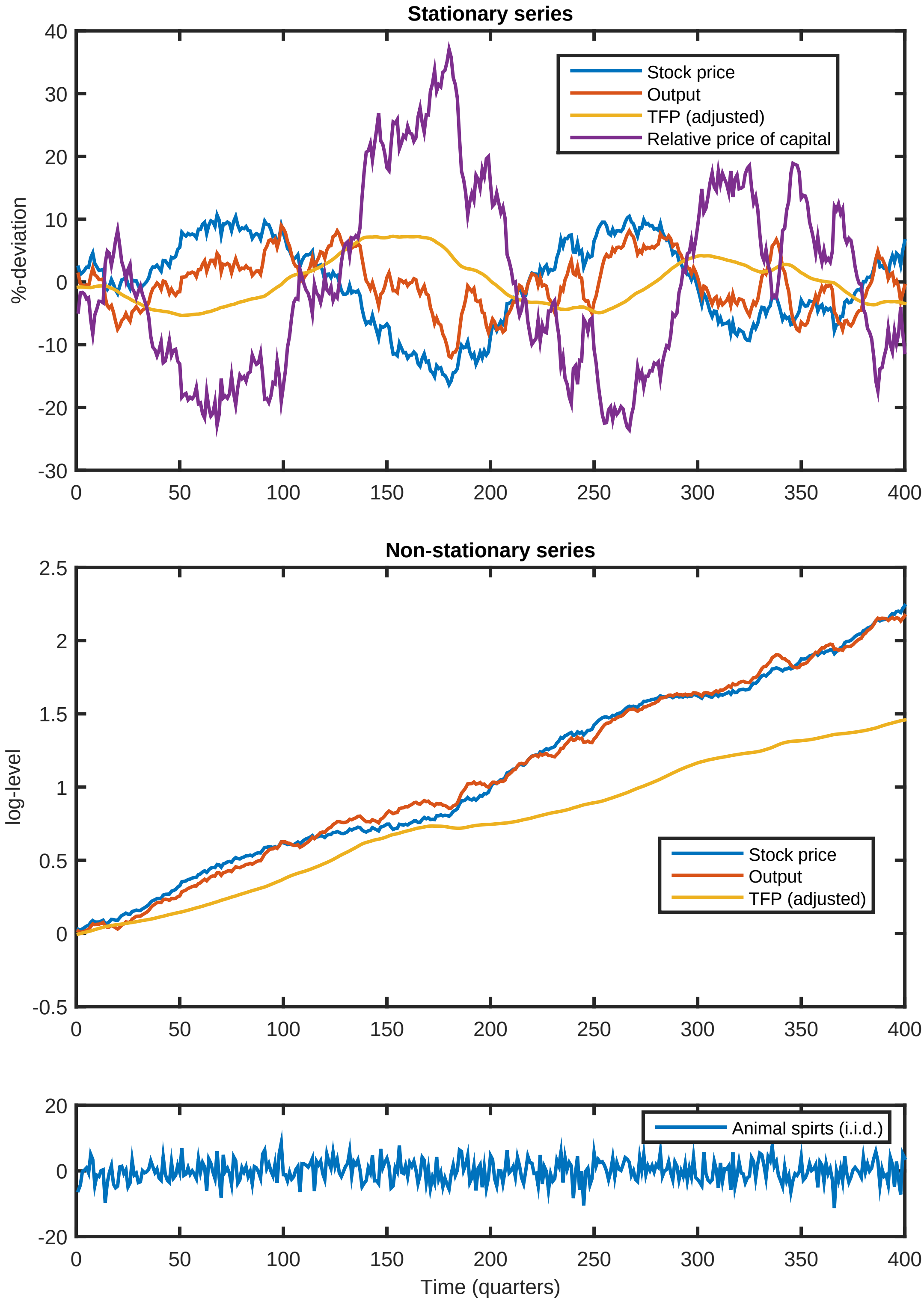

Figure 12: 100-year simulation (stationary & non-stationary series)

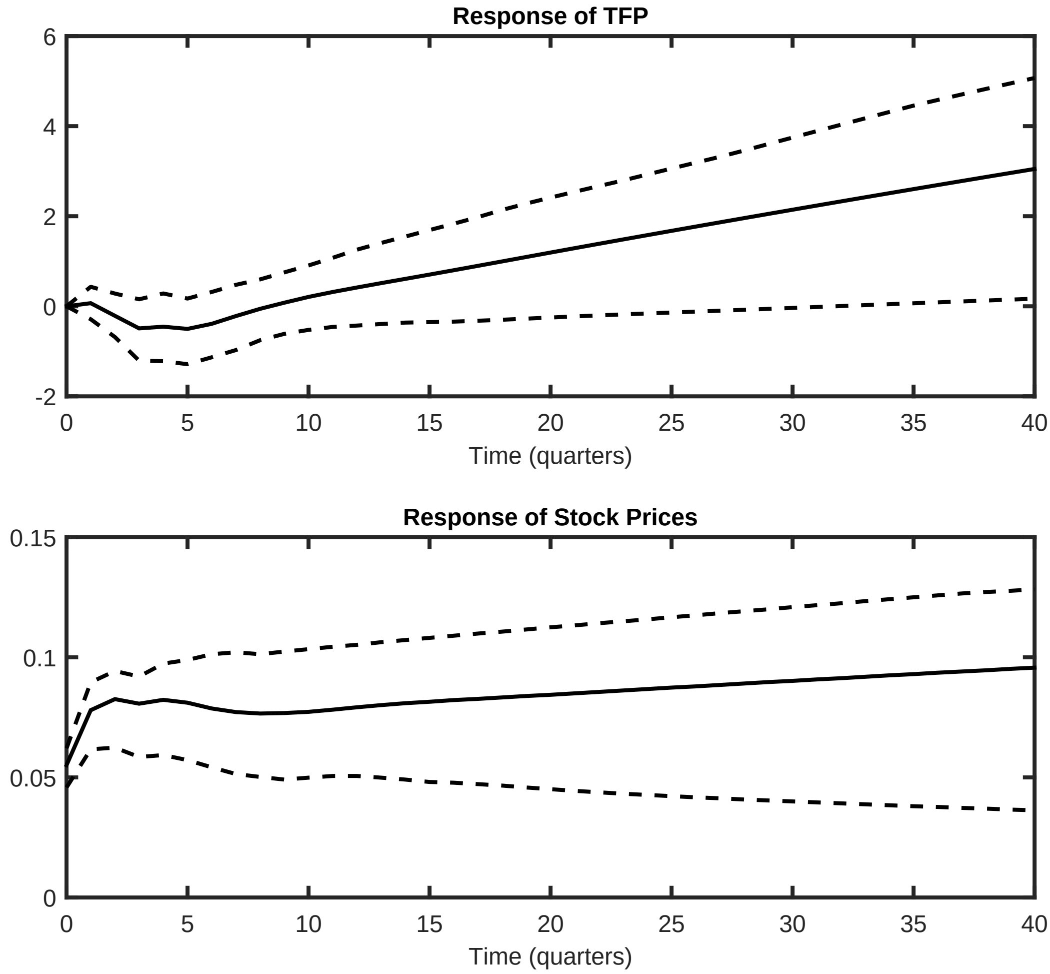

Figure 13: Beaudry-Portier-style VECM on **data**: quarterly observations, 1948 - 2014. *Both panels show response to identified one-standard deviation "news shock". TFP series is adjusted-TFP (per Fernald). Error bands are bootstrapped 95% Efron Percentiles.*

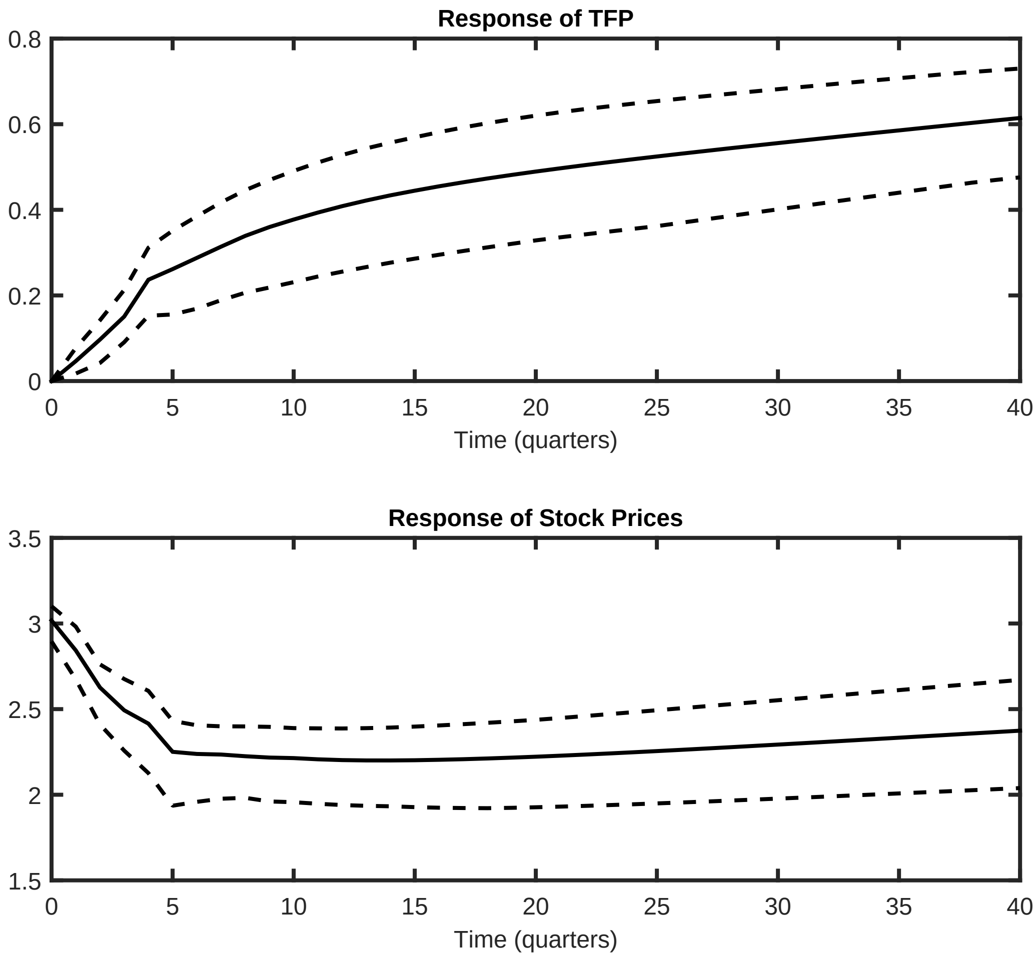

Figure 14: Beaudry-Portier-style VECM on **model simulation, sunspot shock & TFP shock:** sample size 2000 quarters. *Both panels show response to identified 1 standard deviation "news shock", in units of* 10^3. *Sample data are non-stationary log-levels from model simulation. Error bands are bootstrapped 95% Efron Percentiles.*