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Identifying Extreme Values of Exchange Market Pressure

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Abstract: This paper contributes to the existing literature on dating currency crisis in three ways. First, we combine the Monte Carlo simulation with a modified Hill's estimator method to obtain more robust results and efficiently deal with bias variance tradeoff in identifying extreme values. Second, we propose a systematic way to choose the reference country in building the Exchange Market Pressure index rather than arbitrary or descriptive reasoning. Third, different data frequencies are applied and the results are evaluated. Our finding suggests that higher frequency data are more appropriate while applying Extreme Value Theory. It urges researchers to be more cautious in applying EVT and interpreting tail incidences that are obtained from lower frequency data.

JEL Classification: F31, F47, G01

Introduction

Currency crisis is one of major types of financial crises and has caused devastating impacts on the affected economies. In response, numerous empirical and theoretical studies have developed to investigate currency crises, factors that induce them, and their consequences. A large number of these studies investigate timing of crises and attempt to devise an *early warning system* that signals precisely the likelihood of an upcoming crisis, mostly, by relying on macroeconomic fundamentals. Identification of true crisis periods is a vital step in these studies and reliability of their estimations and the relevant policy implications depend on accuracy of the detected crisis episodes.

The origins of current empirical studies on dating currency crises episodes stem from Eichengreen *et al.* (1995 and 1996). They introduced an index for currency pressure that consists of changes in exchange rates, reserves, and interest rates. They define a crisis period when this index exceeds a threshold in that period of time. Eichengreen *et al.* and many researchers who followed them (*e.g.*, see Kaminsky *et al.*,1998), date crises periods by putting priori assumptions and by using arbitrary thresholds. However, there are several concerns regarding the validity of the priori assumptions and arbitrary thresholds (see Abiad, 2003). Alternatively, Pozo and Amuedo-Dorantes (2003) suggest a more objective statistical method to identify crises: Extreme Value Theory (EVT).

The advantage of EVT over the traditional methods is that EVT does not require knowing the exact distributional form of the index of currency pressure. Instead, it determines crises episodes by exploiting information at the tails of the distribution. A shape parameter – called the "tail index" – characterizes the appropriate type of EVT to be applied. In finance and economics, the benchmark estimator for the tail index is Hill's estimator. While it is a consistent estimator, in small samples Hill's estimator is biased and has to deal with the classical bias-variance tradeoff. In the literature there are number of methods to treat the bias-variance tradeoff concern, such as: Monte Carlo simulation (Koedijk *et al.*, 1990), Hill's plot (Embrechts *et al.*, 1997), recursive least squares (Diebold *et al.*, 1999), and a modified Hill's estimator (Huisman *et al.*, 2001). Among these solutions, the Monte Carlo method appears to be more rigorous and has received more attention. Nevertheless, there are two concerns associated with this method: one is conceptual and the other is computational. These concerns will be discussed in detail in the following pages and we will present our solutions, using the modified Hill's estimator as a benchmark.

This paper contributes to the dating of currency crises in three areas. First, we combine Monte Carlo simulation with a modified Hill's estimator method to minimize the bias-variance tradeoff and estimate

more robust results and precise dating. Second, we select the reference country, which a country's currency pressure index should be built around, in a more systematic way rather than by arbitrary choice or descriptive reasoning. Third, we find that higher frequency data are more appropriate for applying EVT compare to lower frequency data. Thus researchers should be cautious in interpreting time aggregation of the tail indices and applying EVT to quarterly and lower frequency data. Furthermore, this paper attempts to improve the existing literature by: dating crises episodes with both monthly and quarterly data, covering a wider sample of countries, and applying more sophisticated statistical tests and methods. Comparing our results with other methods shows substantial differences for a number of countries.

The main objective of this paper is to date currency crises as accurately as possible for 20 OECD members and South Africa during 1970-1998. The identified crisis episodes are to be used in a following paper that studies the empirics of currency crises with the help of duration models.

We proceed as follows. Section 2 presents the currency crisis definition, reviews different methods in dating currency crises, and points to the main concerns regarding the dominant method. Section 3 briefly introduces extreme value theory. Section 4 discusses different methods for the estimation of the tail index. Section 5 describes the data, reports some of their empirical time series properties, and estimates the Hill's index by combining Monte Carlo and the modified Hill's estimator. Section 6 compares our results with some other methods on both monthly and quarterly basis. Section 7 concludes. Some detailed technical results are presented in the appendices.

2. Crisis definition

The first step in the analysis of currency crisis is to identify periods of speculative attacks. Currency crises are not restricted to the *events* of realignments of fixed exchange rates or floating a currency that used to be pegged.[1] Although there are overlapping parts between the *events* and crises, currency crisis is a broader concept.

In the simplest approach, a crisis can be defined as a large movement in nominal exchange rates. Frankel and Rose (1996), define an exchange rate depreciation of 25 percent or more over the last year as a currency crash episode.[2] This approach identifies currency crashes but currency crises are not confined only to the crash periods. Massive sell off of a local currency for foreign exchange is called speculative

1. Interestingly, governments deliberately do the realignments in tranquil periods to avoid future crises.
2. To adjust for instances where countries have high inflation, they also require that the depreciation be at least 10 percent higher than the previous year.

attack, which could lead to a crisis. If the attack is successful it can result in a large depreciation of the exchange rate. But not all attacks are successful. Authorities can alternatively repel attacks by using foreign reserves, hiking interest rates, imposing capital controls, or the combination of all of these methods. Thus, devising a broader definition that includes both successful and unsuccessful attacks would be more useful and will help to better understand and identify the origins of currency crises.

Eichengreen, Rose, and Wyplosz (1995 and 1996) introduce an index to capture both successful and unsuccessful speculative attacks. As Eichengreen *et al.* point out, an ideal index of speculative pressure would be obtained by employing a structural model of exchange rate determination, from which one would derive the excess demand for foreign exchange rate. However, most of empirical studies show little success for structural models to forecast foreign exchange in short and intermediate horizons. Consequently, Eichengreen *et al.* choose an *ad hoc* approach on the basis of Girton and Ropper (1977) to build their index of currency pressure. Girton and Ropper, in an exchange rate determination model, define excess demand for foreign exchange to construct an index – called the Exchange Market Pressure (EMP) – for measuring the volume of intervention that is necessary to achieve any desired exchange rate target. The idea is that an excess demand in foreign exchange can be met through several channels that are not necessarily mutually exclusive. Eichengreen *et al.* exploit Girton and Ropper's index and modify it by adding interest rates. Their speculative pressure measure is a weighted average of exchange rate changes, international reserves changes, and interest rate changes.[3] The weights are set in such a manner to equalize the volatility of all three components. All of these variables are measured relative to a reference foreign currency. A logical choice for the reference country would be a country with a fairly strong and stable currency.

Eichengreen *et al.* index of exchange market pressure is defined as:

$$\mathrm{EMP}_{j,t} \equiv [(\omega\ \%\Delta e_{j,t}) + (\lambda\Delta(i_{j,t} - i^{*}_{t})) - (\theta\ (\%\Delta r_{j,t} - \%\Delta r^{*}_{t}))], \qquad (1)$$

where $\mathrm{EMP}_{j,t}$ stands for exchange market pressure for country j at time t, $e_{j,t}$ denotes the price of one unit of the reference foreign exchange rate in j's currency, $i_{j,t}$ is the money market interest or similar rates for country j during period t, $r_{j,t}$ indicates the ratio of international reserves of country j in domestic currency to its narrow money (M1) at time t; and ω, λ, and θ are the weights. All factors with an asterisk represent similar variables of the reference country.

3. Interest rates can affect capital flows and speculative attacks.

Eichengreen *et al.* choose large positive values of the EMP index and define crises as periods when the index reaches extreme values.[4] Formally:

$$\begin{aligned} \text{Crisis}_{j,t} &= 1 \text{ if } \text{EMP}_{j,t} > \mu_{EMP} + \delta\sigma_{EMP}, \\ &= 0 \text{ otherwise.} \end{aligned} \tag{2}$$

where μ_{EMP} and σ_{EMP} represent the mean and standard deviation of the entire sample of the $\text{EMP}_{j,t}$ and δ is a threshold to be chosen. The result is a binary crisis variable that can be analyzed using limited dependent variable models.[5] In reality, extreme values of EMP date periods of macroeconomic instability that could represent periods of currency crises. Therefore, it seems more appropriate to call the identified periods as currency crises episodes rather than currency crises events. We use both terms interchangeably.

This methodology has received extensive attention from other researchers and on its basis a variety of different versions of EMP has been devised to identify currency crisis episodes. For instance, Kaminsky, Lizondo, and Reinhart (1998), and Sachs, Tornell, and Velasco (1996) modify the EMP index by dropping the interest rate component, arguing lack of availability or reliability of data for countries and time periods used in their sample research. They design their own *early warning system* that is called *signaling approach.* A comprehensive literature review on the empirical studies in the field can be found in Kaminsky *et al.* (1998) and Abiad (2003).[6]

Yet, there are a few concerns regarding the identification of crises with this methodology. First, as Eichengreen *et al.* point out, the data are not pure and are subject to some issues and limitations. Available data for international reserves are imperfect. For instance, some technicalities – balance sheet transactions, third-party intervention, stand-by credits and foreign liabilities that are relevant for exchange market intervention – are usually omitted or incompletely reported. Furthermore, not all changes in international reserves are due to intervention in exchange markets. On the other hand, the availability of market-determined data of interest rates for developing countries is rare. Hence, for this group of countries, EMP is built with the first two components.

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4. Negative values show large appreciations or large increases in reserves that are considered fundamentally different from depreciation pressure crises.
5. Some researchers argue that transforming continuous variables into binary variables may result in loss of information. Thus, they treat EMP as a continuous dependent variable (see *e.g.* Eliasson and Kreuter, 2001).
6. In the literature, there also exists another systematic methodology that identifies currency crises with help of Markov switching models (see *e.g.*, Martinez-Peria, 2002; and Abiad, 2003). In this methodology, states of crises are determined endogenously.

Second, weighting the three components of the index is critical.[7] A simple and easy option for weighting the components can be an un-weighted scheme. However, the volatilities of exchange rates, international reserves, and interest rates are very different and in case of un-weighted components the index will be heavily dominated by the international reserves that have higher volatility compared to exchanges rate and interest rates, respectively. Consequently, it seems more plausible to weigh the components in a manner that their volatilities are equalized and EMP index is not dominated by only one of the components. One common approach is to weigh each component by the inverse of its own standard deviation. Although this approach adjusts each component's weight with its own volatility, it does not exactly equalize the share of the components. Nevertheless, the weights can be normalized to sum up to unity if each weight is divided by the sum of the inverse of standard deviations of all variables. In this case, for instance ω will be:[8]

$$\omega = \frac{1/\sigma_e}{1/\sigma_e + 1/\sigma_i + 1/\sigma_r} \tag{3}$$

This weighting technique is called "precision weight". However, this technique may not be successful in dating periods of crisis, if the volatilities of the components are mostly driven by the policy reaction function of the central banks rather than being market determined. In "precision weighting", higher volatility will result in lower weight, which can potentially lead to biased identifications of the crisis episodes. If the volatility of a component is pretty low, the weights of the other components will be close to zero and the EMP index would be dominated by the stable component. Willett *et al.* (2004) present two instances, Argentina in1995 and Hong Kong in 1998, that the EMP index failed to identify the attacks. In these two cases, the monetary authorities could manage to keep the exchange rates fixed by spending large amount of their international reserves and increasing the interest rates.[9]

In order to avoid averaging and weighting issues, Zhang (2001) breaks down the EMP index into its components and treats each component separately. He identifies crisis episodes when one of the

7. As Eichengreen *et al.* mention the ideal weights should be the slope coefficients that reflect how much official intervention (change in international reserves and/or interest rate) would be required to avoid one percent change in the exchange rate. However, there is no reliable theoretical model for the foreign exchange that the profession agrees upon and the reduced form models provide a good fit.
8. Angkinand, Li, Willett (2006).
9. The weights can be driven from either each country-specific sample [own country precision weights; see *e.g.* Eichengreen *et al.* (1995) and Aziz *et al.* (2000)], or entire sample of countries [pooled precision weights; see *e.g.* Kaminskyand Reinhart (1998) and Glick and Hutchison (2001)]. Some researchers believe pooled weights may lessen this problem but at the cost of probable heterogeneity.

components crosses a sample-dependent threshold.[10] He uses a three-year moving window to compute the standard deviations of the thresholds.

Third, the arbitrary choice of crisis-identification thresholds and their underlying priori assumptions are problematic. While large deviations of the EMP index from its mean is defined as extreme values, the selection of the threshold is fairly arbitrary. Obviously, different choices result in different crisis episodes. In the literature, range of the threshold varies from 1.5 to 3 standard deviations. More surprisingly, as Abiad (2003) notes, some researchers, Kamin *et al.* (2001) and Caramazza *et al.* (2000), treat the threshold as a free parameter to fulfill their research objectives. Furthermore, use of the mean and standard deviation approach to pick up extreme observations is based on that the underlying assumption series of EMPs are well behaved and normally distributed. However, it is well known that speculative price series turn out to be more compatible with fat-tailed distributions than normal ones (Jansen and de Vries, 1991). Therefore, the arbitrary choice of thresholds in picking up extremely large values of EMPs becomes even more inappropriate. Accordingly, we apply an alternative methodology to capture the dispersion of the series and label their extreme values in a rigorous manner.

3. Extreme value theory

The dispersion of the EMP index determines periods of successful and unsuccessful speculative attacks. As stated above, well behaved normality does not necessarily hold due to fat tails and skewness in the series. Alternatively, Pozo and Amuedo-Dorantes (2003), following Koedijk *et al.* (1990), suggest applying extreme value theory (EVT) to exploit information in the tails of the series. EVT identifies crisis dates with the help of more rigorous statistical methods and there is no need to set arbitrary assumptions and/or thresholds.

In this section we briefly introduce EVT, its different types, fat-tailed distributions, and tail indices. A comprehensive, detailed and technical introduction can be found in de Haan and Ferreira (2006) and Embrechts, Kluppelberg and Mikosch (1997).

EVT provides a framework to study the behavior of the tails of a distribution. It enables us to apply extreme observations to measure the density in tails and build statistical models for rare phenomena like

10. He also claims his method can overcome Flood and Marion's (1999) argument. Following Krugman (1979), Flood and Marion argue since the interest rate falls back and reserves flow back right after the devaluation, these two effects may cancel out some part of changes in exchange rates and dampen the EMP index. So, in the case of predictable devaluations, the EMP index may fail to identify a crisis.

stock-market crashes or speculative attacks. EVT is quite similar to the *central limit theorem* and both have common mathematical backgrounds. As the limiting distribution of sample averages is a normal distribution, the limit laws of order statistics are characterized by a class of EVT. This theory deals with asymptotic distribution of maxima without generalizing about the distribution of the whole series. It only studies the tails' distribution. Fortunately, analogous to the *central limit theorem* the limit laws provided by EVT do not require a detailed knowledge of the original distribution that extreme observations belong to. There are two approaches to study the extreme events by EVT. One is direct modeling of either maximum or minimum realizations. The other one is modeling of the exceedances of a certain threshold.

Consider $X_1, X_2, \ldots, X_n$ to be a sequence of stationary random variables that may be either *i.i.d.* or dependent with a distribution function $F(x)$.[11] We are interested in the probability that the maximum[12]

$$M_n = \max \{X_1, X_2, \ldots, X_n\}, \tag{4}$$

of the first n random variables be less than a certain level x. This probability is given by:

$$\Pr\{ M_n \leq x\} = \Pr\{ X_1 \leq x, X_2 \leq x, \ldots, X_n \leq x\} = [F(x)]^n. \tag{5}$$

Unfortunately, in most cases $F(x)$ is not known and for most cases that it is known, it is not practical to calculate $[F(x)]^n$ even for moderate values of n. However, EVT is able to appropriately provide the limiting distribution of the order statistic M_n. One can normalize M_n by a location parameter (b_n) and a scale parameter ($a_n>0$) as:

$$\Pr\{a_n (M_n - b_n) \leq x\} \xrightarrow{w} G(x); \tag{6}$$

and in the case that the X_i are *i.i.d.*

$$[F((x/ a_n) + b_n]^n \xrightarrow{w} G(x), \tag{7}$$

where $G(x)$ is the limit law of M_n and is a *max-stable* distribution and w stands for weak convergence. If equation (5) holds, F will belong to the domain of attraction of G. *Max-stable* distributions are the possible class of limit laws for equation (5). A non-degenerate density function $G(x)$ is called *max-stable*, if there exist real constants $A_n > 0$ and B_n such that for all real x and positive integer n:

11. This part is heavily drawn from Hols and de Vries (1991).
12. Since by changing the sign of the Xs one can switch from the study of maxima to minima, we just concentrate on positive random variables.

$$[G(A_n\, x + B_n)]^n = G(x). \tag{8}$$

Based on the *Fisher-Tippet theorem*, every *max-stable* distribution is one of the following types:

$$\begin{array}{lll}
\text{Type I (Gumbel):} & \Lambda(x) = \exp(-e^{-x}) & x \in R; \\
\text{Type II (Frechet):} & \Phi_\alpha(x) = 0 & x \leq 0, \\
 & \quad = \exp(-x^{-\alpha}) & x > 0, \\
 & & \alpha > 0; \\
\text{Type III (Weibull):} & \Psi_\alpha(x) = \exp(-(-x)^{\alpha}) & x < 0, \\
 & & \alpha > 0; \\
 & \quad = 1 & x \geq 0.
\end{array} \tag{9}$$

The theorem suggests that the asymptotic distribution of the maxima belongs to one of the three distributions above, regardless of the original distribution of the observed data. While α goes toward ∞ or $-\infty$, Frechet and Weibull distributions attain the shape of a Gumbel distribution, respectively. Weibull family tails decline with a finite tail index. They are thin-tailed distributions with a finite upper endpoint.

The possible limit laws for $G(x)$ can also be represented in a unified model with a single parameter. This presentation is known as the Generalized Extreme Value distribution (GVE):

$$\begin{aligned}
H_\gamma(x) &= \exp\{-(1+\gamma x)^{-1/\gamma}\} \quad \text{if } \gamma \neq 0 \text{ and } 1+\gamma x > 0, : \\
&= \exp\{-\exp(-x)\} \qquad \text{if } \gamma = 0.
\end{aligned} \tag{10}$$

where $\gamma = 1/\alpha$ is the shape parameter and α is the tail index.

Intuitively, these functions represent three possibilities for decaying of density functions in the tail. Loosely speaking, the tails of the distribution fall in three different categories:

1) They decline exponentially and all of their finite moments exist. Cases like normal, log-normal, and gamma distributions lie within Gumbel type tails.
2) Tails can also decay by power but not quick enough when weighted by the tail probabilities and consequently cannot be integrated. This type of distributions, like Stable, Paretian, and Student's *t* are said to be fat-tailed and are among Frechet type tails.
3) Weibull family tails fall within a finite tail index. They are thin-tailed distributions with a finite upper endpoint.

Economic theory tends not to be informative about the specific density function that applies. However, the qualitative characteristic of an economic process may point to the relevant limit law. On the basis of the strong fat-tailed and the unbounded nature of exchange rate returns[13], as well as the EMP index, the possibility of type I and III distributions can be ruled out, leaving type II limit laws as the relevant one (if the maxima distribution converges at all). Thus, we will concentrate on the Frechet domain of attraction that includes a range of distributions: Student's *t*, the stable distributions, and ARCH type process.

The tail index (α) is the unifying feature across the limit laws distributions and is used to capture the weights of the tail in the distribution of X_i. In different cases, the scaling parameter (a_n) and the location parameter (b_n) may need to be modified but, since α is the unifying feature, it remains unaffected. The tail index also indicates the number of bounded moments of the distribution that exist; the moments less than α are finite and well defined while those bigger than α are infinite.

For *i.i.d.* stable random variables[14] (not to be confused with *max-stable*) that have an invariant density function under addition, tail index α equals the characteristic exponent (shown by β).[15] The effect of dependency would be larger values that tend to come in clusters. Student's *t* is also in the domain of attraction of type II. Degrees of freedom are equal to the tail index ($\alpha \geq 2$). Student's *t* has well defined mean and variance while stable distributions have a finite mean but no finite variance.

For the ARCH or GARCH processes, though their building blocks can be normal variates, the unconditional distribution of the realizations are fat-tailed. In the case of ARCH (1), the tail index is equal or greater than two. Formal generalizations to higher-order processes are nonexistent, but some generalizations can be easily obtained.

As shown above, having a fat (heavy) tail is the main characteristic of type II limit law. But how to distinguish fat tails? Loosely speaking fat tail or *leptokurtic* distributions exhibit extremely large kurtosis particularly with respect to normal distribution and follow power-law decay.[16]

Formally, one can say there is a heavy upper tail for the positive X_i, if for large x:

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13. See, for example, Boothe and Glassman (1987), Jansen and de Vries (1991), and Koedijk and Kool (1994).
14. Normal distribution belongs to the stable class but it has all moments and is not fat tailed.
15. For Cauchy distribution $\alpha=\beta=1$, for normal distribution $\beta=2$, and for chi square distribution $\beta=1/2$.
16. In a few cases this measure might be misleading. For instance, discrete mixtures of the normal, mixed jump processes, and the power exponential all exhibit higher kurtosis but nevertheless possess all moments and are thin tailed. Anyhow, there is no unique definition for fat-tailed distributions in the literature.

$$1\text{-}F(x) = x^{-\alpha} L(x) \text{ as } x \to \infty,\ \alpha > 0, \tag{11}$$

where L(x)is such that for any $x>0$

$$\lim_{t\to\infty} \frac{L(tx)}{L(t)} = 1. \tag{12}$$

As shown in equation (11), the tail of a distribution can be divided into two parts: the L(x) function and the power part. The L(x) function is asymptotically unimportant since L(tx)$\approx$ $L(x)$ for large t. The sufficient and necessary condition for a distribution to be fat-tailed is the property of regular variation that means L(·) varies regularly at infinity. Therefore, the tail of the distribution is dominated by the power part $x^{-\alpha}$. Due to the power part, the tail of F(x) always falls more slowly than the tails of distributions that decline exponentially, like normal distribution. From (10) it is obvious that there is an inverse relation between α and the size of a fat tail; larger α results in a lower fat tail. When L(x) is constant then F(x) is the Pareto distribution.

From the above discussion, it is obvious that competing fat-tailed density functions are nested within their limit law G(x), and are distinguished by different values for the tail index (α). In fact, the tail index characterizes the limit law. Now we examine how to estimate tail indices.

4. Tail index estimation methods

In this section we review some parametric and nonparametric estimators of the tail index with more stress on the type II limit law. We introduce Hill's estimator as the benchmark estimator of the tail index and consider its bias-variance tradeoff in small samples. As a solution for bias problem the modified Hill's estimator is introduced.

In general, there are two different procedures for estimating the tail index (α). The first class of estimators follows parametric approach and directly estimates α with maximum likelihood or regression techniques. Jansen and de Vries (1991) show that under the type II limit law, direct estimation of α by maximum likelihood is consistent but not the most efficient. This approach assumes each period's maximum follows exactly one of the three limit laws. Obviously, this assumption is too strong and can cause misspecification bias. Furthermore, parametric approach requires estimation of an extra scale parameter, which can be interpreted as another drawback of this approach.

An efficient approach for estimation of the tail parameter is to use all realizations from a single sub-sample that are above a certain high threshold (*exeedance*). Some efficient semi-parametric estimators have been proposed on this basis. These estimators use the largest order statistics and all they require is the original distributions that generate the observations be well behaved. It implies that the remaining estimation errors can be attributed to the use of finite samples.

Suppose $X_1, X_2, \ldots, X_n$ is a stationary sequence such that M_n has a type II distribution. By arranging the observations in an ascending order $X_n \geq X_{n-1} \geq \ldots \geq X_m \geq X_{m-1} \geq \ldots \geq X_1$, two alternative estimators for γ – based on the largest order statistics X_i – are introduced as follows:

Pickands' estimator:

$$1/\hat{\alpha} = \hat{\gamma}_{\mathrm{P}} = (\ln \frac{X_{(m)} - X_{(2m)}}{X_{(2m)} - X_{(4m)}}) / \ln 2. \tag{13}$$

This estimator has been shown to be weakly consistent. Its strong consistency and asymptotic normality has also been obtained when the maximum order value *m* rises rapidly enough relative to the sample size *n*. Pickands' estimator is a general estimator and can provide estimations for all three types of limit laws.

Hill's estimator:

$$1/\hat{\alpha} = \hat{\gamma}_H = \frac{1}{m}\sum_{j=1}^{m} \ln X_{n-j+1} - \ln X_{n-m}. \tag{14}$$

First presented by Hill (1975), has been proven to be a consistent estimator of γ with $(\hat{\gamma}_H - \gamma)m^{1/2}$ being asymptotically normal with mean zero and variance γ^2. Hill's index is more efficient than the maximum likelihood estimator since it has smaller variance and beats the Pickands' estimator on the consistency basis. It is the bench-mark estimator for the tail index of type II limit law.

In both nonparametric estimators, the final tail index estimate relies heavily on the choice of cut-off point *m*. While all estimation procedures require that *m* goes to infinity at a lower rate than the sample size *n*, there is little instruction on how to choose *m* optimally. Like other upper-order observations that deviate from an exact Pareto-tail, the choice of *m* ultimately involves the classic tradeoff. This problem would

even be more severe in small samples. If *m* is chosen conservatively with few observations from the tail, then the tail index estimate will be sensitive to outliers in the distribution and will have larger variance. On the other hand too many observations from the tail and too few from the central part of the distribution can result in a more stable index, but of course, with a higher degree of bias.

There are a number of solutions to deal with the sensitive tradeoff issue. Danielsson *et al.* (2001) use a two-step subsample bootstrap method to estimate the number of order statistics (*m*). Their approach does not require prior knowledge of second-order parameters and is a statistically optimal solution. Unfortunately, it is only appropriate for large enough samples. Another possibility is what Embrechts *et al.* (1997) call Hill's plot. In this method α is estimated for different values of *m* and then an optimal value of *m* will be chosen from the region where the estimated tail parameter (α) is stable. Even if such a region exists, however, selecting the specific *m* within this region may not be precise. Koedijk *et al.* (1992), Longin and Solnick (2001), Haile and Pozo (2006 and 2008), among others, use the asymptotic properties of Hill's estimator to choose *m*. By exploiting asymptotic normality property of Hill's estimator they apply Monte Carlo simulation method to find the optimum level of *m*. They minimize the mean square error (MSE) of the estimated γ, conditional upon a sample size *n* and degrees of freedom of F(*x*). The Monte Carlo simulation appears to be rigorous and helpful in optimizing the tradeoff between bias and inefficiency.

Huisman *et al.* (2001) apply the weighted least square (WLS) method to solve the problem. While the Hill's index is asymptotically unbiased, it is shown to suffer from bias in small sample estimates. They exploit information obtained from a set of Hill's estimates, each conditioned on a different number of tail observations. They calculate a weighted average over a range of estimated Hill indices where weights are measured by simple least square techniques. A brief review of this approach comes as follows.

It is shown that for a general class of distribution represented by:

$$F(x) = 1 - ax^{-\alpha}(1+bx^{-\beta}). \tag{15}$$

depending on the parameters values, F(·) can represent some specific distributions, e.g. $F(x)= 1-x^{-\alpha}$ for $a=1$ and $b=0$. Hall (1990) show that in most cases the expected value of the Hill index for given *m* is:

$$E(\gamma(m)) \approx \frac{1}{\alpha} - \frac{b\beta}{\alpha(\alpha+\beta)}\alpha^{-\frac{\beta}{\alpha}}\left(\frac{m}{n}\right)^{\frac{\beta}{\alpha}}. \tag{16}$$

It is clear that bias is increasing in *m* and Hill's index would be a biased estimator for any *m* greater than zero in small samples. To approximate bias and make it linear in *m*, Hall imposes an $\alpha = \beta$ condition that is warranted not to be harmful.

Hall also derives the asymptotic variance of Hill's estimator for the above class of distribution as:

$$\mathrm{var}(\gamma(m)) \approx \frac{1}{k\alpha^2}. \tag{17}$$

As it is clear from (15) and (16), a small *m* is desirable from the perspective of unbiasedness, while a large *m* is preferred for the sake of efficiency.

Huisman *et al.* (2001) claim that for values of *m*, which are smaller than a threshold value *M*, the γ estimates are seen to increase almost linearly in *m* and for larger values of *m*, the pattern of γ depends on values of β/α exponents. Therefore, they suggest approximating the bias term for small enough values of *m* by:

$$\gamma(m) = \beta_0 + \beta_1 m + \varepsilon(m), \qquad m=1,2,3,\ldots, M. \tag{18}$$

They propose to estimate the tail index of the distribution by computing Hill's index for *m* from 1 to *M*. The intercept value or β_0 in equation (17) should be an unbiased estimator of γ while *m* approaches zero. This approach may solve bias-variance tradeoff by exploiting information from the certain range of conventional Hill's estimators based on values of *m*, where γ varies linearly. They show that estimates of tail index are quite robust with respect to the choice of *M*, as long as $M \leq n/2$.

Huisman *et al.* choose to apply WLS instead of OLS to estimate (17) to overcome two problems. First, from equation (16) it is clear that the variance of Hill's estimator is not constant and varies based on values of *m*, consequently, $\varepsilon(m)$ in equation (17) is heteroscedastic. Second, different estimates of γ are correlated through different values of *m*. $\gamma_{(m)}$ and $\gamma_{(k)}$, where $m \neq k$, are based on $1+min\,(m,k)$ common observations. In a matrix notation, equation (17) can be shown as:

$$a\gamma^* = Z\beta + \varepsilon. \tag{19}$$

where γ^* is a vector of γ for different values of *m* from 1 to *M*, *Z* is a $(M \times 2)$ matrix with ones in the first column and a vector of $\{1,2, \ldots, M\}$ in the second. To apply WLS they propose a $(M \times M)$ weighting

matrix W that has { $\sqrt{1}, \sqrt{2}, ..., \sqrt{M}$ } as the main diagonal elements and zeros elsewhere. WLS estimates of β are:

$$b_{wls}=(Z'W'WZ)^{-1}Z'W'W\gamma^{*}, \tag{20}$$

The estimated tail index γ would be equal to the first element of the vector b_{wls}. Consequently, their modified Hill estimator is a weighted average of the traditional Hill's estimators:

$$\gamma(M) = \sum_{m=1}^{M} w(m)\gamma(m). \tag{21}$$

In order to minimize the bias-variance tradeoff and obtain an unbiased and robust estimation of Hill's estimator, we combine Monte Carlo methods suggested by Longin and Solnik (2001) and the modified Hill's estimator by Huisman, Koedijk, Kool, and Palm (2001) approach to estimate Hill's index.

5. Empirical estimation

5.1. Data. The source of all data is the International Financial (IFS) of the IMF. Available monthly and quarterly data from January 1967 to the end of 1998 (when 10 countries in our sample left their own national currencies and joined the Euro currency system) are collected for the period average of the exchange rate (IFS, line rf.), total reserves minus gold (IFS, line 1l.d), M2 or money plus quasi money (IFS, line 34 plus IFS, line 35), and short term interest rates given by a money market or a similar rate (IFS, line 60b and if not available IFS, line 60). It should be mentioned that short duration attacks may not be evident (especially unsuccessful ones) even for monthly data, which is the highest available frequency. If an attack takes place and be fended off within a few days the average interest rate and international reserves data may not be able to show the intensity of speculative pressure.

Our sample includes 21 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, South Africa, Spain, Sweden, Switzerland, and the UK. Availability of higher frequency data (monthly) and greater chance of reliability are the determining factors for selecting these countries.

In order to compute the EMP index we need to select an appropriate currency of reference for each country. Some studies (for example, Eichengreen *et al.*, 1995) choose one currency while other (for example, Tudela, 2003) chooses two currencies – either the DM or the USD – as the reference. In some

cases, for example the US Dollar (USD) for Canada or the Deutsche Mark (DM) for Austria, choice of the reference currency seems straightforward. But in other cases this choice is not clearly exclusive. For example, in case of UK or Greece both of the DM and the USD look equally good candidates. In previous studies, to the best of our knowledge, choice of the reference currency has been an arbitrary selection between the USD and the DM; with or without descriptive reasoning. However, we attempt to select the reference currency in a more systematic approach. The key criterion that we use to fulfill this task is the stability of the exchange rate during the sample period. For each local currency, we select the reference currency based on the lowest volatility of the local currency against potential candidate references. To conserve the degrees of freedom, we confine potential candidates to the USD and the DM, which during our sample period have been consistently strong and very well accepted internationally. [17]

As the first step, we compute variations of the local currency in terms of both candidates to capture the stability of the local currency against the DM and the USD. In this regard, the simplest approach is to compare the unconditional time invariant variances during the whole sample period and choose the currency with the smaller variance as the reference. Nevertheless, the volatility of economic and financial time series are barely independent during the time. To overcome this concern, we employ the Auto Regressive Conditional Heteroscedasticity (ARCH) models, which are the most efficient method in the literature to capture the time-dependent volatility.

There are several different types of ARCH models that estimate time-dependent volatility as a function of prior volatilities. Specifically, two versions of ARCH models – General ARCH (GARCH) and Exponential GARCH (EGARH) models – are widely applied in financial time series. While GARCH models assume symmetric impacts for the innovations (the good and bad news have the same impacts) and can only account for bounded parameters, EGARCH models relax the symmetric assumption and consider asymmetric impacts of innovations and can also handle unbounded parameters.[18] Hence, we use EGARCH models to capture time-dependent volatilities. Our index in capturing volatility of the exchange rate series is unconditional standard deviation. However, in several cases EGARCH models do not converge. As an alternative method, we also calculate a three-year rolling standard deviation of the exchange rate series to deal with the time-varying volatility. The average of the three-year moving window standard deviations is used as an alternative for the unconditional standard deviation of each country. Results are reported in Table 1.

17. During our sample period, the USD has experienced weak periods in the 1970's (1971-1973) and mid 1990's.
18. For a comprehensive introduction and comparison of ARCH family models, see Enders (2004).

Table 1. Volatility of exchange rates in terms of the DM and the USD

	monthly				*quarterly*			
	EGARCH unconditional std. dev.		*three-year window standard deviation*		*EGARCH unconditional std. dev.*		*three-year window standard deviation*	
Country	*DM*	*USD*	*DM*	*USD*	*DM*	*USD*	*DM*	*USD*
Australia	3.81	…	3.13	1.96	…	…	5.48	3.39
Austria	…	3.29	0.42	2.44	…	…	0.49	4.35
Belgium	…	4.09	0.71	2.43	2.73	5.25	1.12	4.34
Canada	3.49	1.34	2.72	0.93	6.83	…	4.79	1.42
Denmark	…	2.71	0.87	2.35	…	…	1.34	4.24
Finland	…	…	1.76	2.2	…	…	3.01	4.13
France	…	…	1.18	2.5	3.17	…	2.02	4.37
Greece	2.94	…	1.86	1.94	…	…	3.32	3.57
Iceland	…	…	3.36	3.31	…	13.13	5.43	5.46
Ireland	…	…	1.53	2.29	…	…	2.45	4.24
Italy	2.77	…	1.7	2.18	…	…	2.88	4.14
Japan	2.63	5.24	2.37	2.5	5.7	6.99	4.38	4.44
Netherlands	…	4.35	0.52	2.43	…	…	0.8	4.3
New Zealand	…	…	2.89	2.19	…	…	4.74	3.77
Norway	…	…	1.27	2.05	2.75	7.42	2.19	3.73
Portugal	…	…	1.57	2.34	4.14	…	2.53	4.15
South Africa	…	4.99	2.98	2.39	7.13	3.41	5.06	4.63
Spain	…	…	1.98	2.25	…	…	3.4	4.16
Sweden	2.06	6.84	1.74	2.15	…	13.01	3.19	4.15
Switzerland	1.31	3.67	1.38	2.78	…	…	2.3	4.94
UK	…	1.37	2.23	2.34	…	…	4.08	4.49

… implies that either there exists no ARCH effect in the series or EGARCH model does not converge.

Based on presented results in Table 1, the USD is chosen as the reference currency for Australia, Canada, New Zealand, and South Africa while the DM is selected as the reference for all other countries in our sample.[19] The results are in line with previous studies with more than one reference country (for example, Tudela, 2003; or Haile & Pozo, 2006), except for Japan. In these studies, the USD is the reference currency for Japan but our results indicate that the DM is a more appropriate reference currency for Japan. As mentioned earlier, the selected reference currencies are based on the stability of exchange rate relation during our sample period, therefore, the reference currency will be different if the period horizon changes.

19. Case of Iceland can be a little confusing. While monthly data show Krona is marginally more stable in term of the USD, quarterly data indicate different direction and show that Krona is more stable in term of the DM. Considering its major economic partners, our final selection is made based on quarterly data.

5.2. Empirical EMP series. EMP indices are built following Eichengreen *et al.* (1995) as a weighted average of three components: exchange rate changes, variations of the ratio of international reserves in local currency to M2 (money plus quasi money), and money market or similar interest rate changes. All components are measured with respect to the related reference country. The weights are calculated from each country's specific sample such that to equalize the volatilities of the components. Since the conditional standard deviations of three components of the EMP index are not constant, the weights are chosen to be time-varying. This technique will help to prevent the dominance of high volatility periods over the whole sample (Zhang, 2001). Finally, in order to account for conditional time-varying standard deviations and capturing the potential asymmetries of crises, weights are estimated by applying EGARCH models to the series of each component.[20]

Before starting EVT analysis, it is important to check statistical properties of EMP series and verifying that these series are fat-tailed. Some statistics of the monthly and quarterly EMP indices are reported in Tables 2 and 3, respectively. Although both monthly and quarterly series present similar patterns, there are some differences. For monthly series, Shapiro-Wilk test results suggest that none of monthly EMP indices are normally distributed.[21] All of these series exhibit excess kurtosis and most of them are skewed to the right – all series except for the Netherlands and Switzerland. These features indicate that monthly EMP series are fat-tailed and fall asymptotically within the domain of attraction of the Frechet distribution. For quarterly series, all indices exhibit excess kurtosis too, however, less than the case of monthly series. Also, fewer indices are skewed to the right. The Shapiro-Wilk test results show that five quarterly EMP series – Canada, Finland, Greece, Ireland, and Switzerland – are normally distributed.

Stationarity is a vital condition in EVT analysis. For robustness purposes, we use both parametric and non-parametric tests to verify stationary condition of the series. First, for each series the Dickey-Fuller Generalized Least Square (DFGLS) test is run and the optimum number of lags that minimizes the Modified Akaike Information Criterion (MAIC) is obtained. Then, the Dickey-Fuller (DF) test is run with the optimum number of lags that are attained through DFGLS. Second, as a non-parametric test, the Phillips-Perron test for unit root is run. Results are shown in Tables 2 and 3.

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20. Each conditional variance of the three components are estimated based on the EGARCH(1,1) model: $y_t = x_t\varphi + \varepsilon_t$ where $\ln\sigma_t^2 = \nu_0 + g(z_{t-1}) + \nu_1 \ln\sigma_{t-1}^2$, $\varepsilon_t = z_t\,\sigma_t$, and $g(z_t)$ is a well defined function of z_t. In the mean equation, y_t represents one of the three components and x_t is a lagged values of y_t. The fitted conditional standard deviation (σ^h_t) is used to generate weights in the EMP index. For conditional volatility, there is no concern about non-convergence.
21. In order to make the outcomes visually evident, histograms of monthly and quarterly EMP series are overlaid by the standard normal distribution density and are reported in Appendix B. Generally, centers of histograms are more peaked and there is a greater mass in the tails that confirms they have fat tails.

Table 2. Statistical properties of the monthly EMP series

Country	*N*	*mean*	*sd.*	*skew.*	*kurt.*	*norm.*[1]	*station.*[2]	*staion.*[3]	*indep.*[4]	*ARCH*[5]
Australia	348	0.08	1.08	0.41	5.5	0	0	0	0	0
Austria	346	0.03	0.53	0.83	13.61	0	0	0	0	0
Belgium	345	0.08	0.8	1.12	7.54	0	0	0	0	0.02
Canada	348	0.47	0.71	0.48	4.36	0	0.15	0	0	0.13
Denmark	348	0.11	1.06	0.65	8.48	0	0	0	0	0.05
Finland	348	0.05	0.91	1.24	9.88	0	0	0	0	0.04
France	342	0.05	0.95	0.52	6.56	0	0	0	0	0
Greece	348	0.15	0.63	0.91	7.58	0	0.05	0	0	0.61
Iceland	348	0.36	1.29	0.85	6.95	0	0	0	0.21	0.76
Ireland	348	0.1	0.69	1.78	11.13	0	0	0	0.33	0
Italy	348	0.15	1.32	1.05	9.95	0	0	0	0.03	0
Japan	348	-0.02	0.86	0.5	6.99	0	0	0	0	0
Netherlands	348	0.02	0.68	-0.31	15.88	0	0	0	0	0.19
New Zealand	348	0.06	1.48	3.85	62.25	0	0	0	0.04	0
Norway	348	0.09	0.64	0.93	5.88	0	0	0	0	0
Portugal	348	0.29	1.39	1	14.58	0	0.03	0	0	0
South Africa	348	0.14	1.37	0.25	5.71	0	0	0	0	0
Spain	348	0.17	1.48	0.69	6.73	0	0	0	0.39	0
Sweden	348	0.12	1.53	2.26	24.88	0	0	0	0.25	0
Switzerland	348	-0.06	0.8	-0.31	5.28	0	0	0	0	0.01
UK	348	0.15	1.3	0.75	5.46	0	0	0	0	0.57

1. p-value of Shapiro-Wilk test for normality. (null of normality)
2. Mackinnon approximate p-value of Dickey-Fuller test for stationarity. (null of having uint root test)
3. Mackinnon approximate p-value of Phillip-Perron test for stationarity. (null of having uint root test)
4. p-value of White noise test for autocorrelation. (null of no aurocorrelation)
5. p-value of Lagrange Multiplier (LM) test for ARCH. (null of no ARCH effect)

Based on the Philips-Perron test, all monthly and quarterly series are stationary. However, the DF test shows that Canada in monthly series and six other countries: Belgium, Denmark, Greece, Iceland, Ireland, and Switzerland in quarterly series, have non-stationary EMP series. This unit root problem may stem from structural changes in theses series.

We apply the Lee and Strazicich (2004) Lagrange Multiplier (LM) unit root test to account for structural breaks in the non-stationary diagnosed series. This powerful test is able to allow for one-break in intercept and/or trend without showing size-distortions in the presence of a break under the null. [22] Lee and

21. Having correct size and high power results are two main desired factors in every statistical test. In unit root tests, presence of cross-sectional correlation causes size distortion that leads to over-reject of the unit root null.

Strazicich test accounts for the endogeneity of the time break (TB) by minimizing a LM statistic. The test results show that the null of "non-stationary" can be rejected for all of the series that are diagnosed with unit root. Table 4 reports the results.

Table 3. Statistical properties of the quarterly EMP series

Country	*N*	*mean*	*sd.*	*skew.*	*kurt.*	*norm.*[1]	*station.*[2]	*staion.*[3]	*indep.*[4]	*ARCH* [5]
Australia	116	0.23	1.84	0.37	5.69	0.00	0.00	0.00	0.22	0.00
Austria	115	0.03	0.59	-1.85	14.79	0.00	0.00	0.00	0.00	0.59
Belgium	115	0.14	1.11	0.46	4.84	0.00	0.14	0.00	0.56	0.11
Canada	116	0.13	1.20	-0.08	3.33	0.49	0.01	0.00	0.07	0.44
Denmark	116	0.28	1.65	0.61	3.85	0.00	0.21	0.00	0.93	0.00
Finland	116	0.24	1.49	0.48	3.68	0.13	0.05	0.00	0.24	0.66
France	114	0.12	1.48	0.43	4.45	0.00	0.00	0.00	0.02	0.00
Greece	116	0.56	1.13	0.30	3.04	0.50	0.60	0.00	0.41	0.45
Iceland	116	1.14	2.40	0.58	4.03	0.00	0.40	0.00	0.00	0.65
Ireland	116	0.42	1.63	0.40	3.23	0.12	0.30	0.00	0.29	0.52
Italy	116	0.45	1.82	1.71	10.65	0.00	0.00	0.00	0.32	0.00
Japan	116	-0.05	1.41	0.84	6.00	0.00	0.00	0.00	0.39	0.21
Netherlands	112	0.06	1.11	-0.20	11.57	0.00	0.00	0.00	0.00	0.02
New Zealand	116	0.15	2.64	2.60	19.94	0.00	0.00	0.00	0.59	0.24
Norway	116	0.23	1.09	0.48	3.36	0.05	0.00	0.00	0.29	0.04
Portugal	116	0.72	2.01	0.64	5.74	0.00	0.01	0.00	0.01	0.69
South Africa	116	0.34	2.22	0.34	3.88	0.02	0.00	0.00	0.02	0.92
Spain	116	0.36	2.36	0.28	5.91	0.00	0.00	0.00	0.04	0.18
Sweden	116	0.32	1.90	2.35	14.58	0.00	0.00	0.00	0.39	0.76
Switzerland	116	-0.12	0.95	0.23	3.30	0.51	0.17	0.00	0.00	0.73
UK	116	0.18	7.22	-1.90	14.99	0.00	0.07	0.00	0.01	0.48

1. p-value of Shapiro-Wilk test for normality. (null of normality)
2. Mackinnon approximate p-value of Dickey-Fuller test for stationarity. (null of having uint root test)
3. Mackinnon approximate p-value of Phillip-Perron test for stationarity. (null of having uint root test)
4. p-value of White noise test for autocorrelation. (null of no aurocorrelation)
5. p-value of Lagrange Multiplier (LM) test for ARCH. (null of no ARCH effect)

We also test the series for serial correlation and ARCH effects. The White noise and ARCH LM tests results are shown in Tables 2 and 3. Monthly series contain more cases that are diagnosed with serial correlation than quarterly series. In addition, monthly series are more diagnosed with ARCH-type dependence than quarterly series. Hence, one may expect larger values of EMP indices to come in clusters more often in monthly than in quarterly series. As Wagner and Marsh (2000) point out using higher frequency data to have more observations may come at the cost of greater bias-variance trade-off.

Table 4. Lee and Strazicich LM structural break unit root test

Country	*model*	*min test statistics*	*break point*	*lambda*	*result*
BelgiumQ	A	-9.36	67		*no unit root at 1%*
	C	-9.71	44	0.4	*no unit root at 1%*
CanadaM	A	-17.66	186		*no unit root at 1%*
	C	-17.64	162	0.5	*no unit root at 1%*
DenmarkQ	A	-11.1	49		*no unit root at 1%*
	C	-10.82	73	0.6	*no unit root at 1%*
GreeceQ	A	-4.38	60		*no unit root at 1%*
	C	-5.29	83	0.7	*no unit root at 1%*
IcelandQ	A	-8.29	74		*no unit root at 1%*
	C	-8.42	74	0.6	*no unit root at 1%*
IrelandQ	A	-10.54	67		*no unit root at 1%*
	C	-10.45	73	0.6	*no unit root at 1%*
SwitzerlandQ	A	-3.05	64		*unit root*
	C	-6.54	46	0.4	*no unit root at 1%*

Model A (crash model): break in intercept only (critical value at 1%, 5%, and 10% are: -4.239, -3.566, and -3.211). Model C: break in intercept and time trend. Critical values in model C (intercept and trend break) depend on the location of the break (λ = TB/T) and are symmetric around λ and (1-λ). Model C critical values additional break locations can be interpolated. Critical values at 1%, 5%, and 10% are: (for λ=.3) –5.05, -4.50, and -4.18, (for λ=.4) –5.05, -4.50, and -4.18, (for λ=.5) -5.11, -4.51, and -4.17.

From the presented statistics of EMP series, we can conclude that monthly series are more likely to be non-normally distributed, stationary, and dependent while there are more incidences of normally-distributed and independent indices in quarterly series. Thus, one may conclude that EMP indices that are constructed from lower frequency data are less likely to be fat-tailed and quarterly series compared to monthly series are less appropriate for applying EVT. It suggests that, we should be cautious in interpreting the tail indices that are obtained from time aggregation of quarterly and lower frequency data while relying on EVT.

5.3. Hill's index estimation. As mentioned earlier, Hill's estimator is optimal under independent draws from an exact Pareto distribution. Even if EMP indices are exactly from a Pareto distribution, bias-variance tradeoff plays an important role in the estimation of tail thickness for small samples. We combine two methods – Monte Carlo simulation and the Modified Hill's estimator – to deal with the bias-inefficiency tradeoff. These two methods are discussed in the following.

The Monte Carlo method: We design the simulation in a way to minimize mean square error (MSE) of the given sample size n with density function F(x). MSE combines both bias and inefficiency to capture the

tradeoff elements carefully.[23] Monte Carlo method exploits the asymptotic normality of Hill's index to select the optimum cutoff value (m). We follow the simulation steps as provided in Longin & Solnik (2001) and Haile & Pozo (2008) to estimate the tail index. A summary of the adopted steps is presented in the Appendix A.

Nevertheless, there are two concerns with the Monte Carlo method: one conceptual and one computational. First, it only considers Student's t as the possible class of distributions to simulate the tail index while there are two other possibilities – stable laws and ARCH type distributions. Second, selection of the optimum cutoff value is based on the results of a statistical test, which determines the degrees of freedom of the Student's t that the actual data are from. For each $\gamma^h(m^*(\alpha))$ there are several highly significant test results, nevertheless, there is no systematic way to rigorously distinguish the most reliable test result. For instance, in case of the monthly EMP of Austria, the null of "actual data are not from Student's t distribution with α degrees of freedom", has several results that are significant at lower than one percent. Its two smallest p-values are: 1.8e-5 and 6.3e-4, which represent α equal to 5.2 and 3.3, respectively. While it is not very clear that how much 1.8e-5 and 6.3e-4 are statistically different, α equals to 5.2 implies that the optimum cutoff value will be 11 and α equals 3.3 implies that the optimum cutoff value will be 19. We apply the Huisman *et al.* (2001) method to circumvent these two concerns.

Modified Hill's estimator: Huisman *et al.* (2001) show that their modified Hill's estimator can equally perform well under the GARCH (1,1) model. It is quite common in financial econometrics to capture second-order (ARCH type) dependence as reflected in clusters of high and low volatility by GARCH models. Thus, the modified Hill's estimator should be capable of replacing the ARCH type distributions.[24] We also attempt to overcome the computational concern by a screening process. The modified Hill's estimator is insensitive with respect to the choice of the maximum number of tail observations to include, as long as it is less than half of the sample size. Therefore, having the idea of Hill's plot in mind, the modified Hill's estimator of each EMP series are computed, to roughly obtain the stable region for the estimated modified Hill's indices. Then, we go back to simulation results and select the most significant optimum cutoff value (m^{**}) within the region of stable modified Hill's estimations.

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23. MSE of S simulated observations of the estimator $X_s^{\sim}$ can be decomposed as: $MSE((X_s^{\sim})_{s=1,2,\ldots,S}, X) = (X^{-} - X)^2 + 1/S\, \Sigma_{s=1}^{S}(X_s^{\sim} - X)^2$, where X^{-} represents the mean of S simulated observations. The first part measures the bias and second part the inefficiency.

24. The other class of distributions that can account for fat tails are stable laws. But as Wagner & Marsh (2005) argue, although symmetric stable laws with $\alpha<2$ are theoretically justified for extreme value theory, applications of Hill's estimator do not seem promising for stable laws in small samples.

Table 5. Hill's index and number of tail observations

	Monthly			*quarterly*		
Country	*Hill's index*	*degree of freedom**	*tail observations*	*Hill's index*	*degree of freedom**	*tail observations*
Australia	0.19	5.3	13	0.21	4.8	7
Austria	0.3	3.3	19	0.24	4.2	8
Belgium	0.34	2.9	22	0.22	4.6	7
Canada	0.28	3.6	18	*0.5*	*normal*	9
Denmark	0.26	3.9	17	0.23	4.3	9
Finland	0.3	3.3	19	*0.5*	*normal*	11
France	0.42	2.4	25	0.4	2.5	15
Greece	0.4	2.5	24	*0.5*	*normal*	10
Iceland	0.5	2	32	0.36	2.8	14
Ireland	0.34	2.9	20	*0.5*	*normal*	10
Italy	0.26	3.8	17	0.27	3.7	9
Japan	0.2	5	11	0.23	4.4	7
Netherlands	0.26	3.8	17	0.29	3.5	10
New Zealand	0.33	3	21	0.26	3.8	10
Norway	0.29	3.5	19	0.26	3.9	8
Portugal	0.34	2.9	22	0.27	3.7	9
South Africa	0.25	4	15	0.26	3.9	8
Spain	0.28	3.6	18	0.32	3.1	11
Sweden	0.34	2.9	22	0.38	2.6	10
Switzerland	0.27	3.7	17	*0.5*	*normal*	8
UK	0.32	3.1	21	0.37	2.7	13

* degree of freedom of the Student's *t* that the actual data are from.

Estimated tail indices and number of tail observations for monthly and quarterly series are reported in Table 5. Hill's index, γ, is the inverse of the tail index, α, or the degrees of the freedom of the closest Student's *t*. For those quarterly series that are well behaved and normally distributed, the number of observations is estimated from conventional methods. Hence, any observation larger than the mean of the series plus one and half of its standard deviation is counted as a currency crisis episode.

In the literature, some studies put exclusion window to avoid counting the same crisis more than once. In different studies, width of the window varies from one quarter to two years. However, as mentioned earlier, in our research project recognizing the number of crisis periods is a vital step. It is more important to correctly identify the crisis periods than distinguishing whether the period is a new crisis or it is the

continuation of the previous one. Furthermore, exclusion windows can cause potential problems. First, it introduces artificial serial correlation (see, *e.g.* Abiad 2003). Second, it equalizes the length of all spells and eliminates any information that the duration of spells may contain. Finally, same as the choice of threshold level, it requires another arbitrary selection. Hence, we choose not to have exclusion window.

It is important to recognize how the crisis episodes are scattered through the time. Figures 1 and 2 show the percentage of the countries that experience a currency crisis (monthly or quarterly) over the period of

Figure 1. Percentage of crises episodes per month

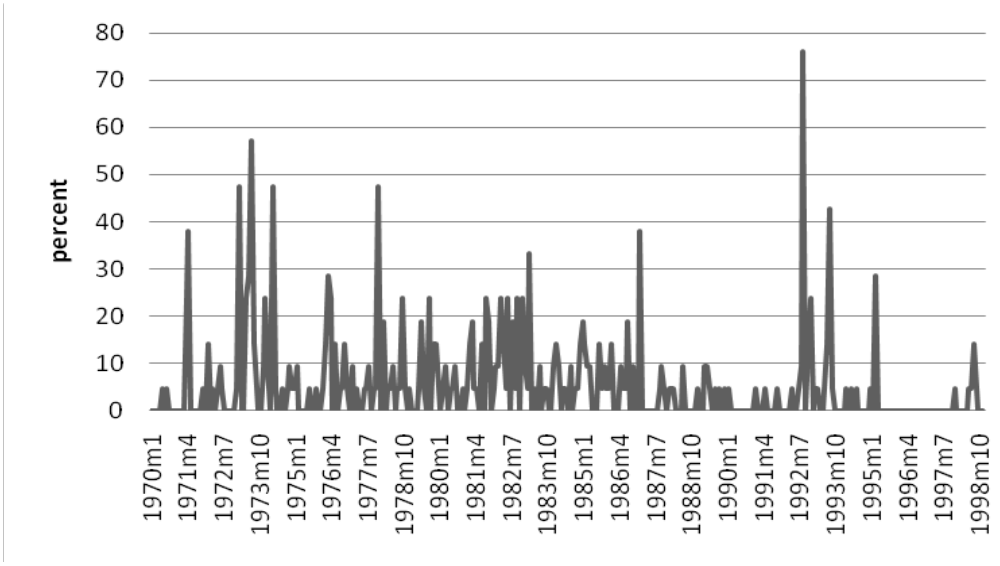
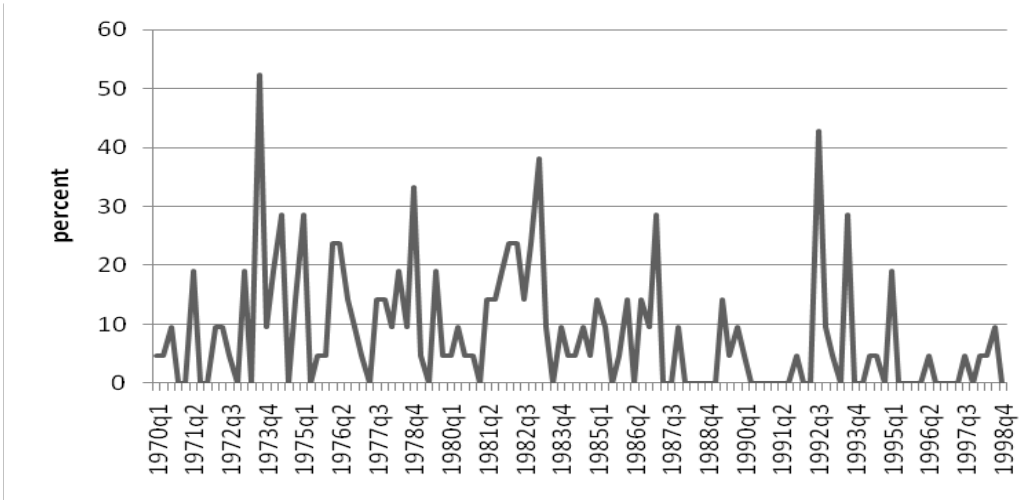


Figure 2. Percentage of crises episodes per quarter



1970-1998. As it is evident, the number of crisis episodes peaks in some specific periods. Two periods have the largest number of crisis incidences: the third quarter of 1973 (the breakdown of the Bretton Woods system) and September 1992 (the collapse of the British pound and crisis in the EMU). It can potentially be interpreted as a sign of the contagious nature of currency crisis. We will come back to this point and will pay close attention to it in the next chapter.

6. Comparison

In this section we compare the results of our approach with four other methods. First, currency crisis episodes are estimated by the time varying conventional Eichengreen *et al.* (1995) method, for a threshold equal to 1.5 and 2 standard deviations (only corresponding results to 1.5 standard deviations that produces more significant results are reported). Next, the crisis periods are recognized with the method that is introduced by Zhang (2001). We also identify the crisis episodes by Haile and Pozo (2006) approach. They apply EVT and select the optimum cutoff values with Monte Carlo simulation method.

Finally, following Pozo and Dorantes (2003), Lestano and Jan (2007), and Pontines and Siregar (2008), we employ the recursive least squares method to estimate the number of tail observations. In the recursive method, first, each series is arranged in descending order and the Hill's estimator are computed for the first thirtieth percentile. In the next step, the computed amounts for Hill's index are regressed on a constant and time trend variable successively. Then, the recursive residuals are derived to find the structural break. The optimum *m* is picked where the value of recursive residuals lies outside of the two-standard errors band. Table 6 reports the results.

Total number of the identified crisis episodes by Eichengreen *et al.* (1995) method is almost close to that of this paper. However, distributions of these two types of crises are substantially different.

Results of the Zhang's method seem inconsistent with our data and are not reported. This method, in one hand, does not identify any crisis episodes for most of the countries. On the other hand, for those crisis periods that it recognizes, crisis periods all occur consecutively.

The recursive method outcomes are different. In general, it identifies much more crisis periods compared to other approaches. Similar to Zhang's method, this method identifies a small number of crisis episodes for some countries while finds a very large amount of crisis periods for the others. In some cases, it cannot recognize any optimum cutoff value (*m*) at all.

Table 6. Number of currency crisis episodes with different methods

	Monthly				*Quarterly*			
Country	*Mod. EVT*	*Eichen.**	*Rec.*	*Haile.*	*Mod. EVT*	*Eichen.**	*Rec.*	*Haile.*
Australia	13	23	13	6	7	9	…	6
Austria	19	15	…	34	8	7	11	21
Belgium	22	21	12	42	7	9	44	26
Canada	18	23	…	23	9	9	21	17
Denmark	17	24	53	19	9	10	12	14
Finland	19	21	88	18	11	11	18	11
France	25	25	18	39	15	8	24	20
Greece	24	23	12	27	10	10	..	18
Iceland	32	26	50	38	14	10	6	24
Ireland	20	22	98	34	10	10	8	24
Italy	17	20	69	13	9	5	37	8
Japan	11	21	59	9	7	8	35	7
Netherlands	17	16	…	41	10	6	40	24
New Zealand	21	8	…	27	10	1	30	16
Norway	19	21	7	29	8	9	37	18
Portugal	22	14	5	28	9	7	…	11
South Africa	15	22	12	21	8	6	37	12
Spain	18	26	15	24	11	7	42	14
Sweden	22	18	83	36	10	5	27	22
Switzerland	17	16	…	29	8	8	6	14
UK	21	23	60	36	13	8	…	18

** Reported numbers correspond to threshold equals to1.5 of standard deviations.*
… implies that there exists no structural break in the selected range.

Haile and Pozo (2006) method generally identifies more episodes of crisis compared to our approach. It indicates that determining extreme values of the EMP series just by relying on Monte Carlo simulation (not combining with the modified Hill's estimator) can potentially increase number of the crisis episodes by picking the less significant EMP indices. However, recognizing higher number of crisis periods comes at a cost of more wrongly determined episodes of crisis. While our approach mostly identifies severe crisis periods as episodes of currency crisis, the mentioned method besides the severe crisis times determines slight and mild stressed periods of macro economy as crisis.

It is a very difficult task to officially evaluate performance of the different methods and approaches for identification of currency crisis periods since there is no consensus on a formal definition of currency crisis in the profession. Furthermore, no international organization systematically categorizes currency

Table 7. Identified Canadian currency crises episodes through 1970-1998

year	*crises episodes*		*chronology of economic and political events**
	month	*quarter*	
1976	11		Political uncertainties following the Parti Québécois win in Quebec election on November 15 and softening prices for non-energy commodities.
1977	10		Rising cost and wage pressures and substantial current account deficit.
1978	2 and 9		Inflation pressure leads to increase in interest rates and tight monetary policy by the Bank of Canada.
1979	1 and 5	I	The Bank of Canada Rate rises by 375 basis points to 11.25 per cent in the beginning of January 1979.
1980	3 and 11	II and IV	Quebec Referendum and its political concerns, weakening prices for non-energy commodities, and the introduction of the National Energy Program by the federal government in October.
1981	7	III	Sharp rise in short term interest rates through 1980 and into summer 1981. The Bank of Canada Rate touches an all-time high of 21.24 per cent in early August.
1982	2,3, and 6	II	Growing concerns about the commitment of Canadian authorities to an anti-inflationary policy stance and cancellation of a number of large energy projects. Bank of Canada allows the short term interest rates rise to prevent from turning into a speculative rout. Bank of Canada abandons M1 as an anchor against inflation.
1984	3 and 6	II	The high interest rates and favourable investment opportunities in the United States attract funds.
1985	2		Outflow of funds into the US.
1986		I	Weak economic and financial prospects, esp. following the failure of Canadian Commercial Bank and Northland Bank. The CAD depreciates to US$0.6913 on February 4.
1992	9 and 11	IV	The ERM in Europe comes under repeated attack. Defeat of Charlottetown Accord on October 26.
1995		I	Mexican Peso crisis and weakness of Canada's fundamental, esp. large fiscal and current account deficit.
1998	8		Crisis in emerging-markets economies widens and intensified by the debt default in Russia. The CAD falls toUS$0.6311 on August 27.

* Source: Powell, J. (2005). A History of the Canadian Dollar. Bank of Canada, Ottawa.

crisis periods. An alternative solution is to verify the reliability of the identified crisis episodes with chronology of the economic and political events. Unfortunately, a full chronology of the events for countries of our sample is not available. Consequently, as an example, we present the identified crisis episodes for Canada in Table 7 and validate their accuracy with chronology of the economic and political events over the 1970 to 1998 time period. The identified crisis episodes match very well with the chronological events. Although there is a good overlap between the monthly and quarterly crisis periods, there are a few episodes that are only identified by one set of data – either monthly or quarterly. Nevertheless, it can be attributed to the nature of data and severity of the crises.

7. Conclusion

Identifying crisis periods is a substantial step in most of empirical studies in the field. This paper analyzes and estimates the dating of currency crises for 21 countries from 1970 to 1998.

In our approach, we constructed EMP series from monthly and quarterly data and defined those large values that lie in tail of the EMP distribution as episodes of currency crisis. In order to identify the tail observations, we applied a more objective statistical method based on extreme value theory rather than conventional methods which is based on arbitrary thresholds and priori assumptions. We showed EMP series – especially for higher frequency data – are fat-tailed and are more appropriate for applying EVT than assuming they are well behaved and normally distributed series. However, the EVT method for low frequency data should be applied cautiously.

We combined Monte Carlo simulations with a modified Hill's estimator method to carefully minimize the bias-variance tradeoff and overcome the related concerns with Monte Carlo application. This paper also attempted to introduce a systematic way to select the reference country around which a country's currency pressure index should be built. This approach can help us to identify the currency crisis periods more precisely and in the following chapters it will result in a better understanding of the roots and determinants of the currency crises.

References

Abiad, A. (2003). Early-warning systems: A survey and a regime-switching approach. IMF Working Paper 03/32. Washington: International Monetary Fund.

Andersen, T.G., Davis, R.R., Krei, J.-P., Mikosch, T. (2009). *Handbook of Financial Time Series*, Springer, Berlin.

Angkinand, A., Li, J., Willett, T.D. (2006). Measuring currency crises. Presented at the annual meetings of the Western Economic Association.

Aziz, J., Caramazza, F., Salgano, R. (2000). Currency crises: In search of common elements. IMF Working Paper 00/67. Washington: International Monetary Fund.

Boothe, P.M., Glassman, D.A., (1987). The statistical distribution of exchange rates: Empirical evidence and economic implications. *Journal of International Economics* 22, 297–319.

Caramazza, F., Ricci, L., Salgado, R. (2000). Trade and financial contagion in currency crises. *Journal of International Money and Finance* 23(1), 51-70.

Danielson, J., de Haan, L., Peng, L., de Vries, C.G., (2001). Using a bootstrap method to choose the sample fraction in tail index estimation. *Journal of Multivariate Analysis* 76, 226–248.

De Haan, L., Ferreira, A. (2006). *Extreme Value Theory: An Introduction*. Springer, New York.

Diebold, F., Schuermann, T., Stroughair, J., (1999), Pitfalls and opportunities in the use of extreme value theory and risk management, Mimeo, University of Pennsylvania.

Eliasson, A.C., Kreuter, C. (2001). On currency crisis models: A continuous crisis definition. Deutsche Bank Research Quantitative Analysis unpublished working paper; available online at: http://www.economia.uniroma2.it/ceis/conferenze_convegni/banking2001/papers/mercoledi/Eliasson-Kreuter.pdf

Eichengreen, B., Rose, A., Wyplosz, C. (1995). Exchange market mayhem: The antecedents and aftermath of speculative attacks. *Economic Policy* 21 (21), 249–312.

Eichengreen, B., Rose, A., Wyplosz, C. (1996). Contagious currency crises: First tests. *The Scandinavian Journal of Economics* 98, 463–484.

Embrechts, P., Kluppelberg, C., Mikosch, T. (1997). *Modelling Extremal Events for Insurance and Finance*. Springer, Berlin.

Enders, W. (2004). *Applied Econometric Time Series.* Hoboken, NJ: Wiley.

Flood, R.P., Marion, N.P. (1999). Perspectives on the recent currency crisis literature. *International Journal of Finance and Economics* 4(1), 1-26.

Frankel J.A., Rose A.K. (1996). Currency crashes in emerging markets: an empirical treatment. *Journal of International Economics* 41(3–4), 351–366.

Glick, R., Hutchison, M. (2001). Capital controls and exchange rate stability in developing countries. *Economic Letter* 21.

Girton, L., Roper, D. (1977). A monetary model of exchange market pressure applied to postwar Canadian experience. *American Economic Review* 67, 537-548.

Haile, F.D., Pozo, S. (2008). Currency crisis contagion and the identification of transmission channels. *International Review of Economics and Finance* 17, 572-588.

Haile, F.D., Pozo, S. (2006). Exchange rate regimes and currency crises: an evaluation using extreme value theory. *Review of International Economics* 14(4), 554-570.

Hill, B.M. (1975). A simple general approach to inference about the tail of a distribution. *Annals of Statistics* 3(5), 1163–1174.

Hols, M.C.A.B., de Vries, C.G. (1991). The limiting distribution of extremal exchange rate returns. *Journal of Applied Econometrics* 6, 287–302.

Huisman, R., Koedijk, K.G., Kool, C.J.M., Palm, F. (2001). Tail-index estimates in small samples. *Journal of Business and Economic Statistics* 19*(*1), 208–216.

International Monetary Fund, International Financial Statistics: http://www.imfstatistics.org/imf/

Jansen, D.W., De Vries, C. G. (1991). On the frequency of large stock returns: Putting booms and busts into perspective. *The Review of Economics and Statistics* 73, 18−24.

Kamin, S.B., Schindler, J.W., Samuel, L.S. (2001). The contribution of domestic and external factors to emerging market devaluation crises: An early warning system approach. Board of Governors of the Federal Reserve System. International Finance Discussion Papers 711.

Kaminsky, G., Lizonado, S., Reinhart, C.M. (1998). Leading indicators of currency crises. International Monetary Fund Staff Papers 45, 1–48.

Kaminsky, G., Reinhart, C.M. (1998). Financial crises in Asia and Latin America: Then and now. *American Economic Review* 88, 444–448.

Koedijk, K.G., Stork, P.A., de Vries, C.G. (1992). Differences between foreign exchange rate regimes: The view from the tails. *Journal of International Money and Finance* 11, 462–473.

Koedijk, K.G., Schafgans, M.M.A., de Vries, C.G. (1990). The tail index of exchange returns. *Journal of International Economics* 29, 93–108.

Krugman, P. (1979). A Model of Balance of Payments Crises. *Journal of Money, Credit and Banking* 11, 311-325.

Lestano, J., Jan, P.A.M. (2007). Dating currency crises with *ad hoc* and extreme value-based thresholds: East Asia 1970-2002. *International Journal of Finance and Economics* 12, 371-388.

Lee, J., Strazicich, M. (2003). Minimum LM unit root test with two structural breaks. *Review of Economics and Statistics* 85, 1082-1089.

Lee, J., Strazicich, M. (2004). Minimum LM unit root test with one structural break. Working Paper, Department of Economics Applachian State University.

Longin, F., Solnik, B. (2001). Extreme correlation of international equity markets. *Journal of Finance* 56(2), 649–676.

Pozo, S., Amuedo-Dorantes, C. (2003). Statistical distributions and the identification of currency crises. *Journal of International Money and Finance* 22(4), 591–609.

Martinez-Peria, M. S. (2002). A regime switching approach to studying speculative attacks: A focus on European Monetary System crises. *Empirical Economics* 27, 299-334.

Pontines, V., Siregar, R. (2008). Fundamental pitfalls of exchange market pressure-based approaches to identification of currency crises. *International Review of Economics and Finance* 17(3), 345-365.

Powell, J. (2005). *A History of the Canadian Dollar.* Bank of Canada, Ottawa. (http://www.bank-banque-canada.ca/en/dollar_book/)

Sachs, J.D., Tornel, A., Velasco, A. (1996). Financial crises in emerging markets: The lessons from 1995, Brooking Papers on Economic Activity 1, 147-215.

Tudela, M. (2003). Explaining Currency Crises: a duration model approach, *Journal of International Money and Finance 23,* 799-816.

Wagner, N., Marsh, T. A. (2005). Measuring tail thickness under GARCH and an application to extreme exchange rate changes. *Journal of Empirical Finance 12,* 165-185.

Wagner, N., Marsh, T. A. (2000). On adaptive tail index estimation for financial return models. Research program in finance working paper RPF-295 U.C. Berkeley: Haas School of Business.

Willett, T., Nitithanprapas, E., Nitithanprapas, I., Rongala, R. (2004). The Asian crises reexamined. *Asian Economic Papers* 3, 32-87.

Zhang, Z. (2001). Speculative attacks in the Asian crisis. IMF Working Paper 01/189. Washington: International Monetary Fund.

Appendix A

To determine the optimum cutoff value by Monte Carlo simulations, we adopt the simulation steps from Longin & Solnik (2001) and Haile & Pozo (2008) and run them with R software. These steps are:

1. *S* time series containing *n* observations of EMP are simulated. Each S is derived from a known Student's *t* distribution with α degrees of freedom, where α ranges from 1 to *K*. The class of the Student's *t* distribution is chosen to consider different degrees of tail fatness. Since the tail index γ is inverse of α ($\gamma=1/\alpha$), the lower the degree of freedom is, the fatter the distribution will be. In our simulation α is allowed to take values from 1 to 10 with increment of 0.1 and number of replications (*S*) equals 1000.

2. For different numbers of *m* of the extreme EMPs, a tail index $\gamma_s^h(m,\alpha)$ corresponding to the *s*th replication from the Student's *t* with α degrees of freedom is estimated. Values of *m* can vary from %1 to %30 of *n*, where *n* is the sample size of the actual EMP data.

3. For a particular Student's *t* distribution with α degrees of freedom and for each value of *m*, MSE of the *S* tail index estimates, which is denoted by MSE $(\gamma_s^h(m,\alpha)_{s=1,2,\ \ldots,S})$, is computed. This computation repeatedly continues for different values of *m* and particular Student's *t* with α degrees of freedom. Then the optimal *m*, denoted by $m^*(\alpha)$, which minimizes MSE for the particular Student's *t* distribution with α degrees of freedom is selected.[25] Optimum values of *m* for different Student's *t* distributions are repeatedly selected. A total of *K* optimal values of m^*, $(m^*(\alpha))_{\alpha=\ 1,2,\ \ldots,K}$, are subsequently selected for *K* possible theoretical distributions.

4. Using each of *K* optimum values of *m* that are obtained in last step, the Hill index, $\gamma^h(m^*(\alpha)$, is estimated from actual EMP series. For all α from 1 to 10, the tail indices, γ^h, are estimated from actual EMP series.

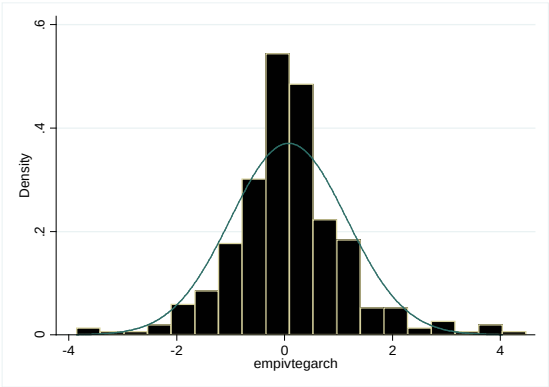
As the final step, we select one single number (say m^{**}) from the *K* optimum tail indices, m^*, for each EMP series such that the estimated tail index from the actual data (step 4) is statistically the closest to the corresponding tail index of the theoretical distribution. The main objective of the whole exercise is to

25. As explained by Jansen and de Vries (1991), there is a U-shaped relation between MSE$(\gamma_s^h(m,\alpha)_{s=1,2,\ldots,\ s})$ and *m* that expresses the tradeoff between bias and inefficiency. Choosing a few observations from the tail makes the bias part of MSE dominant over the inefficiency part. On the other hand, including too many observations from the tail makes the inefficiency part of the MSE dominant over the bias part.

determine the number of extreme observations for each EMP series. This value, m^{**}, that is corresponding to the optimum tail index, specifies the number of observations as the largest EMPs or episodes of currency crises.

Appendix B

Australia M



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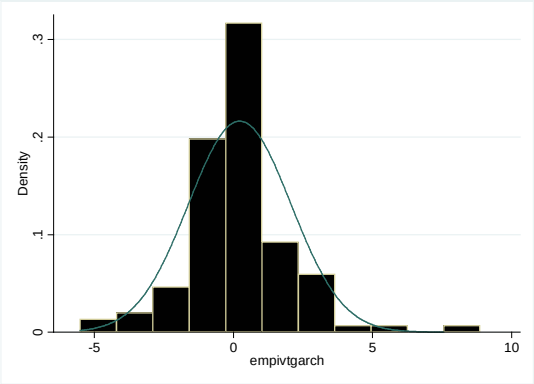

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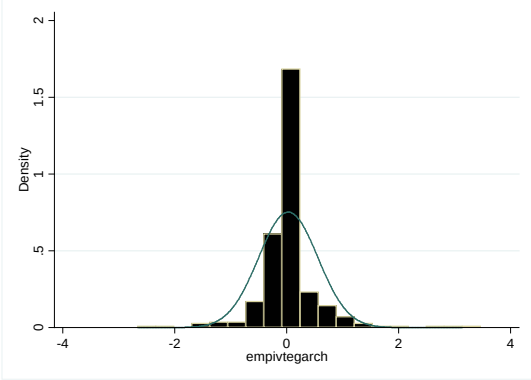

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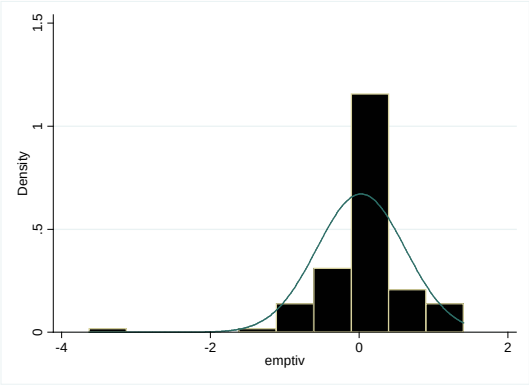

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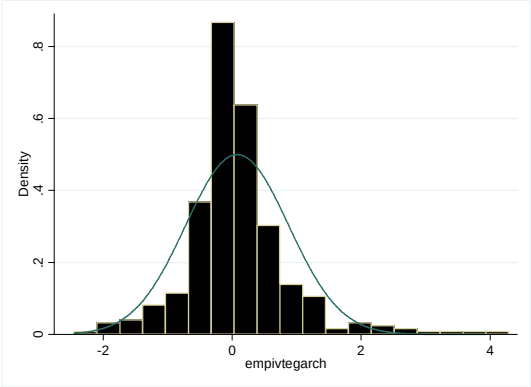

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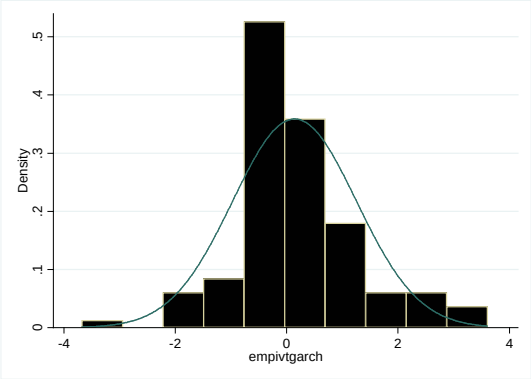

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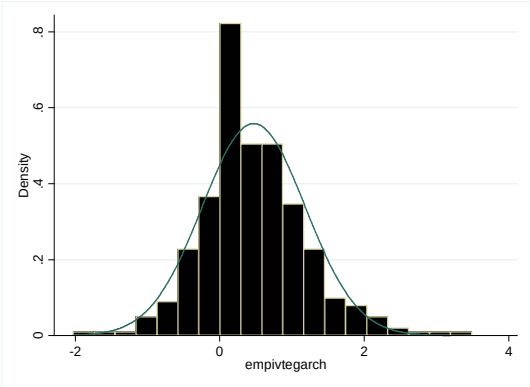

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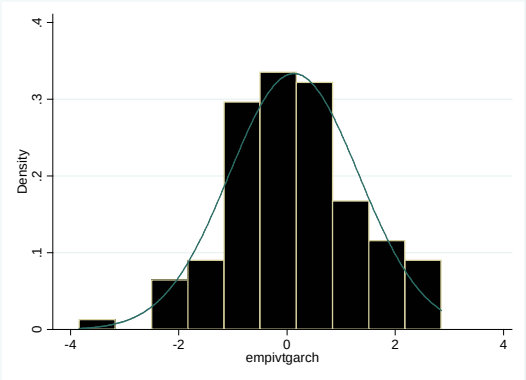

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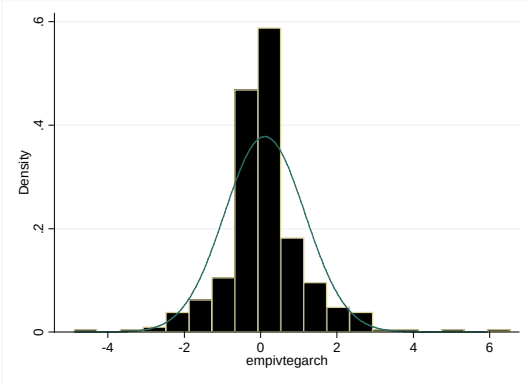

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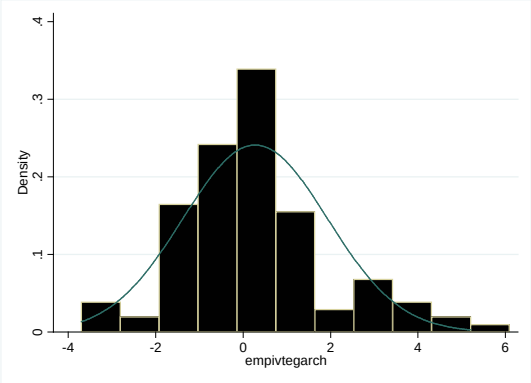

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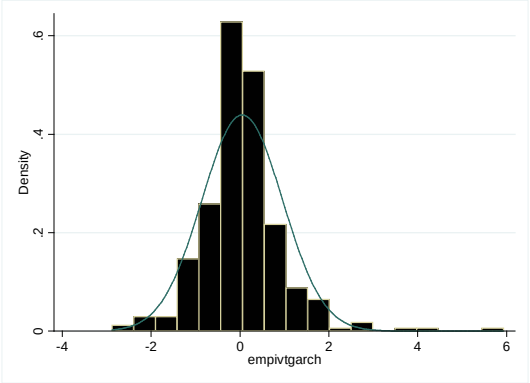

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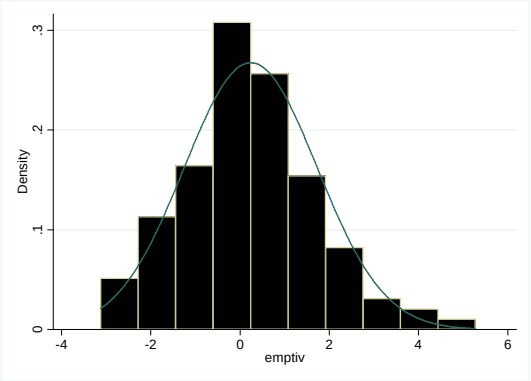

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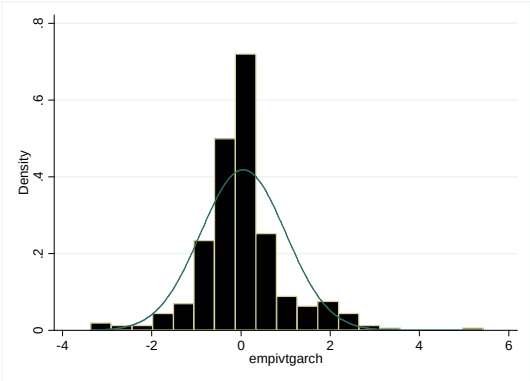

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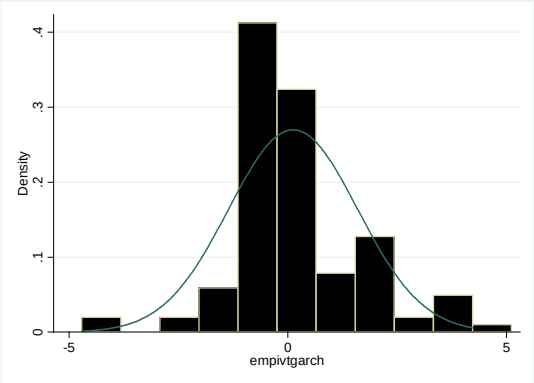

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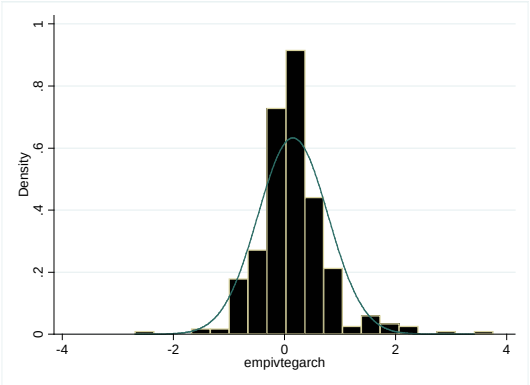

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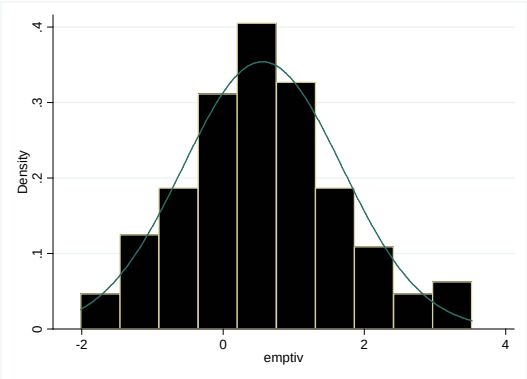
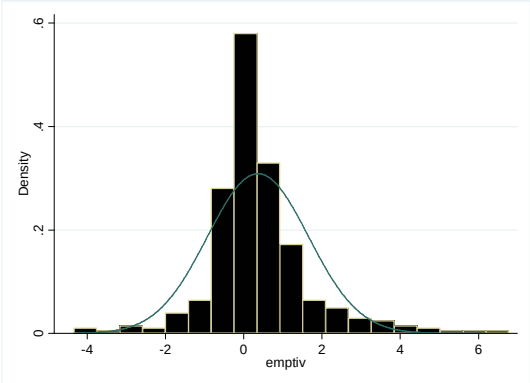


Iceland m



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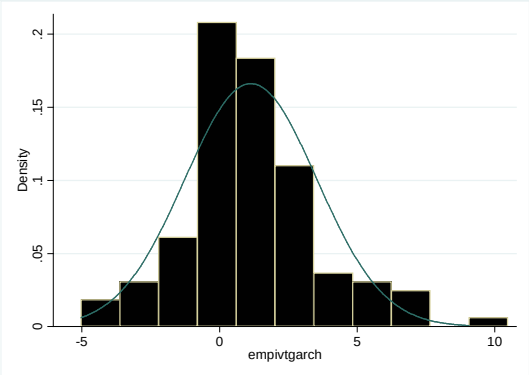
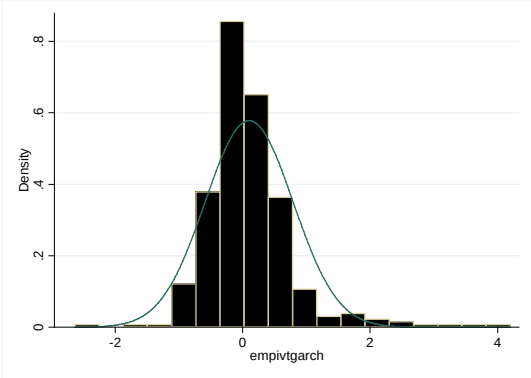


Ireland M



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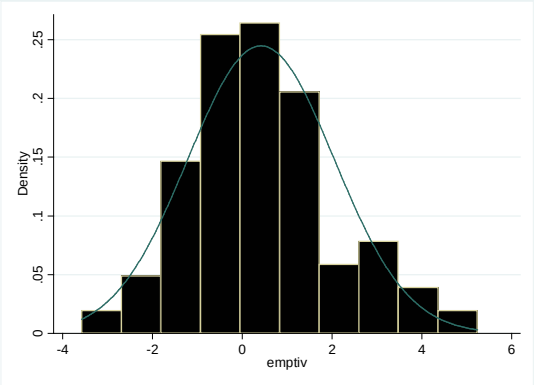

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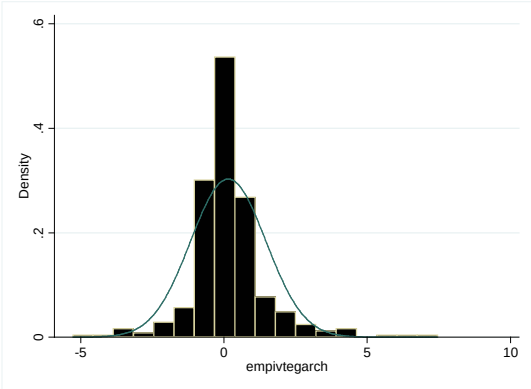

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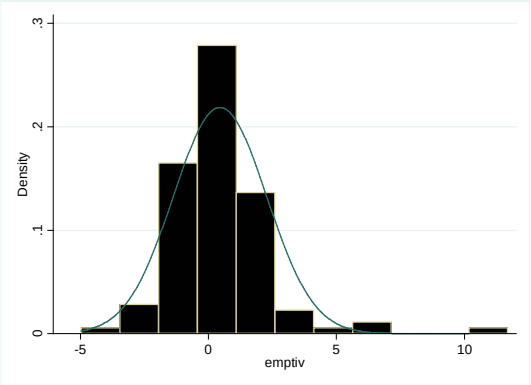

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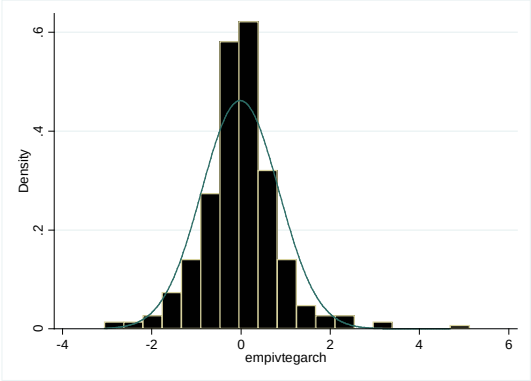

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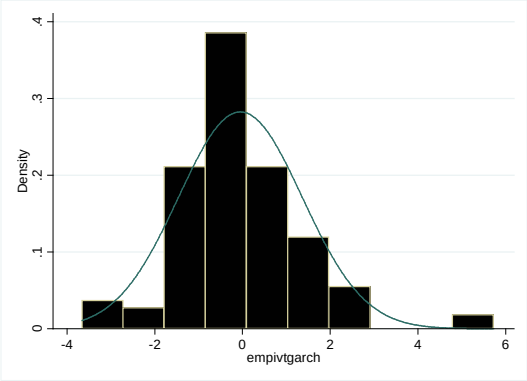

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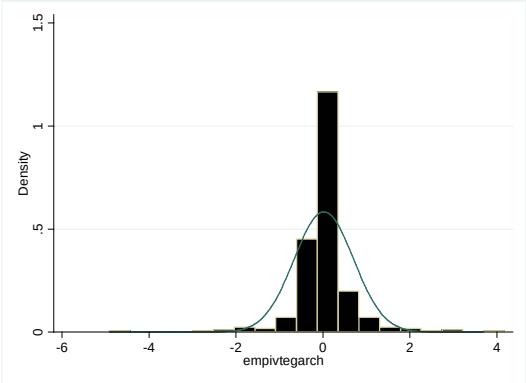

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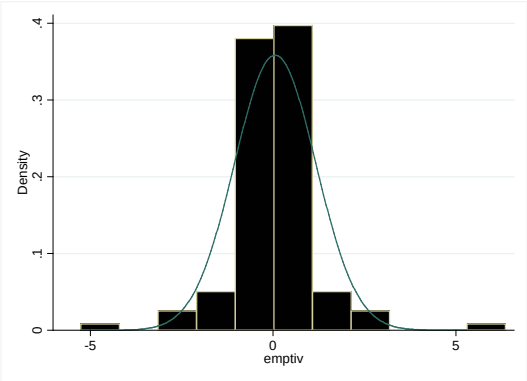

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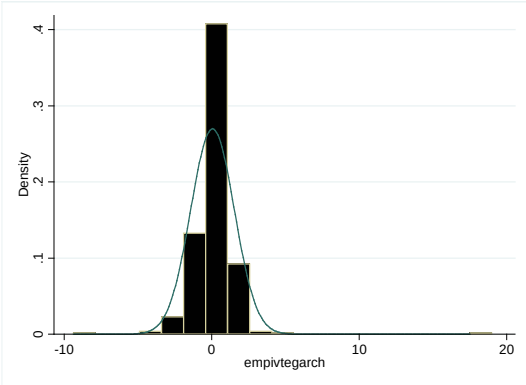

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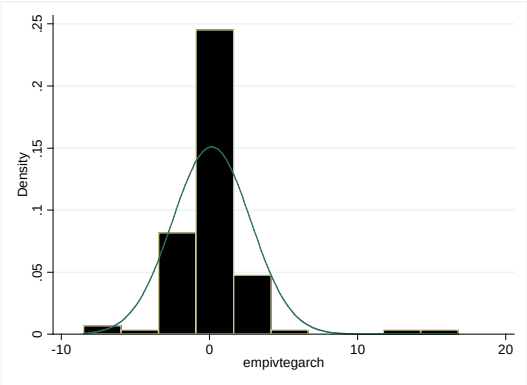

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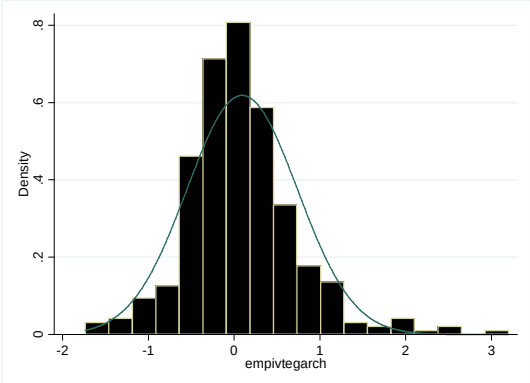

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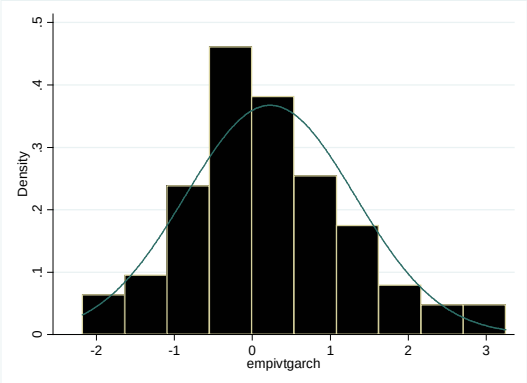

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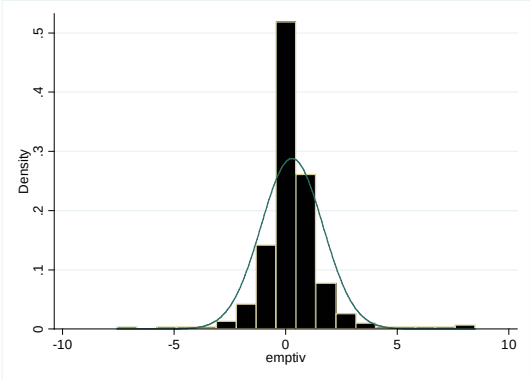

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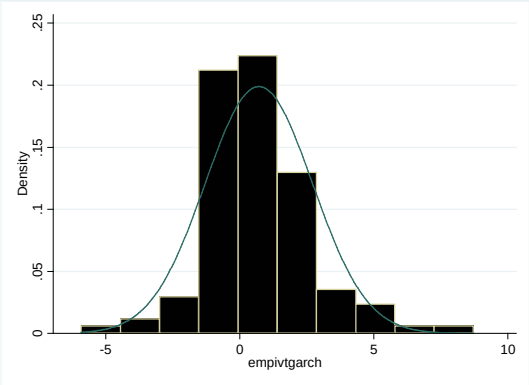

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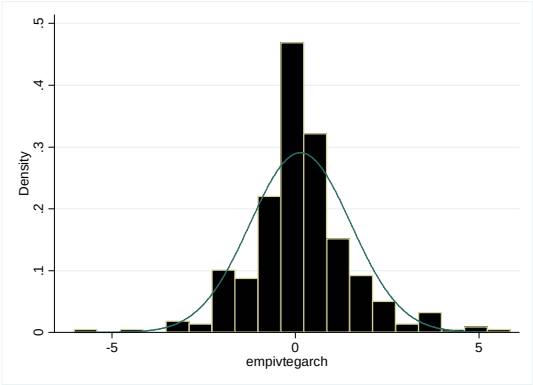

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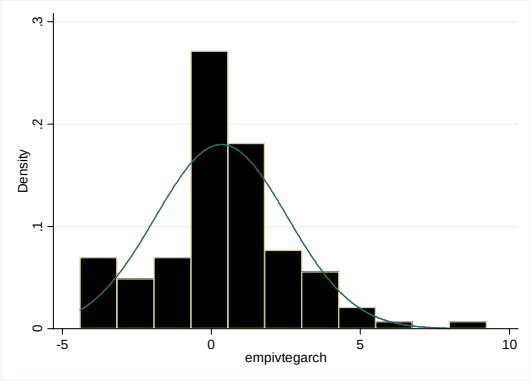

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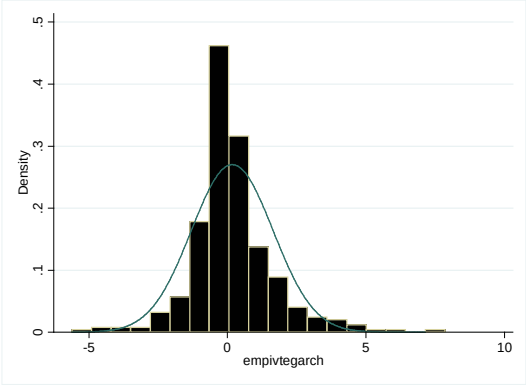

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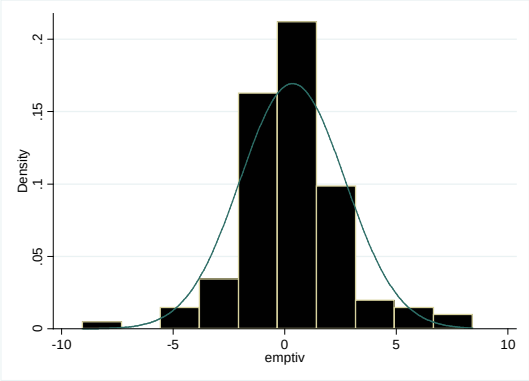

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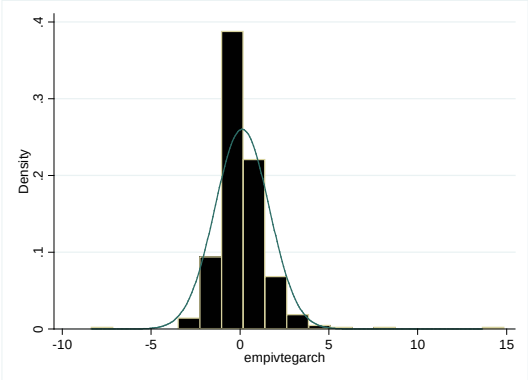

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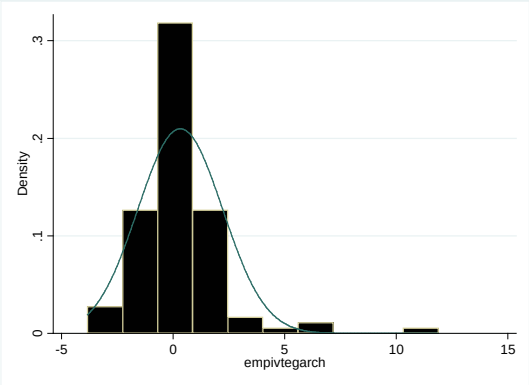

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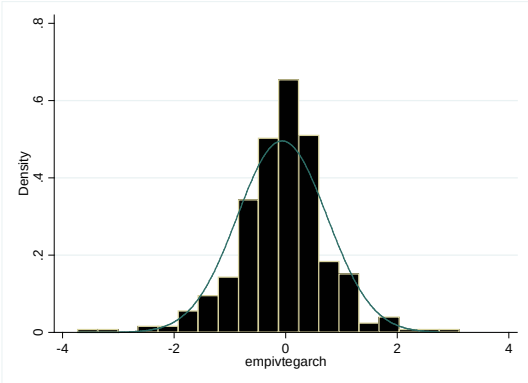

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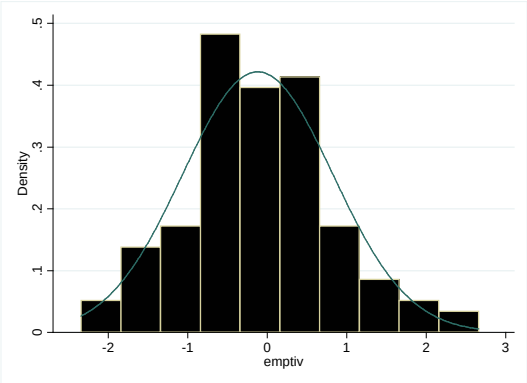

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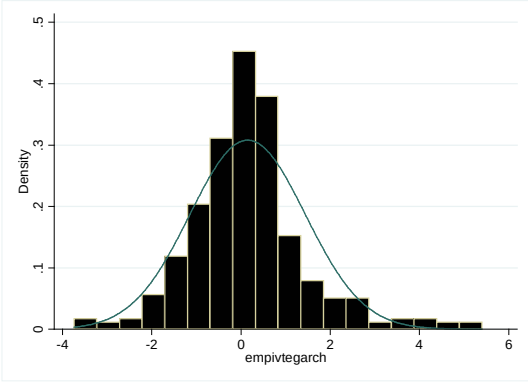

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