Debt discharge in personal bankruptcy and the role of collateral as a sorting device*

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Abstract

We analyze debt discharge in personal bankruptcy – according to which some of the borrower's assets are exempt from liquidation in the event of default – in the context of a competitive credit market characterized by adverse selection. In particular, we study how the level of such exemption affects the role of collateral as a sorting device. The decision to post collateral results in a lower cost of credit and a lower level of credit rationing, to the extent to which in equilibrium, safer borrowers self-select into contracts different from those chosen by riskier ones. An increase in the level of exemption strengthens the role of collateral as a sorting device: 1) the effect of posting collateral on cost of credit is enhanced, and; 2) The decision to post collateral results in a lower probability to be rationed. We exploit cross State variability in the level of asset exemption from liquidation – according to personal bankruptcy US State laws prior to 2005 federal reform – in order to test some of the implications of the model in a sample of american small business taken from the Survey of Small Business Finances.

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1 Introduction

According to the US personal bankruptcy law, the process of liquidation in the event of personal bankruptcy is governed by the so called, Chapter 7, which is indeed the most common bankruptcy procedure in the US.[1] If an individual files under Chapter 7, her unsecured debt would be mostly discharged.[2] At the same time, the trustee will liquidate individual's non-exempt assets to repay creditors.

Types of exempt assets and levels of exemption are decided by individual States. There exists a widely variation across states. Exemptions can be classified on the basis of the type of asset they apply to. Homestead exemption is the exemption on the individual's equity in owner-occupied principal residence. Differently, non-homestead exemption includes individual's equity in cars, cash, and other goods such as furniture, clothing, cooking utensils, farm implements, family bibles, and tools for trade, etc. In most States, the level of homestead exemption is larger that of the non-homestead one. Furthermore, non-homestead exemption is generally low across states. Finally, homestead-exemption is unlimited in some State, and zero in some others.

According to chapter 7, bankrupt individuals could benefit from a fresh start opportunity to the extent that they will keep some assets, while their debt obligation will be partially redeemed. This has a number of potential consequences on the functioning of credit markets.

A crucial feature of Chapter 7 is that [...] "Although a debtor is not personally liable for discharged debts, a valid lien (i.e., a charge upon specific property to secure payment of a debt) that has not been avoided (i.e., made unenforceable) in the bankruptcy case will remain after the bankruptcy case. Therefore, a secured creditor may enforce the lien to recover the property secured by the lien. [...] ".[3]

As a consequence of that, borrowers could in principle undo the effects of bankruptcy exemption by posting enough collateral. In this paper we analyze how such possibility affects the significance of collateral as a sorting device in the context of a competitive credit market characterized by adverse selection. We derive a set of implications, which we then test on a sample of small businesses in the

[1] Whenever it might be relevant, please note that we refer to the bankruptcy law in place prior to the reform which took place in 2005, since the data we use in the empirical analysis are from 2002.

[2] Most of the (unsecured) debt is discharged. According to the US Courts official website, [..] An individual receives a discharge for most of his or her debts under chapter 7 [..] , %99 of cases that are not dismissed or converted receive a discharge.

[3] This quote is taken from the following webpage of the official US federal courts website: http://www.uscourts.gov/FederalCourts/Bankruptcy/BankruptcyBasics/DischargeInBankruptcy.aspx. As a clarifying example of the fact that exemption does not protect assets voluntarily posted as collateral, read about the case of Minnesota: http://www.legalconsumer.com/bankruptcy/bankruptcy-law.php?ST=MN.

US based on 2002 SSBF data.

We spell out the intuition behind the theoretical model in the following session. After discussing the related literature in session 3, we then present the model and discuss its main implications in terms of the effects of exemption on access to credit and the cost of it, in sessions 4 to 6. Session 7 contains the empirical analysis and session 8 concludes the paper.

2 Sorting role of collateral: An informal discussion

Consider a competitive credit market populated by (1) Entrepreneurs endowed with illiquid assets, who need to borrow to finance their business, and; (2) Competitive lenders who face an opportunity cost equal to zero, and make zero profits in equilibrium. Entrepreneurs are of two types: risky (R), who have a high probability of default, and safe (S), with a low probability of default. While each entrepreneur knows her type, lenders only know the distribution of types.

In a pooling equilibrium where all entrepreneurs are financed, safe entrepreneurs would be subsidizing risky ones. Hence, they could benefit from separation by means of a signaling device. Collateral could be such device.

There are two identical credit markets, one located in State 1, and the other located in State 2, where,

1. in State 2's, no assets are exempt from liquidation in the event of bankruptcy;
2. in State 1's, all uncollateralized assets are exempt in the case of default.

in State 1, in the event of bankruptcy, entrepreneurs' assets are liquidated anyway; independently of whether they were posted as collateral or not. Posting collateral makes no difference. Neither the cost of credit nor access to credit can be affected by the decision to post collateral. Accordingly, in State 1, the prevailing equilibrium should be a pooling equilibrium where, if financial exchange takes place, the same contract applies to all entrepreneurs, independently of whether they post any collateral.

Consider now State 2. In the event of bankruptcy, entrepreneurs' assets will be liquidated if and only if they were posted as collateral. Hence, if going bankrupt, an entrepreneur who has posted collateral suffers a greater loss than an entrepreneur who has not. Posting collateral has now an opportunity cost. Crucially, such opportunity cost is, in expected terms, type-dependent. Risky entrepreneurs have a higher probability to fail than safe entrepreneurs, so that their expected opportunity cost from posting collateral is higher than that of safe entrepreneurs. This provides the sorting condition for a separating equilibrium in which safe entrepreneurs self-select into contracts characterized by higher collateral requirements. In such equilibrium, posting collateral results in a lower interest rate for two reasons:

1. Posting collateral increases the cash flow available to financiers (Direct effect);

2. Posting collateral signals a lower probability of default (Signaling effect).

An example. Assume S -type entrepreneurs are endowed with a project that requires 100 units of finance and delivers 360 with probability 1 and zero otherwise. R-type entrepreneurs are endowed with a project that requires 100 and delivers 360 with probability 1/2 and zero otherwise. An entrepreneur is of type $i = R, S$ with probability 0.5. Each entrepreneur is endowed with a house which is worth \$110 dollars to him and whose liquid value is \$100. Lenders opportunity cost of lending is normalized to zero. As before, in State 1 there is no exemption, while exemption is unlimited in State 2.

Let us assume that lenders issue contracts specified in terms of cost of credit, R_L and collateral, C. Let us analyze the case of an entrepreneur operating in State 1. Consider first candidate pooling equilibria (PE). Since there is no exemption, and the liquid value of the house is less than the value of debt, the entrepreneur's asset will be liquidated entirely in the event of bankruptcy, independently of whether the asset had been posted as collateral or not by the entrepreneur. Therefore, the guarantees offered by an entrepreneur is $\max(100, C) = 100$. Given lenders' participation constraint, the value of R_L associated with the contract must satisfy:

$$\frac{3}{4}R_L \times \$100 + \frac{1}{4}\$100 = \$100 \Rightarrow R_L = 1. \tag{1}$$

Borrowers' participation constraint is satisfied:

$$\begin{aligned} type - S \quad &: \qquad \$260 > 0 \qquad (2) \\ type - R \quad &: \qquad \frac{1}{2}\$260 - \frac{1}{2}\$110 > 0. \qquad (3) \end{aligned}$$

Hence a PE always exists.

Consider now a candidate separating equilibrium (SE) whereby type-S entrepreneurs signal themselves by posting their house as collateral, $C = 100$ while type-R do not. Then, by observing collateral, lenders infer an entrepreneur is safe with probability one. The values of the cost of credit, R_i , associated with the contract designed for entrepreneurs of type i, with $i = R, S$, satisfy:

$$R_S \;:\; R_S \times \$100 = \$100 \Rightarrow R_S = 1 \tag{4}$$
$$R_R \;:\; \frac{1}{2}R_R \times \$100 + \frac{1}{2}100 = \$100 \Rightarrow R_R = 1 \tag{5}$$

Note that if type-R entrepreneurs mimic type-S ones, they get the same payoff. The same result holds starting from a pooling equilibrium. Hence, pooling and separating yield the same equilibrium outcome in terms of cost of credit.

Consider now the case of an entrepreneur who operates in State 2. Let us focus on SE whereby S type self-select themselves into a lending contract that requires collateral, C=100 , while risky self-select into a contract with no collateral. Then, by observing an entrepreneur posting collateral, lenders infer an entrepreneur is safe. Correspondingly, the cost of credit for safe and risky are respectively given by:

$$R_S \times \$100 = \$100 \Rightarrow R_S = 1, \tag{6}$$
$$\frac{1}{2}R_R \times \$100 = \$100 \Rightarrow R_R = 2. \tag{7}$$

The payoff of type-R entrepreneurs who are not posting collateral is:

$$\frac{1}{2}\$160 = \$80. \tag{8}$$

If they were to mimic type-S entrepreneurs they would be worse off:

$$\frac{1}{2}\$260 - \frac{1}{2}110 = \$75. \tag{9}$$

Hence, the SE exists. Summarizing the results, with no exemption, collateral has no effect on the cost of credit; while with unlimited exemption, posting collateral might results in a lower cost of credit due to the information content of the decision to self-select into contracts characterized by higher collateral requirements. We study the above intuition in the context of a general model of a competitive market with adverse selection, which is presented in session 3. By developing such model, we will see that by affecting the information content of the decision of posting collateral, the level of exemption does not only influence the cost of credit

3 Related literature

Cross-State variability in exemption levels associated with US State bankruptcy laws prior to the 2005 reform, is key to most empirical investigations on the effects of exemption. Various papers have examined the effects of exemption rates on interest rates and credit rationing. Gropp, Scholz and White (1997) found that interest rates on car loans where higher in states with higher exemption levels. Lin and White (2001) found that potential borrowers are more likely to be turned down by banks the higher is the level of exemption. Berkowitz and White (2004) found that small businesses borrow less and pay higher interest rates in states with higher exemption levels. In a similar vein, Berger, Cerqueiro and Penas (2011) find that borrowers have lower access to credit in states with more debtor-friendly levels of exemption. They also find that in such states borrowers are more likely to pledge collateral and have generally tighter loan terms.

Fan and White (2003) investigate the effects of the bankruptcy system on entrepreneurial behavior. States with unlimited homestead exemption are found to have one-third more entrepreneurs than states with low exemptions. Armour and Cummings (2005) find that countries in which the post-bankruptcy period for which filers are obliged to repay from earnings is shorter have more entrepreneurs.

Fay, Hurst and White (2003) tested where pro-debtor bankruptcy laws encourage borrowers' opportunistic behavior. Their evidence is that for every \$1000 increase in debtors' potential gain from bankruptcy, the filing rate raises by 7%.

Finally, Grant and Koeniger (2005) investigate the insurance effects. They find that the variance of consumption over time is lower in states with higher exemption levels.

4 The model

We consider a competitive market populated by a large number E of entrepreneurs and a large number L of lenders. The set of entrepreneurs, $\mathcal{E}$, and that of lenders, $\mathcal{L}$, are indexed by $e = 1, ..., E$, and $l = 1, ..., L$, respectively. Both entrepreneurs and lenders are risk-neutral. Lenders are endowed with one unit of financial resources, each, and face an opportunity cost of capital, $r > 0$. Each entrepreneur, is endowed with an investment opportunity of size equal to one, and an amount of illiquid and pledgeable wealth, $w \in [\underline{w}, \overline{w}]$. $F(w)$ is the distribution of entrepreneurs with respect to wealth, where for any $w_1 \in [\underline{w}, \overline{w}]$ $F(w_1)$ is the fraction of entrepreneurs endowed with an amount of wealth $w \leq w_1$. With no loss of generality, we set $L/E > 1$, so that financial resources are abundant. For any given level of wealth, w, we define $\mathcal{E}_w \subseteq \mathcal{E}$ the subset of entrepreneurs endowed with illiquid wealth w, and $E(w) = |\mathcal{E}_w|$ the corresponding number of entrepreneurs.

Investment lasts one period and delivers $R > 0$ with probability p and 0 otherwise, where p is a function of entrepreneur's type, q, with $p_H > p_L$. A fraction λ of the population of entrepreneurs is of type H (safe) and a fraction $1-\lambda$ is of type L (risky). We assume $p_L R > (1+r)$, which means that all entrepreneurs, irrespectively of their type, are worth financing.

Entrepreneurs have no liquid resources, so that they need to borrow from lenders in order to finance their investments. One unit of entrepreneurial wealth is worth β to a lender, with $\beta < 1$, so that liquidating entrepreneurial wealth in order to repay lenders is inefficient.

Entrepreneurs can credibly disclose the true value of their wealth at zero cost if they want to. Ex post, in the event of default, wealth is observable and verifiable. If borrowers fail, lenders can appropriate 100% of collateralized assets. However, in the event of bankruptcy, borrowers' uncollateralized assets are exempt from liquidation up to the value, η.

Timing. The market functions as follows:

Stage 0 : Lenders simultaneously offer credit contracts;

Stage 1 : Entrepreneurs decide whether to disclose information about their wealth or not, whether to apply for credit or not, and under which contract;

Stage 2 : Lenders decide whether to reject or approve each loan application they receive;

Stage 3 : Exchange, if any, takes place.

Contracts. A lending contract $\mathcal{C}$ is defined as a triplet, $\mathcal{C} = (R^B, C, \pi)$, where R^B is the cost of credit, C is the amount of collateral, and π is the probability to be financed. Given the level of asset exemption, η, define $w_\eta \equiv \max(w-\eta, 0)$ the value of non-exempt entrepreneurial wealth. Then, the level of real guarantees implicitly offered by an entrepreneur endowed with a generic amount of wealth, w, if applying for the contract $\mathcal{C} = (R^B, C, \pi)$ is

$$G = \max(\min(w_\eta, \frac{R^B}{\beta}), C), \tag{10}$$

where we note that, other things equal, G is weakly increasing in C, and weakly decreasing in η.

Agents' strategies and payoff. The expected payoff of an entrepreneur of type, q, who signs a generic contract, $\mathcal{C}$, is

$$u_q = \pi[p_q(R - R^B) - (1-p_q)G] + w. \tag{11}$$

Correspondingly, the expected payoff of the lender who finances that entrepreneur is

$$v_q = p_q R^B + (1 - p_q)\beta G, \tag{12}$$

where we note that, $u_H > u_L$ and $v_H > v_L$ hold.

Sorting condition. Let $\mathcal{C}_1$ and $\mathcal{C}_2$ two contracts with $\pi_1 = \pi_2 = 1$, $C_1 > C_2$ and $R_1^B < R_2^B$, such that the levels of guarantees associated with the two contracts, given by equation (10), satisfy $G_1 > G_2$.[4] Then,

$$p_L(R - R_1^B) - (1 - p_L)G_1 \geq p_L(R - R_2^B) - (1 - p_L)G_2, \tag{13}$$

implies

$$p_H(R - R_1^B) - (1 - p_H)G_1 > p_H(R - R_2^B) - (1 - p_H)G_2. \tag{14}$$

This follows directly from $p_H > p_L$. That is, whenever an entrepreneur of type L prefers the contract characterized by a higher level of real guarantees, other things equal, an entrepreneur of type H strictly prefers such contract. This leads to the possibility that entrepreneurs of type H could signal their type by self-selecting into a contract characterized by a level of guarantees sufficiently high. Since guarantees are a weakly increasing function of collateral, this means that collateral has a potential role as sorting/signaling device.

Sorting role of collateral as a function of exemption, η. As we shall see, the effectiveness of collateral as a signaling/sorting mechanism depends upon the level of exemption, η. The intuition is as follows. Under no exemption, i.e. if $\eta = 0$, independently of whether they post collateral or not, in the event of default, entrepreneurs' wealth is liquidated. Hence, posting collateral does not provide any meaningful signal. In the opposite extreme case rof unlimited exemption, i.e. if $\eta \to \infty$, entrepreneurs' wealth is liquidated in the event of default if and only if they post it as collateral, which implies that –to the extent that the above described sorting condition holds– type L dislike posting collateral more than type H do – the decision to post collateral has now a sorting/signaling role.

5 Equilibrium analysis

Definition 1. *An equilibrium is a strategy profile and a set of beliefs such that at each node of the game:*

[4] Note that $C_1 > C_2$ implies $G_1 > G_2$, if η is sufficiently high and β is sufficiently low, relative to the other parameters' values.

i. Players' strategies for the remainder of the game are best replies given the strategies of the other players;

ii. Beliefs are derived using Bayes's rule whenever possible.

We first characterize the possible separating and pooling equilibria, and then study existence thereby identifying the equilibrium that prevails in the credit market depending upon parameters' values. With no loss of generality, we focus on parameter configurations such that,

$$p_q R - (1+r)\left[p_i + \frac{1-p_q}{\beta}\right] > 0 \tag{15}$$

which implies that both risky and safe entrepreneurs are willing to demand credit even when the ex post level of non-exempt wealth imply that they would be repaying the loan in full irrespectively of whether their investment is successful or not.

5.1 Separating equilibria (SE)

In any SE, lenders will be offering a menu of contracts such that borrowers of different types will be self-selecting into different contracts. A separating equilibrium (SE) is defined as a set of contracts,

$$\mathcal{C}^{SE} = \{\mathcal{C}_{q,w} = (R^B_{q,w}, G_{q,w}, \pi_{q,w}); R^B_{q,w} \geq 0, G_{q,w} \geq 0, w \in [0, \overline{w}]; q = H, L\},$$

contingent on borrowers' wealth, w, such that,

1. Borrowers' incentive constraints are satisfied:

$$(ICC_H) \;:\; \pi_{H,w}[p_H(R - R^B_{H,w}) - (1-p_H)G_{H,w}] \geq \pi_L[p_H(R - R^B_{L,w}) - (1-p_H)G_{L,w}] \tag{16}$$

$$(ICC_L) \;:\; \pi_{L,w}[p_L(R - R^B_{L,w}) - (1-p_L)G_{L,w}] \geq \pi_H[p_L(R - R^B_{H,w}) - (1-p_L)G_{H,w}]; \tag{17}$$

2. Borrowers' participation constraints are satisfied:

$$(PC_H) \;:\; \pi_{w,H}[p_H(R - R^B_{H,w}) - (1-p_H)G_{H,w}] \geq 0 \tag{18}$$

$$(PC_L) \;:\; \pi_{w,L}[p_L(R - R^B_{L,w}) - (1-p_L)G_{L,w}] \geq 0; \tag{19}$$

3. Lenders make zero profits:

$$p_q R^B_{q,w} + (1-p_q)G_{q,w}\beta = (1+r), \quad \forall q = H, L; \tag{20}$$

4. Feasibility constraints are satisfied: $G_{q,w} \geq 0$, $\pi_{w,q} \in [0,1]0$, $\forall q = H, L$.

The following result holds,

Proposition 1 (Separating equilibrium: Characterization)**.** *The separating equilibrium, if it exists, yields a unique outcome characterized as follows:*

i. Lenders offer the following menu of contracts with positive probability,

$$\mathcal{C}^{SE} = \{\mathcal{C}_{q,w} = (R^B_{q,w}, G_{q,w}, \pi_{q,w}); R^B_{q,w} \geq 0, G_{q,w}, \pi_{q,w} \in [\underline{w}, \overline{w}]; \forall w \in [\underline{w}, \overline{w}], q = H, L\},$$

where

$$R^B_{q,w} = \frac{(1+r)}{p_q} - \frac{(1-p_q)\beta G_{q,w}}{p_q}, \tag{21}$$

$$G_{H,w} = \min\left(\frac{(1+r)(p_H - p_L) + p_H(1-p_L)(1-\beta)G_{L,w}}{(1-p_L)p_H - p_L(1-p_H)\beta}), w\right), \tag{22}$$

$$G_{L,w} = \min(w_\eta, \frac{1+r}{\beta}), \tag{23}$$

$$\pi_{L,w} = 1, \tag{24}$$

$$\pi_{H,w} = \min\left(\frac{p_L R - (1+r) - (1-p_L)(1-\beta)G_{L,w}]}{[p_L R - \frac{p_L}{p_H}(1+r) - (1-p_L)\left[1 - \frac{p_L}{p_H}\frac{1-p_H}{1-p_L}\beta\right] w}, 1\right). \tag{25}$$

ii. Borrowers of type $q = L, H$ self-select into contract $\mathcal{C}_{q,w}$.

Proof. See appendix.

For both contracts $\mathcal{C}_{i,w}$, figure 1 describes the levels of real guarantees , $G_{H,w}$ and, $G_{L,w}$, as a function of wealth w. Similarly, figure 2 describes the probabilities to be financed, $\pi_{H.w}$ and $\pi_{L.w}$, and figure 3 describes the costs of credit, $R^B_{L,w}$ and $R^B_{H,w}$. Safe borrowers endowed with a value of wealth w such that $w > \widehat{w}$, where

$$\widehat{w} \equiv \begin{cases} \frac{(1+r)(p_H-p_L)-(1-p_L)(1-\beta)p_H\eta}{(p_H-p_L)\beta} & \text{if } \widehat{w} > \eta \\ \frac{(1+r)(p_H-p_L)}{(1-p_L)p_H-(1-p_H)p_L\beta} & \text{if } \widehat{w} \leq \eta \end{cases} \tag{26}$$

are *rich* in the sense that they can afford the level of real guarantees necessary to self-select into contracts designed for H-type borrowers, and characterized by a probability of access to credit equal to one. Differently, safe borrowers with $w < \widehat{w}$ are poor, as they cannot afford the level of real guarantees necessary in order to self-select into contracts for type-H borrowers, that assure full access to credit. For any given level of wealth, poor and safe borrowers separate from type L-borrowers by self-selecting into contracts characterized by a higher level of guarantees and a lower probability of access to credit than those associated with contracts for type-L borrowers endowed with the same level of wealth. For poor borrowers, the marginal effect of w on $\pi_{H,w}$ is which is strictly positive for any η.[5] The poorer the safe borrower the higher the probability to be rationed according to the contract designed for the safe type (see figure 2).[6] Finally, conditional on wealth, safe borrowers always post a higher level of guarantees than risky ones (see figure 1), so that they always face a lower cost of credit (see figure 3).

The effects of exemption on guarantees, access to credit and cost are as follows. As exemption increases, the level of guarantees offered by risky borrowers is reduced, so that the critical level of wealth, $\widehat{w}$, that a safe borrower needs to offer in order to be necessary in order to be able to signal his type and be financed with probability one is also reduced (see figure 4). Correspondingly, a higher level of exemption results in a higher probability to access credit for safe borrowers at all levels of wealth., as shown in figure 5. Finally, conditional on wealth, the difference between the cost of credit faced by risky and safe borrowers is also increasing in the level of exemption (see figure 6).

Uniqueness and selection of SE. It should be noted that while the unique equilibrium outcome in terms of guarantees, probability to be financed and cost of credit described in proposition 1 is associated with a unique contract being offered to type H borrowers for any given level of wealth, w, this is not true for type L borrowers. . For risky borrowers, conditional on wealth, there is a continuum of contracts that are all characterized all by the same cost of credit and probability to be rationed, and different levels of C_L, with $C_L : C_L \leq w - \eta$, which yield the same outcome in terms of guarantees. Therefore, the SE is not unique. However, this is due to the fact that –for simplicity– we assume that the cost of posting collateral is exactly zero. More generally, allowing for the possibility that the cost of posting collateral might not be costless, implies that in a SE, lenders of type L would always choose a contract characterized by $C_L = 0$, which yields a unique SE. Accordingly, for simplicity, we refer just to SE characterized by $C_L = 0$.

5.2 Pooling equilibria

In any pooling equilibrium, lenders offer a set of contracts, $\mathcal{C}^P = \{\mathcal{C}_w = (R_w^B, G_w, \pi_w); R_w^B \geq 0, G_w \geq 0, w \in [0, \overline{w}]; w \in [\underline{w}, \overline{w}]\}$, $\pi_w \in [0,1]$, such that:

[5] As shown in appendix, these borrowers are wealth-constraint, i.e. they would like to post more guarantees than what they can afford. The only reason why they would like to do that is to get into contracts characterized by a higher probability to be financed.

[6] Note that the second order derivative of $\pi_{H,w}$ with respect to w is strictly positive.

1. Borrowers' participation constraints are satisfied,

$$(PC_H) \quad : \quad \pi_w[p_H(R - R_w^B) - (1 - p_H)G_w] \geq 0 \tag{27}$$

$$(PC_L) \quad : \quad \pi_w[p_L(R - R_w^B) - (1 - p_L)G_w] \geq 0; \tag{28}$$

2. Lenders make zero profits:

$$p_m R_w^B + (1 - p_m)G_w\beta = (1 + r), \tag{29}$$

where $p_m \equiv \lambda p_H + (1 - \lambda)p_L$.

The following result holds.

Proposition 2 (Pooling equilibrium: Characterization)**.** *The pooling equilibrium, if it exists, is characterized by the fact that lenders offer the following set of contracts with positive probability,* $\mathcal{C}^P = \{\mathcal{C}_w^P = \{R_w^B, G_w, \pi_w), w \in [\underline{w}, \overline{w}]\}$*, where* $\pi_w = 1$*, and*

i. If,

$$\beta\frac{1 - p_m}{p_m} < \frac{1 - p_H}{p_H} \tag{30}$$

$$R_w^B = \frac{(1 + r)}{p_m} + \frac{(1 - p_m)}{p_m}\beta(w - \eta) \tag{31}$$

$$G_w = w - \eta \tag{32}$$

ii. If,

$$\beta\frac{1 - p_m}{p_m} > \frac{1 - p_H}{p_H} \tag{33}$$

$$R_w^B = \frac{(1 + r)}{p_m} + \frac{(1 - p_m)}{p_m}\beta(w - \eta) \tag{34}$$

$$G_w = w. \tag{35}$$

iii. If

$$\beta\frac{1 - p_m}{p_m} = \frac{1 - p_H}{p_H} \tag{36}$$

Then there is a continuum of equilibrium values of $G_w \in [w_\eta, w]$*, and correspondingly, of the interest rate, where*

$$R_w = \frac{1 + r}{p_m} + \frac{1 - p_m}{p_m}\beta G_w$$

Proof. See appendix. According to the above proposition, all entrepreneurs borrow according to the same contract. No rationing takes place. The cost of credit is a decreasing function of wealth. Importantly, so long as condition (30), so that $G_w = w_\eta$, an increase in the level of exemption, η, is always associated with an increase in the cost of credit. Whether in a pooling equilibrium entrepreneurs will enough collateral to undo the effects of exemption or not it will depend upon whether safe borrower prefer to post collateral in order to get a lower cost of credit or not. While under perfect information safe borrowers would never prefer to do so, in a pooling, whereby they are facing a higher cost of credit due to the fact that they are pooled with L type borrowers, they might want to do that. In fact, this is the case if and only if condition (30) hold; in which case the unique pooling equilibrium is unique and characterized by the fact that all borrowers post enough collateral that $G_w = w$. Otherwise, if condition (33) holds, they would never choose to post enough collateral to undo the effects of exemption. Finally, a continuum of pooling equilibrium outcomes, whereby $G_w \in [w - \eta, w]$, if (36) holds so that entrepreneurs are indifferent between posting enough collateral to undo the effects of exemption or not.

5.3 Prevailing equilibrium

The key question is whether pooling and/or separating equilibria as characterized above exist and under which conditions. Let ω_1 and ω_2 two critical levels of wealth, with

$$\omega_1 \ : \ (1+r)\frac{p_H - p_m}{p_m} = (1-\beta)(1-p_H)(G_{H,\omega} - G_{p,\omega_1}) + G_{p,\omega_1}(1-p_H)\beta[\frac{1-p_m}{1-p_H}\frac{p_H}{p_m} - 1] \tag{37}$$

$$\omega_2 \ : \ \pi_{H,\omega_2}[p_H R - (1+r) - (1-p_H)(1-\beta)G_{H,\omega_2}] = p_H R - (1+r)\frac{p_H}{p_m} - G_{p,\omega_2} p_H \left[\frac{1-p_H}{p_H} - \beta\frac{1-p_m}{p_m}\right]$$

so that for $w = \omega_1$, rich entrepreneurs of type H are indifferent between the PE and the SE contract, and the same happens when $w = \omega_2$ for poor entrepreneurs of type H. The following result holds,

Proposition 3 (Credit market quilibrium)**.** *The equilibrium is unique and characterized as follows,*

Case i. *If condition (30) holds, (1) All safe and rich entrepreneurs separate from rich and risky ones; (2) All safe and poor entrepreneurs with $w > \omega_2$ separate from risky ones, while those with $w < \omega_2$ pool with risky ones.*

Case ii. *If (33), the reverse condition holds, then: (1) All rich and safe entrepreneurs with $w > \omega_1$ separate from rich and risky ones, while those with $w < \omega_1$ pool; (2) All safe and poor entrepreneurs with $w > \omega_2$ separate, while those with $w > \omega_2$ pool.*

Proof. See Appendix.

The intuition is as follows. Whenever condition (30) holds, then, the pooling equilibrium would be characterized by a level of guarantees equal to individual wealth, $G_w = w$. Clearly, rich and safe borrowers then, would strictly prefer a SE equilibrium contract designed for them, which would be characterized by a lower level of guarantees{Recall $G_H < w$ for rich and safe borrower. and a lower cost of credit. Therefore, under condition (30) rich and safe entrepreneurs never pool with risky ones. The same argument would hold for poor and safe borrowers, except that for these borrowers, the probability of being financed under a SE contract designed for them would be less than one, and declines with w. Hence, there could be safe borrowers who are that poor that they would be hardly financed under SE. This explains why, so long as $\omega_2 > \underline{w}$, there will be safe borrowers who are poor enough to prefer pooling with risk borrowers rather than separate.

Conversely, the pooling equilibrium would be characterized by a level of guarantees equal to non-exempt wealth, $G_w = w - \eta$ so long as condition (30) holds. In this case, in principle, rich and safe borrowers could prefer a PE to a SE to the extent that the PE involves less guarantees. Yet, in a PE safe borrowers face a higher cost of credit than under the SE, which is why they might actually prefer a SE contract rather than a PE one. However, this latter effect becomes smaller as w increases, which is why so long as $\overline{w} > \omega_1$, there will be safe borrowers who are rich enough that they want to pool rather than to separate. As for poor borrowers, the same argument as in the previous case holds, which implies that so long as $\omega_2 > \underline{w}$, there will be safe borrowers who are poor enough to prefer pooling.

6 Implications: Cost of credit, access to credit and exemption

We now analyze the model's determinants of access to credit and cost of credit, and analyze how these relationships change depending on the level of exemption, η.

i. Cost of credit. Independently of whether borrowers pool or not,

1. the cost of credit is, if anything, negatively associated with entrepreneurial wealth;

2. Posting a *significant* amount of collateral, which we define as an amount of collateral large enough to affect the value of guarantees reduces the cost of credit. For positive values of non-exempt wealth, w_η this effect is larger as the level of non exempt wealth decreases; while the opposite is true whenever non-exempt wealth is zero.

ii. Access to credit. To the extent that borrowers pool together, they are not rationed. When adverse selection is significant enough for safe borrowers to separate from risky ones,

3. Rich and safe borrowers are more likely to access credit than poor ones; that is entrepreneurial

wealth, w is positively associated with access to credit; $\pi_{H,w}$;[7]

4. The decision to post a significant amount of collateral – that is an amount of collateral large enough to affect the value of guarantees – is associated with a lower probability to access credit for poor and safe borrowers. That is, there is a negative relationship between posting collateral and access to credit, for sufficiently low levels of entrepreneurial wealth.

iii. Effects of exemption. The level of exemption, η affects the marginal effects induced by the various determinants of cost of credit and access to credit, as follows. The higher is the exemption level η,

5. The higher is the probability to access credit for those borrowers who are posting collateral.[8] Therefore, to the extent that adverse selection is significant and entrepreneurs separate, higher levels of exemption are – other things equal– associated with less rationing;

6. The higher is the cost of credit differential in favor of entrepreneurs who are posting a significant amount of collateral, for any given positive level of non-exempt wealth, w_η.

7 Empirical analysis

Based upon our theoretical model, we derive structural and reduced form econometric models to test the predictions about the effect of the decision to post collateral on (i) access to credit, and (ii) cost of credit, as well as about how these relationship change depending on whether the borrower posts collateral or not.

7.1 Data

We use the public version of the Survey of Small Business Finances (SSBF), which has been conducted in $2004 - 2005$ for the Board of Governors of the Federal Reserve System. The dataset provides information on a sample of 4240 firms, selected from the target population of all for-profit, non-financial, non-farm, non-subsidiary business enterprises that had fewer than 500 employees and were in operation as of year-end 2003 and on the date of the interview. Information on the availability and use of credit and other financial services, demographic characteristics for up to three of the individual owners, other firm's characteristics such as number of workers, organizational form, location, credit

[7]Note that risky entrepreneurs – who are all credit worth in the model – are always financed.
[8]This effect is on average larger for relatively low levels of wealth than for sufficiently high ones.

history, income statement and balance sheet is also available. The survey asked entrepreneurs, whether their firm applied for credit during the last three years (from 2001 to 2003) and, in that case, whether such loan applications were always denied, always approved or sometimes approved.

In our empirical approach, we stick as close to our theoretical model as possible. Accordingly, Since in our model all firms are credit worth, we restrict our sample to those firms whose loan applications have been approved at least once in the observation period.[9] By doing so, the sample size reduces to 1761 credit worth firms, 96% of which have been always financed. For all these firms, which have been financed at least sometime in $2001-03$, the survey provides some information on the most recent loan contract. In particular, we have information on the loan interest rate and whether the firm had to post some collateral to secure the loan.[10]

We augment the data by including the level of bankruptcy homestead and personal property exemptions according to firm's geographical location. Exemption levels vary across states. However, the public version of the SSBF reports firm's location only for nine census divisions (New England; Middle Atlantic; East North Central; West North Central; South Atlantic; East South Central; West South Central; Mountain; Pacific). Thus, we assign to each firm the average level of exemption of its census division. In the states in which the exemption is unlimited we set it to the average dollar value of firms' assets in the sample. **Q1: Can we use other measures for robustness check? Like median....**

In the empirical analysis we consider two levels of exemption: high exemption for firms in a census divisions in which average exemption is above the median value and low exemption otherwise. Firm's wealth is measured by firm's total asset. We divide firms into two groups according to wealth: one including firms with "high assets", that is asset values above the median, and the other including those with "low assets"; that is asset values below the median value. Thus, based on wealth and exemption, we have four categories of firms.

Furthermore, based on our theoretical model, we distinguish two types of firms, those that post collateral, and those who do not. This is crucial to our empirical analysis since the model yields different predictions for these two types of firms.

Table 1 reports the descriptive statistics about the cost of credit and the probability to be rationed, for the whole sample as well as for the different subsamples defined above. The observed patterns are as follows:

[9] We area ware that this might cause selection bias, and –as detailed below – we take that possibility into account in our econometric exercise.

[10] The dataset does not provide information on the amount of collateral posted.

1. High asset firms face a lower cost of capital and are less rationed. The loan rate and the fraction of rationed firms are, respectively, 1.5% and 3.8% lower than in the low asset group;

2. Firms that post collateral face a lower cost of capital. In the whole sample, firms pcollateral the loan rate is 0.7% lower. Notably, this effect is larger the higher is the exemption level. In the low exemption subsample, the cost of capital differential in favor of firms posting collateral is 0.53% while in the high exemption one, it grows to 1.20%;

3. The correlation between the decision to post collateral and the cost of capital depends on wealth. Low asset firms have a cost of capital 0.9% lower if they post collateral, while for high asset the reduction in the cost of capital for posting collateral is much smaller (0.04%);

4. Firms that post collateral are more rationed. The fraction of rationed firms among those that post collateral is 1.5% higher than that in the group of firms that do not post collateral;

5. The association between rationing and posting collateral depends on wealth. In the group of low asset firms, the fraction of those rationed is 4.4% higher in the subsample firm posting collateral, while in the high asset group there is no difference in rationing depending on collateral;

6. For firms that post collateral, the fraction of those that are rationed falls by 1.1% moving from low to high exemption levels. This effect is larger (-1.9%) for the low asset firms compared to those with high asset (-0.5%).

We note consistently with the model: i. the loan rate differential in favor of firms posting collateral grows with exemption; ii. a lower fraction of firms posting collateral is rationed in high exemption census divisions than in low exemption ones. We now proceed to test some of the model's predictions. As described below, in doing so we follow the model closely and use it in order to derive the appropriate econometric models.

7.2 Access to credit, and the cost of credit

We first discuss how we derive our econometric models for access to credit and cost of credit, respectively, and then present the results.

Access to credit. According to the theoretical model, in any separating equilibrium, at structural level, the probability of being financed is: i. (Weakly) decreasing as we move from firms of type L to firms of type H as, only poor firms of type H are rationed, if any; ii. Increasing in the level of entrepreneurial wealth; iii. decreasing in the cost of credit. Moreover, the effect associated with firm's type is declining with exemption as H type firms are less rationed the higher is the level of exemption. Importantly, according to the model, both the cost of credit and firm's type are exogenous with respect to the probability to access credit. That given, model the probability to access credit for firm i as follows,

$$\pi_i = \alpha_1 Z_i + \alpha_2 \eta_i + \alpha_3 C_i + \alpha_4 C_i \eta_i + \alpha_5 R_i^B + u_i, \tag{38}$$

where π takes on two values: 1 if the firm is always financed, 0 if it is sometimes rationed; Z_i is a set of controls that affect bank's decision to finance; α_1 is a vector of parameters; α_2, α_3, α_4 are parameters; η_i is the level of exemption for firm i; C_i is a dummy that equals one if firm i posts collateral; R_i^B is the interest rate on the loan and the error term $u_i \sim N(0, \sigma_1)$ is the error term. We estimate the probability that $\pi_i = 1$ with standard probit.

Following the theoretical model, the variable C_i captures firm's type, while the interaction term captures the fact that the difference in access to credit across firm types should decrease with exemption as firms of type H become less rationed. Therefore, according to our theory, we expect $\alpha_3 < 0$, and $\alpha_4 > 0$.

Cost of credit. In the model, the equilibrium value of the cost of credit depends on: i. Firm's type, as only firms of type H post collateral; ii. The level of exemption, and iii. the level of entrepreneurial wealth. Accordingly, our reduced form regression model for the cost of credit is

$$R_i^B = \beta_1 X_i + \beta_2 \eta_i + \beta_3 C_i + \beta_4 C_i \eta_i + v_i, \tag{39}$$

where X_i is a set of controls, β_1 is a vector of parameters, β_2, β_3, β_4 are parameters, and $v_i \sim N(0, \sigma_2)$ is the error term. We estimate the equation of loan rate by OLS.

Our theory predicts that firms of type H – which are the only to post collateral– face a lower cost of credit. Hence, we expected $\beta_3 < 0$. Furthermore, the cost of credit differential in favor of firms of type H should increase with exemption, so that we expect $\beta_4 < 0$. Finally, we should expect $\beta_2 > 0$ to be positive as our theory tells us that the interest rate goes up with exemption for type L firms who are not posting collateral.

Control variables. Both in the regression model for the probability to be financed and in that for the cost of credit, we control for firm-bank relationship and various firm, entrepreneur and loan characteristics in the and the equation of the interest rate.

Sorensen and Chang (2006) provide wide evidence of the positive relationship between entrepreneur's experience and firm's profitability. To catch entrepreneur's experience, we include the number of years of managing experience of the principal owner. To the extent that greater experience leads to higher profits, it should also imply a higher probability of success for the entrepreneurial activity, and therefore we expect it to be negatively associated with the cost of credit.[**This is not clear cut...should we emphasize this so much?**]

Belonging to a minority group has been found to reduce the probability of obtaining a loan (Cavalluzo and Wolken, 2005; Berkowitz and White, 2002), while Cerqueiro and Penas (2011) find evidence that owners belonging to a minority group rely more heavily on their own funds to finance a start up. We control for minorities by means of two dummies: the first equals one if the principal owner is black and zero otherwise, the second equals one if the owner belongs to other minority groups (asian, hispanic, asian pacific, native american). We also include a dummy indicating whether the owner is a female, to verify possible discrimination effects on the loan price. Firm's proprietorship characteristics may affect access to credit and terms of the loan – including interest rate – due to agency costs compared to those of non-family owned firms [**???**]. Anderson et al (2003) argue that debt holders often establish informed relationships with managers, and the family's presence may foster these relationships to build over successive generations[**Non capisco...???**]. Niskanen et al (2010) find evidence that for small Finnish firms family ownership is associated with lower availability of credit, while managerial ownership leads to lower collateral requirements. On the contrary no proprietorship effect on the loan interest rate has been found, suggesting the presence of more relevant agency cost for family ownership.[**???**]

Firm-bank relationship can be represented by several variables, such as firm distance from the bank, the time length of the relationship with the lender. The structure of local credit markets may also have a role in explaining the cost of credit. To account for bank's local market power we include a dummy equal one if the Herfindahl-Hirschman bank deposit index of local credit market concentration is greater than 1800 [**Why 1800, is this standard?**]. We also include the number of credit applications in the previous three years as a proxy for firm's financial needs. Given other firm's characteristics frequent loan applications may be either a signal of financial distress or greater investment opportunities. To control for borrower's quality we include a dummy equal one if firm credit score is in the top 25% of the distribution.

We also consider that the characteristics of the loan may have an impact on the loan interest rate. Accordingly, we distinguish three typologies: 1.line of credit, 2. mortgage or 3. ffixed interest rate loans.

We control for firm's dimension using the log of sales, and we use the ratio of debt on total asset as a measure of firm's financial structure, i.e. firm's leverage. Finally, firm wealth is proxied by firm's asset.

In the equation relative to the probability to be financed, control variables are mainly related to loan characteristics and firm-bank relationship. We consider the amount of loan granted on the total amount applied as it may positively affect bank profits and hence the bank willingness to finance **?????**. We also include loan maturity, which we expect to have a negative effect on the probability to be financed, as long term loans could be less liquid and therefore more risky from the bank's perspective. A longer firm-bank relationship improves the information flow between lenders and borrowers. We include the numbers of years of the relationship with the lender and we expect a positive effect on the probability to receive a loan. Past delinquencies may represent a bad signal of firm trustworthiness. Thus, we expect a negative sign for the a dummy equal one if the firm has a delinquency record. As in the loan rate equation, we include firm's credit score to proxy its credit quality. We also control for firm's capital structure. The ratio of debt on total asset is expected to have a negative impact on the bank's willingness to finance, because higher leverage may reduce firm's ability to repay. Firm's wealth, as proxied by firm's asset, is expected to have a positive effect on the probability to access credit. Finally, we include a dummy equal one if the firm has limited liability, which might limit banks' ability to seize owners' wealth in the event of default.

Results. The OLS estimation of model (39) for the cost of credit, R^b, is reported in table 2. The sample of 1761 firms that received a loan during the period of the survey, reduces to 1691 observations, due to missing values. Exemption, firm's type as proxied by the decision to post collateral, and the interaction term between the decision to post collateral and exemption are all strongly significant and have the expected sign. In line with the model's prediction, firms' posting collateral face a lower cost of credit and this effect is larger the greater is the level of exemption. All control variables have the expected sign and are significant at least at 10% level except for the dummy indicating a female owner and the number of applications.

In table 3 we report the results of the estimation of model (38) for the probability to access credit. Also in this case, firm's type as proxies by the decision to post collateral, and the interaction term between the decision to post collateral and exemption are highly significant and with the expected sign. Posting collateral is associated with rationing, and firms that post collateral are less likely to be rationed the higher is the level of exemption.

In the light of these results, we conclude that the main predictions of the model regarding access to credit, cost of credit, decision to post collateral and exemption cannot be rejected.

7.3 Structural model: simultaneous determination of guarantees and cost of credit

According to the model the level of entrepreneur's guarantees and cost of credit – none of which is affected by the probability to be financed – are simultaneously determined at structural level. That is, the cost of credit is affected by the guarantees an entrepreneur is able to post l and the amount of guarantees is affected by the cost of credit. Therefore, coherently with the model, we estimate the following system of two equations for the cost of credit as a function of the guarantees and the amount of guarantees as a function of the cost of credit.

$$R_i^B = \gamma_1 G_i + \beta_1 X_{i,1} + \varepsilon_i \tag{40}$$

$$G_i = \gamma_2 R_i^B + \beta_2 X_{i,2} + \mu_i, \tag{41}$$

where R_i^B is a continuos endogenous variable (interest rate), G_i is a dichotomous endogenous variable (dummy equal 1 if firm posted collateral and is always financed), $X_{i,1}$ and $X_{i,2}$ are matrices of exogenous variables, β_1 and β_2 are vectors of parameters of the exogenous variables, γ_1 and γ_2 are the parameters of the endogenous variables, ε_i and μ_i are the error terms.

In attempting this estimation, we are constraint by data availability. The level of guarantees is an unobserved continuos variable. We do observe, however, whether the firm posts collateral or not; and we also observe whether a firm that posts collateral is always or only sometimes financed. The model tells us that firms who post collateral and are always financed are – other things equal – posting more guarantees than those that are financed only sometimes. Based on this, we construct a dummy variable, which we call G, that takes value 1 if the firm posts collateral and it is always financed, and zero otherwise – i.e. if the firm does not post collateral or it posts collateral and it is financed only sometimes. We use this variable as a discrete measure of the level of guarantees.

The estimation method and the routine used to implement a two stage probit least square are presented in Keshk (2003). We expect the following effects:

1. a negative effect of collateral on loan rate;

2. this effect is larger for firms in high exemption areas;

3. a negative effect of loan rate on collateral, i.e. high level of guarantees.

In the second column of tables 4 and 5 we report the estimation results of the simultaneous equation model for the whole sample. We find a negative relationship between $R^b$and C. Other things equal, posting collateral is associated with an average reduction in the cost of credit by 46 base points. Consistently, a higher interest rate is associated to a lower probability to post collateral, as measured by our proxy. Assuming that the decision to post collateral is endogenous we do not to include the interaction term between exemption and collateral in the estimation, which would be endogenous by construction. In order to identify the signaling value of collateral, we estimate the model dividing the sample in two subsamples, one including firms located in groups of states with average homestead and personal property exemption level below the mean (low exemption), and the other including firms located in group of states with exemption level above the mean. The results are reported In the last two columns of table 4. The negative relationship between collateral and cost of credit is significantly higher (59 base points) in the group of the high exempt firms. This offers further support to the model, and to the role of collateral as sorting device.

7.4 Robustness: Selection

As already explained, since in the theoretical model all entrepreneurs are credit worth, we conduct our empirical analysis on the subsample of firms that have needed credit and have been financed at least sometimes. To the extent that credit worth firms are a selected subsample, our estimates might suffer from selection bias. We control for such possibility by estimating a selection model à la Heckman (see appendix, part B). Controlling for selectivity does not halter any of our conclusions.

8 Conclusions

Under chapter 7, bankrupt entrepreneurs benefit from a fresh start opportunity to the extent that they keep "uncollateralized" assets exempted from liquidation according to the exemption levels set by the law of the State in which the entrepreneurs operate. This, of course, at the expenses of creditors. However, ex ante, entrepreneurs could undo the effects of exemption by posting enough collateral. We investigate how such possibility affects the effectiveness of collateral as a sorting device in a competitive credit market characterized by adverse selection, depending on the level of exemption. We find that by increasing the effectiveness of collateral as a sorting device, a higher level of exemption results in a lower cost of credit and a higher access to credit for entrepreneurs posting collateral. We test these implications in a sample of small businesses in the US using 2002 SSBF data. According to empirical results, we cannot reject the theoretical predictions of the model.

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A Appendix

A.1 Proof of proposition 1

We first characterize the equilibrium and then provide conditions for existence.

i. Cost of credit. The following (preliminary) result holds,

Lemma 1. *In any equilibrium, lenders must be making zero profits.*

Proof. Consider a candidate equilibrium E whereby lenders offer $\mathcal{C}_{L,w}$ and $\mathcal{C}_{H,w}$, to risky and safe borrowers respectively, and make strictly positive profits.[11] Since financial resources are abundant, in E, the probability to finance an entrepreneur must be less than one, for at least some of the lenders. Clearly, any lender whose probability to finance an entrepreneur is, in E, less than one, would be strictly better off by deviating and offering a contract $\mathcal{C}'_{L,w}$ to type L borrowers, characterized by $R^{B'}_{L,w} = R^{B}_{L,w} - \epsilon$, with $\epsilon > 0$. Such contract will attract all L-type borrowers – and maybe also borrowers of type H who are of better quality, and it would guarantee to the lender an expected profit strictly greater than the equilibrium one, since the lender would be now able to finance an entrepreneur with probability one. Therefore, in any equilibrium, lenders must be making zero profits.□

According to the above lemma, in any SE,

$$p_H R^B_{H,w} + (1-p_H)G_{H,w}\beta = (1+r) \Rightarrow R^B_{H,w} = \frac{(1+r)}{p_H} - \frac{(1-p_H)\beta G_{H,w}}{p_H} \tag{A.1}$$

$$p_L R^B_{L,w} + (1-p_L)\beta G_{L,w} = (1+r) \Rightarrow R^B_{L,w} = \frac{1+r}{p_L} - \frac{(1-p_L)\beta G_{L,w}}{p_L}. \tag{A.2}$$

ii. Guarantees, collateral and access to credit for type L borrowers. Since asset liquidation is an inefficient way of corresponding cash flows to lenders, due to $\beta < 1$, – in any SE equilibrium, the level of guarantees associated with the contract designed for type-L entrepreneurs should be minimum. If not, there always exist a strictly profitable deviation for lenders, which would be to offer a contract characterized by a higher cost of credit and la lower (implied) level of guarantees. Accordingly,

$$G_{L,w} = \min\left(w_\eta, \frac{1+r}{\beta}\right), \tag{A.3}$$

which means that the level of collateral associated with the contract designed for type-L entrepreneurs, $C_{L,w}$, satisfies $C_{L,w} \leq G_{L,w}$.

As for the probability to be financed, $\pi_{L,w}$, consider a candidate SE in which lenders are making zero profits, and $\pi_{L,w} < 1$. Clearly, lenders' have a strictly profitable deviation, $\mathcal{C}'_{L,w} = \{R^B_{L,w} + \epsilon, G_{L,w}, 1\}$, where we note that such deviation will surely attract at least borrowers of type L so long as $\epsilon > 0$ is sufficiently small. Hence, in any equilibrium, $\pi_{L,w} = 1$.

Summing up, type L borrowers get the same contract they would get under perfect information (no distortion at the bottom).

[11] The candidate equilibrium would be a pooling equilibrium if $\mathcal{C}_{L,w}$ =$\mathrm{C}_{H,w}$, and a separating one, otherwise.

iii. Guarantees, collateral and access to credit for type H borrowers. Given the equilibrium values for $R^B_{H,w}$, $R^B_{L,w}$, and $\pi_{L,w}$, the correspindent equilibrium values for $\pi_{H,w}$ and $G_{H,w}$ are found solving the following maximization problem,[12]

$$\max_{\{\pi_{H,w},G_{H,w}\}} \pi_{H,w}\left[p_H(R-R^B_{w,H})-(1-p_H)G_{H,w}\right]+w \tag{A.4}$$

subject to,

$$\begin{aligned}
\pi_{H,w}\left[p_H(R-R^B_{H,w})-(1-p_H)G_{H,w}\right] &\geq \left[p_H(R-R^B_{L,w})-(1-p_H)G_{L,w}\right] && \text{(A.5)}\\
\left[p_L(R-R^B_{L,w})-(1-p_L)G_{L,w}\right] &\geq \pi_{H,w}\left[p_L(R-R^B_{H,w})-(1-p_L)G_{H,w}\right] && \text{(A.6)}\\
\pi_{H,w} &\in [0,1] && \text{(A.7)}\\
G_{H,w} &\in [w_\eta,\min(w,\frac{1+r}{\beta})] && \text{(A.8)}
\end{aligned}$$

The lagrangean expression associated with the above problem is:

$$\begin{aligned}
\mathcal{L} &= u_H+\overline{\lambda}_\pi(1-\pi_{H,w})+\underline{\lambda}_\pi\pi_{H,w}+\overline{\lambda}_G\left[\min(w,\frac{1+r}{\beta})-G_{H,w}\right]+\underline{\lambda}_G(G_{H,w}-w_\eta) && \text{(A.9)}\\
&+ \lambda_{ICC,H}\{\pi_{H,w}\left[p_H(R-R^B_{H,w})-(1-p_H)G_{H,w}\right]-\left[p_H(R-R^B_{L,w})-(1-p_H)G_{L,w}\right]\}\\
&+ \lambda_{ICC,L}\{\left[p_L(R-R^B_{L,w})-(1-p_L)G_{L,w}\right]-\pi_{H,w}\left[p_L(R-R^B_{H,w})-(1-p_L)G_{H,w}\right]\}
\end{aligned}$$

where we use,

$$u_H=[p_HR-(1+r)-(1-\beta)(1-p_H)G_{H,w}]+w. \tag{A.10}$$

The FOCs are:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\pi_{H,w}} &= p_H(R-R^B_{H,w})-(1-p_H)G_{H,w}-\overline{\lambda}_\pi+\underline{\lambda}_\pi && \text{(A.11)}\\
&+ \lambda_{ICC,H}\left[p_H(R-R^B_{H,w})-(1-p_H)G_{H,w}\right] && \text{(A.12)}\\
&- \lambda_{ICC,L}\left[p_L(R-R^B_{H,w})-(1-p_L)G_{H,w}\right]=0 && \text{(A.13)}\\
\frac{\partial\mathcal{L}}{\partial G_{H,w}} &= -\pi_{H,w}\left[p_H\frac{dR^B_{H,w}}{dG_{H,w}}+(1-p_H)\right]+\underline{\lambda}_G && \text{(A.14)}\\
&- \overline{\lambda}_G-\lambda_{ICC,H}\left[p_H\frac{p_HdR^B_{H,w}}{dG_{H,w}}+(1-p_H)\right] && \text{(A.15)}\\
&+ \lambda_{ICC,L}\left[p_L\frac{dR^B_{H,w}}{dG_{H,w}}+(1-p_L)\right]=0 && \text{(A.16)}
\end{aligned}$$

Case 1: Wealth constraints are not binding, ie. $G_{H,w}\in(w_\eta,\min(w,\frac{1+r}{\beta}))$**.** In this case, $\underline{\lambda}_G=\overline{\lambda}_G=0$. Under the hypothesis (to be later verified) that PC_H is satisfied as a strict inequality, the FOC relative to the choice of $\pi_{H,w}$ implies $\pi_{H,w}>0$. Accordingly, it follows from the FOC

[12]Note that the objective function is obtained by substituting the equilibrium value of $R^B_{H,w}$ as a function of $G_{H,w}$.

relative to the choice of $G_{H,w}$ that $\lambda_{ICC,L} > 0$, i.e. the ICC for type L-borrowers is binding,[13]

$$\left[p_L(R - R^B_{L,w}) - (1-p_L)G_{L,w}\right] = \pi_{H,w}\left[p_L(R - R^B_{H,w}) - (1-p_L)G_{H,w}\right] \tag{A.18}$$

It can be verified that if the ICC_L is binding, then the ICC_H is slack, so that $\lambda_{ICC,H} = 0$. The optimal value of $G_{H,w}$ is then found by substituting for $R^B_{H,w}$, and $R^B_{L,w}$ using equations xxx, and $\pi_{H,w} = 1$, in the above equality and solving it for $G_{H,w}$, which yields,

$$G_{H,w} = \frac{(1+r)\left(\frac{p_H - p_L}{p_L}\right) + (1-p_L)(1-\beta)G_{L,w})}{(1-p_L)\left[1 - \frac{p_L}{p_H}\frac{1-p_H}{1-p_L}\beta\right]} \equiv G_{H,w}(G_{L,w}) \tag{A.19}$$

As shown in figure 1, $G_{H,w} \geq G_{L,w}$ for $w \leq (1+r)p_H/p_L + \eta$.

The above analysis relies on the assumption that the optimal solution satisfied PC_H. Substituting for the equilibrium contract the PC_H reduces to

$$u_H = p_H R - (1+r) - (1-\beta)(1-p_H)G_{H,w} \geq 0 \Rightarrow G_{H,w} \leq \frac{p_H R - (1+r)}{(1-\beta)(1-p_H)} \tag{A.20}$$

where we note that the LHS is strictly decreasing in $G_{H,w}$. In the case we are analyzing, $G_{H,w} < (1+r)/\beta$. Hence, the PC_H is always satisfied so long as

$$\frac{p_H R - (1+r)}{(1-\beta)(1-p_H)} \geq \frac{1+r}{\beta}. \tag{A.21}$$

which is the same conditon as (condition).

Case 2: $G_{H,w} \in [\max(0, w-\eta), \min(w, \frac{1+r}{\beta})]$ **is binding.** In this case, either $G_{H,w}(G_{L,w}) \geq (1+r)/\beta$ or $G_{H,w}(G_{L,w}) \geq w$. If $G_{H,w} \geq (1+r)/\beta)$, then the above analysis applies, and the optimal contract is characterized by $\pi_{H,w} = 1$. If $G_{H,w} = w$, so long as, $p_H(R - R^B_{H,w}) - (1-p_H)G_{H,w} > 0$ holds (to be later verified), ICC_L must be binding. Accordingly, we find,

$$\pi_{H,w} = \frac{p_L(R - R^B_{L,w}) - (1-p_L)G_{L,w}}{p_L(R - R^B_{H,w}) - (1-p_L)G_{H,w}} \tag{A.22}$$

Bisogna verificare che ICC_H verificato

A.2 Proof of proposition 2

Given a candidate PE, the payoff of a borrower of type q and wealth w as a function of the equilibrium contract $\mathcal{C}^P_w = \{R^B_w, G_w, \pi_w)$ is,

$$u^{PE}_q = \pi_w\{Rp_q - (1+r)\frac{p_q}{p_m} + G_w\left[(1-p_m)\frac{p_q}{p_m}\beta - (1-p_q)\right]\} \tag{A.23}$$

[13]Imposing $\underline{\lambda}_G = 0 = \overline{\lambda}_G$ equation A.15 reduces to,

$$(\lambda_{ICC,H} + \pi_{H,w})\left[p_H\frac{dR^B_{H,w}}{dG_{H,w}} + (1-p_H)\right] = \lambda_{ICC,L}\left[p_L\frac{dR^B_{H,w}}{dG_{H,w}} + (1-p_L)\right] \tag{A.17}$$

so that $\lambda_{ICC,L} > 0$ whenever $\pi_{H,w} > 0$.

It can be seen immediately that, the payoff is increasing in π_w for both types. Hence, given a candidate PE such that the equilibrium contract implies π_w profitable deviations exist, which destroy the equilibrium. Hence, $\pi_w = 1$ must hold. Moreover, if

$$\beta\frac{(1-p_m)}{p_m} < \frac{1-p_H}{p_H} \tag{A.24}$$

both types of borrowers prefer less guarantees, so that $G_w = w - \eta$ must hold. Viceversa, if the reverse inequality holds, then safe borrowers prefer more guarantees, so that any pooling contract must be characterized by $G_w = \min(w, \frac{1+r}{\beta})$.

A.3 Incentives to disclose wealth

We characterized SE and PE under the assumption that borrowers are disclosing their wealth. The following result holds.

Lemma 2 (Wealth disclosure)**.** *In any equilibrium, borrowers always disclose their wealth.*

Proof.

Let us first analyze the incentives that safe borrowers have to disclose their wealth in a SE. In any SE, the ICC_L holds as a strict equality. That is,

$$[p_L(R-(1+r)-(1-\beta)(1-p_L)G_{L,w}] = \pi_H[p_L(R-R^H_{H,w})-(1-p_L)G_{H,w}]. \tag{A.25}$$

We note that the LHS of the above constraint is decreasing in $G_{L,w}$. Let $\mathcal{E}'$ be the set of entrepreneurs who are not disclosing their wealth in a candidate equilibrium, and $\overline{w}(\mathcal{E})$ the highest value of wealth of individual wealth of entrepreneurs in such set. Since $G_{L,w}$ is increasing in

w, the contract $\mathcal{C}_H = (\pi_H, R^B_H, G_H)$ offered to any borrower of type H who is not disclosing his wealth must satisfy,

$$[p_LR-(1-p_L)(1-\beta)\min(\max(\overline{w}(\mathcal{E}),0)-\eta,(1+r)/\beta] = \pi_H[p_L(R-R^B_H)-(1-p_L)G_H]; \tag{A.26}$$

Crucially, for a risky borrower with wealth, $w_1 < \eta + (1+r)/\beta$ the above constraint is satisfied as a strict inequality. Hence, borrowers of type H with the same level of wealth equal to w_1, have an incentive to disclose their wealth because in that case they can be offered a contract conditional on the wealth level, which needs to satisfy only the ICC_L for risky borrowers endowed with that level of wealth, that is

$$\pi_L[p_L(R-R^L_L)-(1-p_L)(\max(w_1-\eta),0) = \pi_{H,w_1}[p_L(R-R^B_{H,w_1})-(1-p_L)G_{H,w_1}]; \tag{A.27}$$

which is less strict than the above, and therefore allows either for a greater probability to be financed (for poor and safe), or a lower level of guarantees (for rich and safe). This directly implies that, given a SE in which safe borrowers with wealth w such that $w-\eta < (1+r)/\beta$ are not disclosing their wealth, lenders have the incentive to propose contracts that require safe borrowers to disclose their wealth, as by doing so they can make extra profits and surely attract borrowers.[14]

Let us now turn to the incentives of risky borrowers to disclose their wealth.

Let $\mu(w|e \in \mathcal{E}'$ be the equilibrium expected value of wealth for an entrepreneur who is not disclosing her wealth, with, $\mu(w|e \in \mathcal{E}') < \overline{w}(\mathcal{E}'))$. In equilibrium, lenders break even in expected terms, given the information available. Hence, for each borrower e with $e \in \mathcal{E}'$, the equilibrium contract satisfies,

$$p_H R^B_{L,\mu(w|e\in\mathcal{E}')} + (1-p_H)\beta G_L = 1+r \tag{A.28}$$

where, $G_L = \min(\max(\mu(w|e \in \mathcal{E}') - \eta, 0), \frac{R^B_{L,\mu(w|e\in\mathcal{E}')}}{\beta})$. It is then immediate to verify that – unless $\mu(w|e \in \mathcal{E}') = (1+r)/\beta$, if disclosing her wealth, the richest entrepreneur who is not disclosing it, would be better off by doing so, she will increase the level of expected guarantees she is offering the lenders, thereby reducing the cost of credit, which destroys the candidate equilibrium.

Equivalent arguments can be made for the case of a pooling equilibrium. $\square$

A.4 Proof of proposition 3

We analyze first the case of rich entrepreneurs and then that of poor ones.

a. Safe and rich. The payoff of a safe and rich borrower payoff is

$$p_H R - (1+r) - (1-\beta)(1-p_H)G_{H,w} \tag{A.29}$$

If such a borrower could apply for contract, $\mathcal{C}^P$ is payoff would be,

$$p_H R - (1+r)\frac{p_H}{p_m} - G_w(1-p_H) + \beta(1-p_m)\frac{p_H}{p_m}G_{ww} \tag{A.30}$$

A strictly profitable deviation for lenders would then exists if and only if the following inequality holds,

$$p_H R - (1+r) - (1-\beta)(1-p_H)G_{H,w} < p_H R - (1+r)\frac{p_H}{p_m} - G_w(1-p_H)[\beta\frac{1-p_m}{1-p_H}\frac{p_H}{p_m} - 1], \tag{A.31}$$

which rearranging reduces to,

$$(1+r)\frac{p_H - p_m}{p_m} < (1-\beta)(1-p_H)G_{H,w} + G_w p_H \left[\frac{1-p_H}{p_H} - \beta\frac{1-p_m}{p_m}\right]. \tag{A.32}$$

Let ω_1 be the critical level of entrepreneurial wealth such that the entrepreneur is indifferent between the SE contract and $\mathcal{C}^P$, that is

$$\omega_1 : (1+r)\frac{p_H - p_m}{p_m} = (1-\beta)(1-p_H)(G_{H,\omega} + G_\omega p_H \left[\frac{1-p_H}{p_H} - \beta\frac{1-p_m}{p_m}\right]. \tag{A.33}$$

[14]Note that, the above argument does not hold for safe borrowers endowed with levels of wealth such that $w - \eta \geq (1+r)/\beta$. However, whether these borrowers disclose their wealth or not does not make a difference in terms of the equilibrium outcome.

It is immediate to verify that if

$$\beta\frac{1-p_m}{p_m} < \frac{1-p_H}{p_H} \tag{A.34}$$

the RHS of condition (A.32) is increasing in w. Therefore,

Case i. If $\overline{w} < \omega_1$ then all rich and safe entrepreneurs separate form risky ones, i.e. the SE prevails;

Case ii.] If $\widehat{w} > \omega_1$ then all rich and safe borrowers pool i.e. the PE prevails;

Case iii. Finally if $\omega_1 \in [\widehat{w}, \overline{w}]$, then all borrowers with wealth $w < \omega$ separate, while all borrowers with $w > \omega_1$ pool.

Viceversa, if

$$\beta\frac{(1-p_m)}{p_m} > \frac{1-p_H}{p_H} \tag{A.35}$$

holds, $G_w = w$, so that $G_{H,w} < G_w$. But, then, all safe and rich borrowers always strictly prefer to separate from risky ones, so that the equilibrium will be separating.

b. Safe and poor. The equilibrium payoff for a safe and poor entrepreneur is

$$\pi_{H,w}[p_H R - (1+r) - (1-p_H)(1-\beta)G_{H,w}] \tag{A.36}$$

If the entrepreneur could apply for $\mathcal{C}^P$, the payoff would be,

$$p_H R - (1+r)\frac{p_H}{p_m} - (1-p_H)G_w\left[1 - \frac{1-p_m}{p_m}\frac{p_H}{1-p_H}\beta\right]. \tag{A.37}$$

Therefore there exist a profitable deviation for lenders if,

$$\pi_{H,w}[p_H R - (1+r) - (1-p_H)(1-\beta)G_{H,w}] < p_H R - (1+r)\frac{p_H}{p_m} - (1-p_H)G_w\left[1 - \frac{1-p_m}{p_m}\frac{p_H}{1-p_H}\beta\right]. \tag{A.38}$$

Consider first the case in which,

$$\beta\frac{1-p_m}{p_m} < \frac{1-p_H}{p_H}. \tag{A.39}$$

so that $G_w = w - \eta$. Rearranging terms, the above condition can be written as,

$$\pi_{H,w}[p_H R-(1+r)-(1-p_H)(1-\beta)G_{H,w}] < p_H R-(1+r)\frac{p_H}{p_m} - G_w p_H\left[\frac{1-p_H}{p_H} - \beta\frac{1-p_m}{p_m}\right]. \tag{A.40}$$

Define, ω_2 the value of wealth such that

$$\pi_{H,\omega_2}[p_H R-(1+r)-(1-p_H)(1-\beta)G_{H,\omega_2}] = p_H R-(1+r)\frac{p_H}{p_m} - G_{\omega_2} p_H\left[\frac{1-p_H}{p_H} - \beta\frac{1-p_m}{p_m}\right] \tag{A.41}$$

It is immediate to verify tat the RHS of condition (A.40) is decreasing in w while the RHS is increasing in w.[15] Accordingly,

[15] Poor and safe agents are wealth constrained in a SE, so that increasing w increases their SE payoff.

i. If $\omega_2 > \widehat{w}$, then all poor and safe entrepreneurs pool with risky ones;

ii. If $\omega_2 < \underline{w}$ then all poor and safe separate;

iii If $\omega_2 \in [\underline{w}, \widehat{w}]$ then poor and safe entrepreneurs with $w \in [\underline{w}, \omega]$ pool while the rest separate.

Consider now the case in which

$$\beta \frac{1-p_m}{p_m} > \frac{1-p_H}{p_H} \tag{A.42}$$

so that $G_w = w = G_{H,w}$ holds. In this case, the above condition reduces to:

$$\pi_{H,w} \frac{p_H R - (1+r) - (1-p_H)(1-\beta)G_{H,w}}{p_H R - (1+r)\frac{p_H}{p_m} - (1-p_H)G_{H,w}\left[1 - \frac{1-p_m}{p_m}\frac{p_H}{1-p_H}\beta\right]} < 1 \tag{A.43}$$

Substituting for the equilibrium value of $\pi_{H,w}$ we get,

$$\frac{p_L R - (1+r) - (1-p_L)(1-\beta)(w-\eta)]}{[p_L R - \frac{p_L}{p_H}(1+r) - (1-p_L)\left[1 - \frac{p_L}{p_H}\frac{1-p_H}{1-p_L}\beta\right] w} \frac{p_H R - (1+r) - (1-p_H)(1-\beta)w}{p_H R - (1+r)\frac{p_H}{p_m} - (1-p_H)w\left[1 - \frac{1-p_m}{p_m}\frac{p_H}{1-p_H}\beta\right]} < 1 \tag{A.44}$$

It is immediate to verify that both terms on the LHS are decreasing with $G_{H,G}$ and it is greater than one for $\pi_{H,w} > 1$. Hence, all entrepreneurs with $w > \omega_2$ separate while the others pool. Finally, note also that

$$\pi_{H,w}[p_H R - (1+r) - (1-p_H)(1-\beta)G_{H,w}] > \pi_{H,w}[p_L R - (1+r) - (1-p_L)(1-\beta)G_{H,w}] \tag{A.45}$$

so that when type H prefer pooling type L also prefer pooling. $\square$

B Selection

In order to account for the possibility of sample selection, we model access to credit as a selection process based on the decision tree portrayed in figure 7. In the first stage, an entrepreneur decides if additional credit is needed. If so, in the second step, the potential borrower decides whether to apply or not. If applying –stage 3 – the entrepreneur can be evaluated as credit worth or not by the bank. Finally, stage 4 – among credit worth entrepreneurs some will be always financed and some will be rationed with some positive probability.

The key issue is that rationing, decision to post collateral, and cost of credit are observable only for credit worth firms applying for credit. Credit worthiness is modeled as follows,

$$CW_i = \begin{cases} = 1 \, if & \delta W_i + \xi_i > 0 \\ = 0 \, if & \delta W_i + \xi_i \leq 0 \end{cases} \tag{B.1}$$

where $CW_i = 1$ for credit worth entrepreneurs, W_i is a set of publicly known variables that determine the firm's credit worthiness; δ is a vector of parameters; and ξ_i is the error term.

For credit worth firms, the cost of credit is determined according to the following empirical specification equivalent to model (B.2), which we estimated not accounting for selection,

$$R_i^B = \beta_1 X_i + \beta_2 \eta_i + \beta_3 C_i + \beta_4 C_i \eta_i + v_i. \tag{B.2}$$

where u_i is the error term.

We assume $(u_i, \xi_i) \sim N(0, 0, \sigma_u, \sigma_v, \rho)$, where ρ is the correlation coefficient. Furthermore, $\sigma_v = 1$, is the normalization used to identify a probit model. Taking expectation we obtain the regression model for R^B:

$$E(R_i^B \mid CW_i = 1) = E(R_i^B \mid \xi_i > -\delta W_i) = \beta_1 X_i + \beta_2 \eta_i + \beta_3 C_i + \beta_4 C_i \eta_i + \rho\sigma \frac{\phi(\alpha F_i)}{\Phi(\alpha F_i)} \tag{B.3}$$

Note that, as we are modeling the cost of credit for those entrepreneurs who are selected to be credit worth, $\lambda_i(-\delta W_i) = \frac{\phi(\delta W_i)}{\Phi(\delta W_i)}$ represents the non selection hazard, or the inverse Mill's ratio.

According to the above, the econometric specification for the cost of credit – that accouts for selection – is given by

$$R_i^B = E(R_i^B \mid CW_i = 1) + e_i \tag{B.4}$$

where $e_i \sim iid(0, \sigma_e)$.

To obtain estimates for the vector $\lambda_i(\cdot)$, we run a bivariate probit with selection for CW_i. Firms are considered creditworth by the bank provided they apply for a loan. Hence, we estimate a two equations model in which the first equation represents the firm's decision to apply, while the second equation estimates the probability the firm is creditworth. From the linear prediction of this second equation we obtain the inverse Mill's ratio $\lambda_i(\cdot)$ to be used into the equation for the cost of credit.

We employ maximum-likelihood to estimate the following bivariate probit model with selection:

$$A_i = \alpha F_i + \epsilon_i \tag{B.5}$$

$$CW_i = \delta W_i + \xi_i \tag{B.6}$$

where A_i is a dichotomous variable equal 1 if a firm applied for a loan; F_i are a set of determinants of firm's decision to apply; and ϵ_i and ξ_i are the correlated error terms.

Equation B.5 is the selection equation, while equation B.6 estimates firms' probability to be credit worth. The ML estimation results are not reported and can be provided on request. We reject the null hypotheses of independence of the two equations.

Loan rate equation with sample selection In table 6 we report the results of the estimation of the cost of credit including among the regressor the non selection hazard of being credit worthy given a loan application. The estimation with sample selection confirms the findings about the sorting role of collateral.[16]

[16]Exemption is not significant once we consider sample selection.

Probability to access credit and sample selection. The financing process depicted in Fig.7 shows that a borrower can be always financed or not conditional to the fact that she is credit worth. Once the bank selects the credit worth borrowers, depending on their type, it offers contracts that may involve a probability to be financed less than one. This selection is taken into account estimating the following bivariate probit with selection:

$$
\begin{aligned}
CW_i &= \delta W_i + \xi_i && \text{(B.7)} \\
\pi_i &= \alpha_1 Z_i + \alpha_2 \eta_i + \alpha_3 C_i + \alpha_4 C_i \eta_i + \alpha_5 R_i^B + u_i && \text{(B.8)}
\end{aligned}
$$

where the first equation has been already defined, and the second equation is the equivalent of model (38), which we estimated not accounting for selection.

Simultaneous model with sample selection. Sample selection is also considered in the estimation of the joint determination of the cost of credit and guarantees by augmenting by the inverse Mills ratio the loan rate equation. Results are reported in tables 8 and 9. The marginal effect of guarantees on the cost of credit is substantially unchanged (-0.475 vs -0.468), both in the low exemption (-0.377 vs -0.346) and in high exemption subsamples (-0.9314 vs -0.9210).

Figure 1: Separating equilibrium: Real guarantees by borrower's type

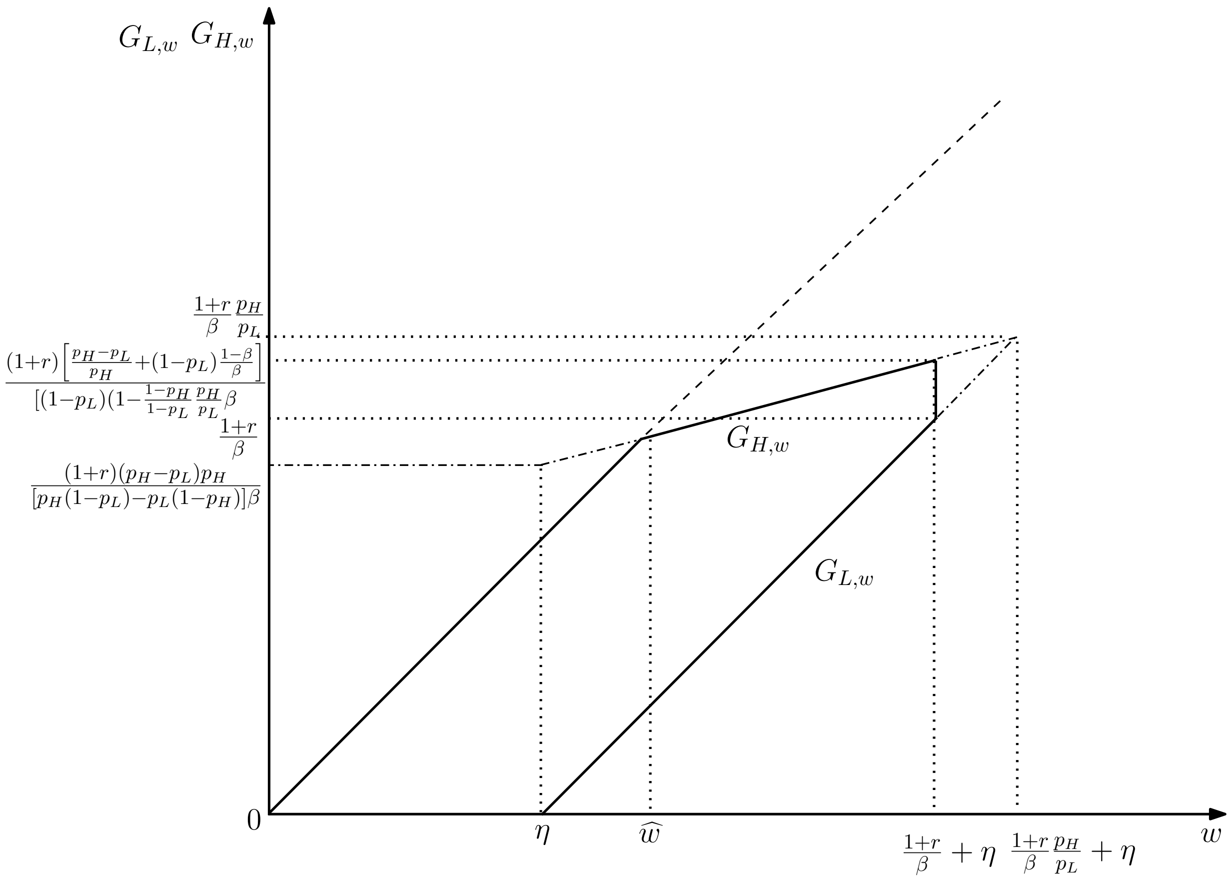

Figure 2: Separating equilibrium: Probability to be financed by borrower's type

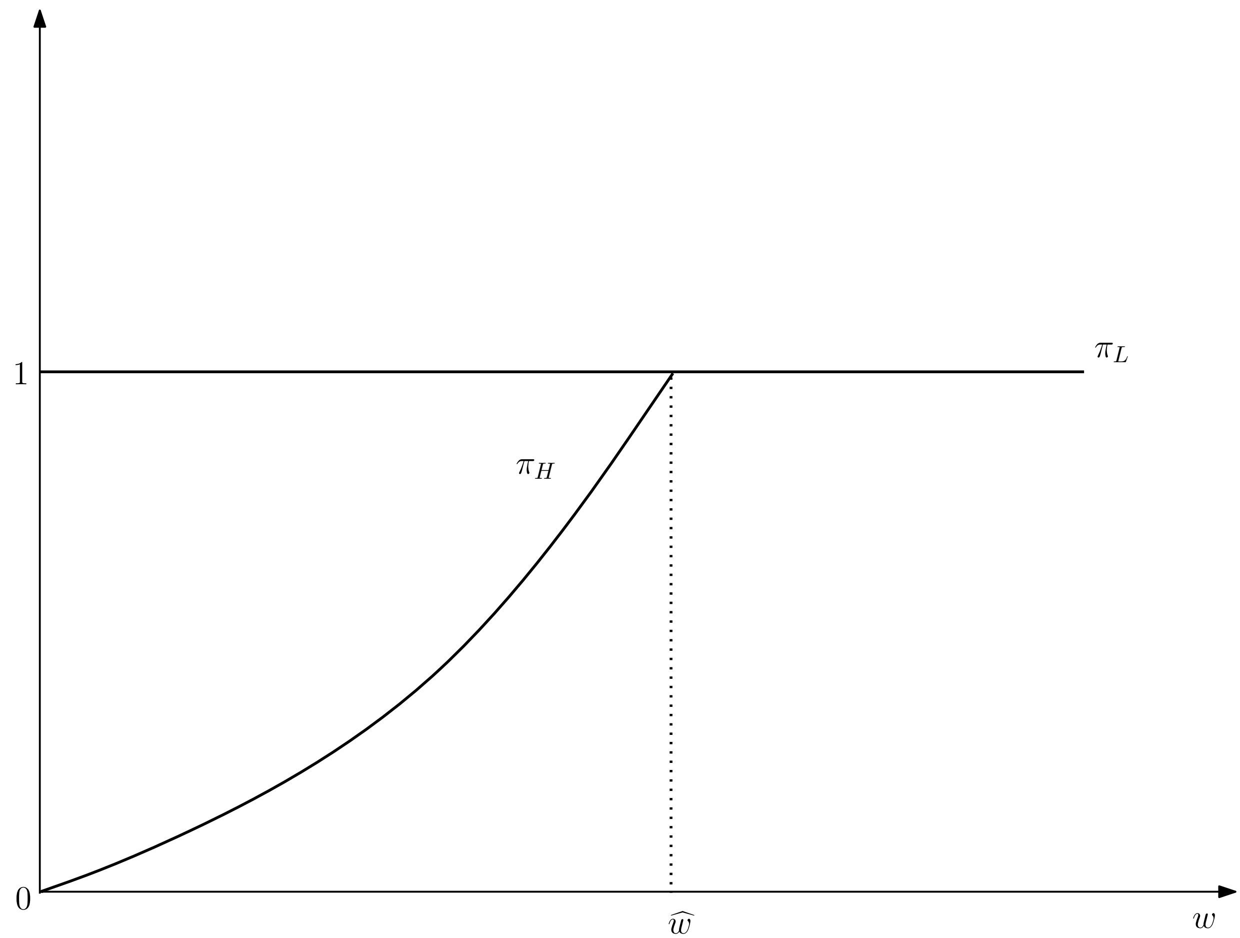

Figure 3: Separating equilibrium: Cost of credit by borrower's type

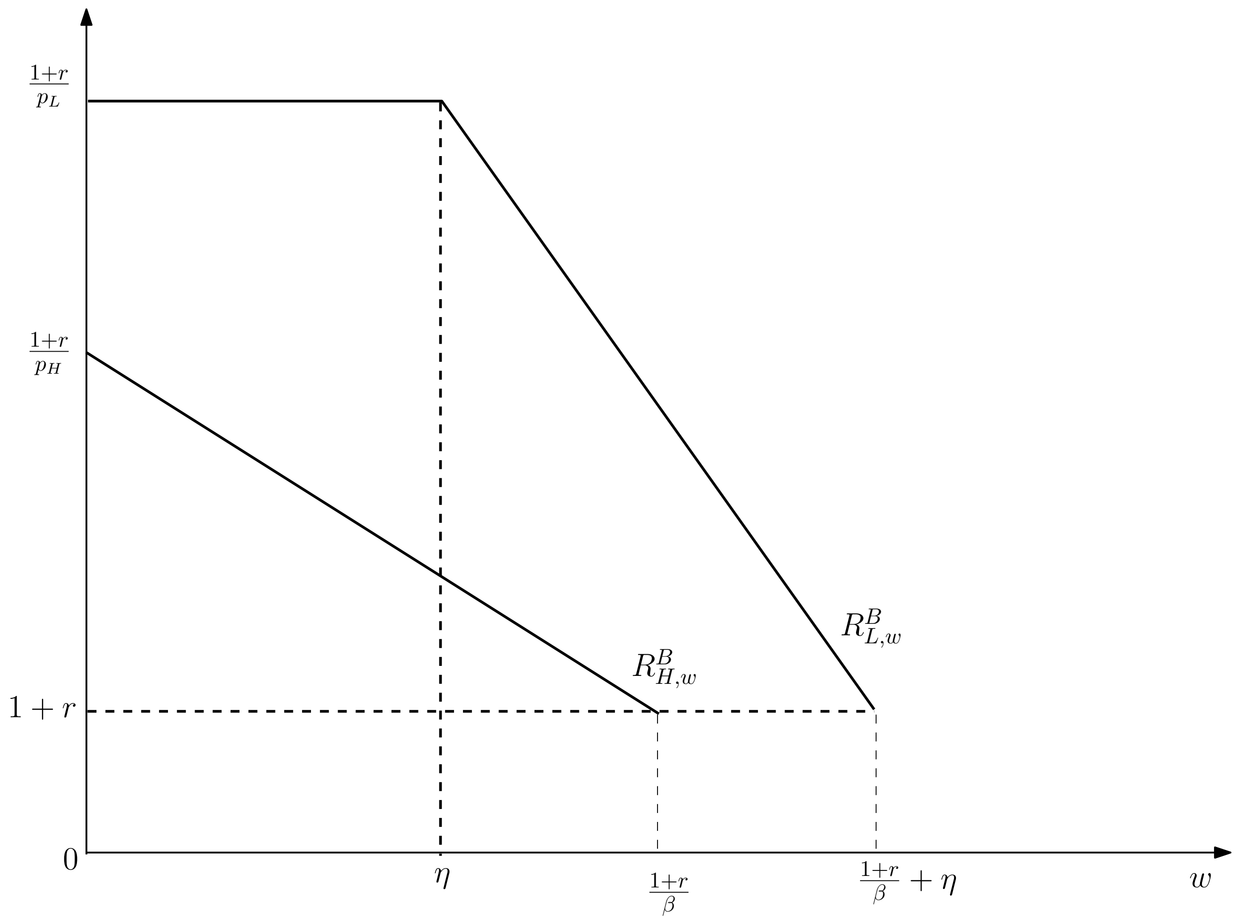

Figure 4: Separating equilibrium: Exemption and levels of guarantees

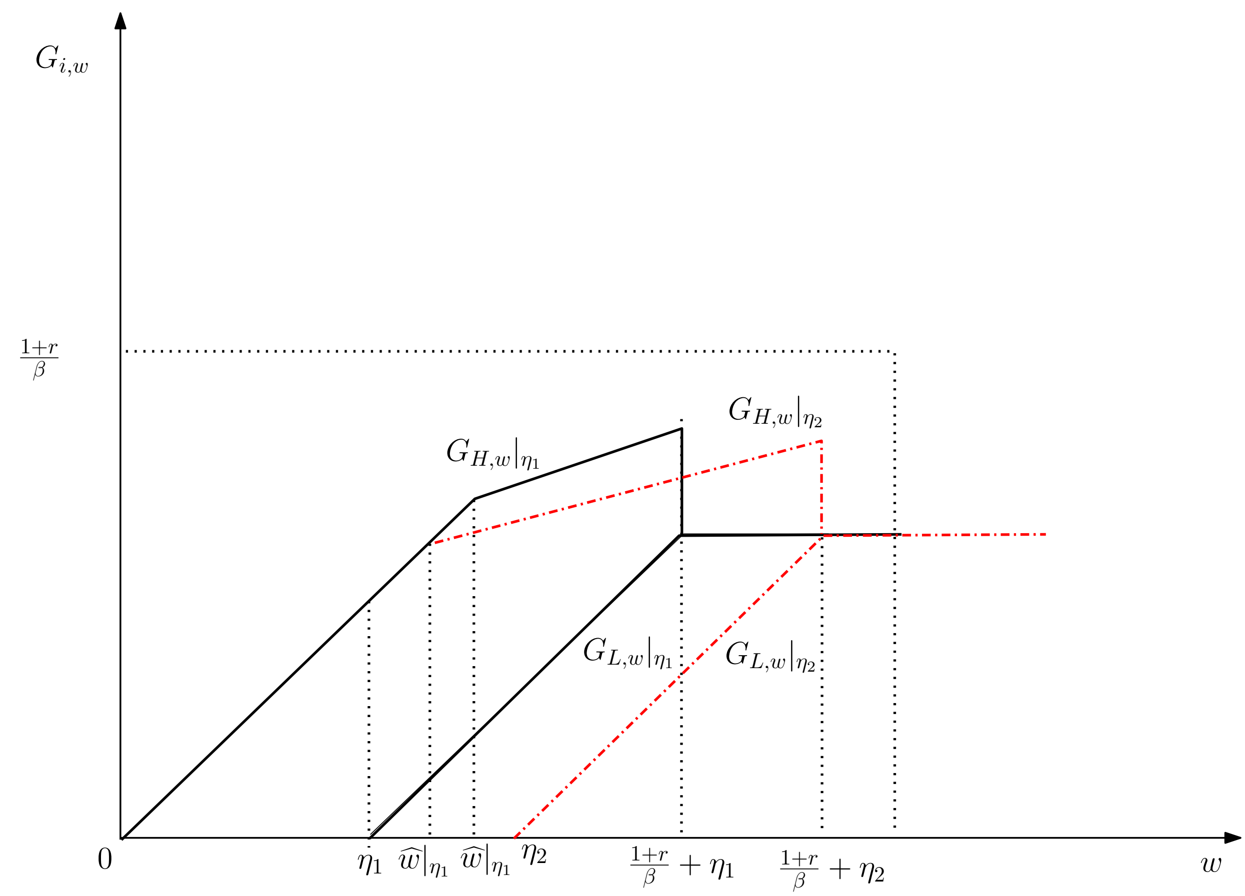

Figure 5: Separating equilibrium: Exemption and levels of guarantees

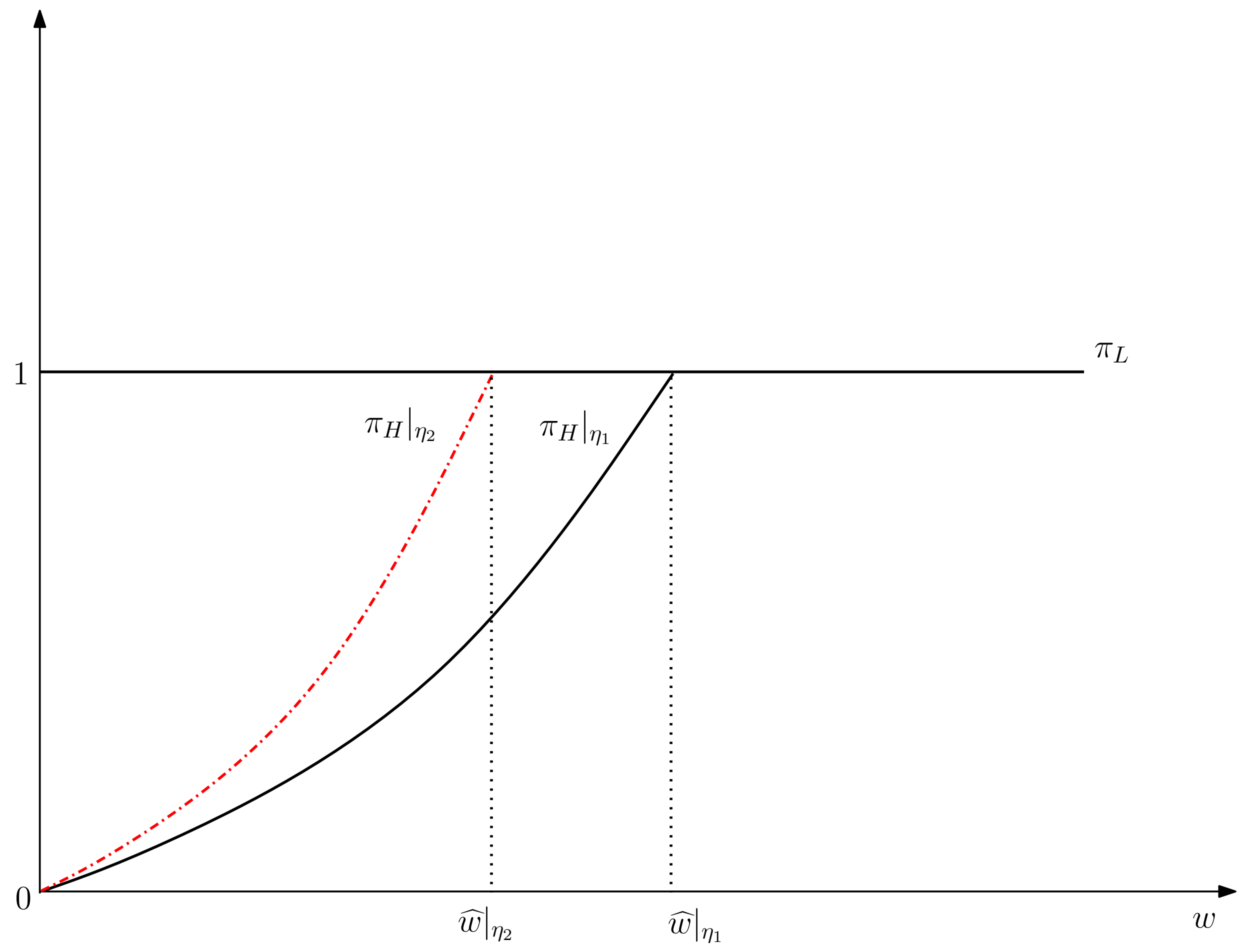

Figure 6: Separating equilibrium: Exemption and cost of credit

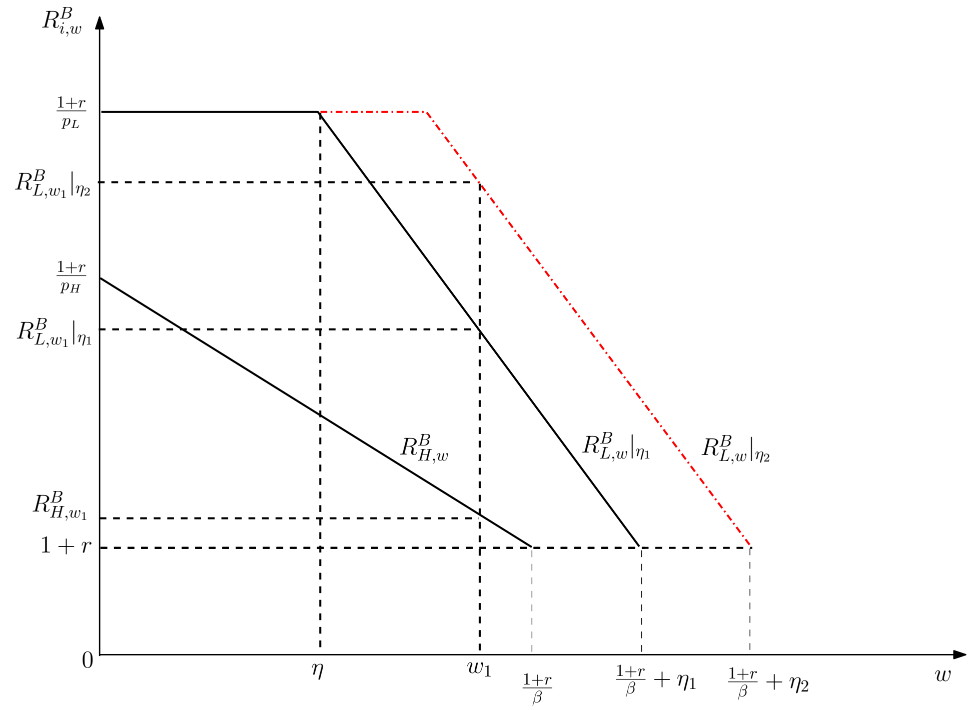

Figure 7: Financing process

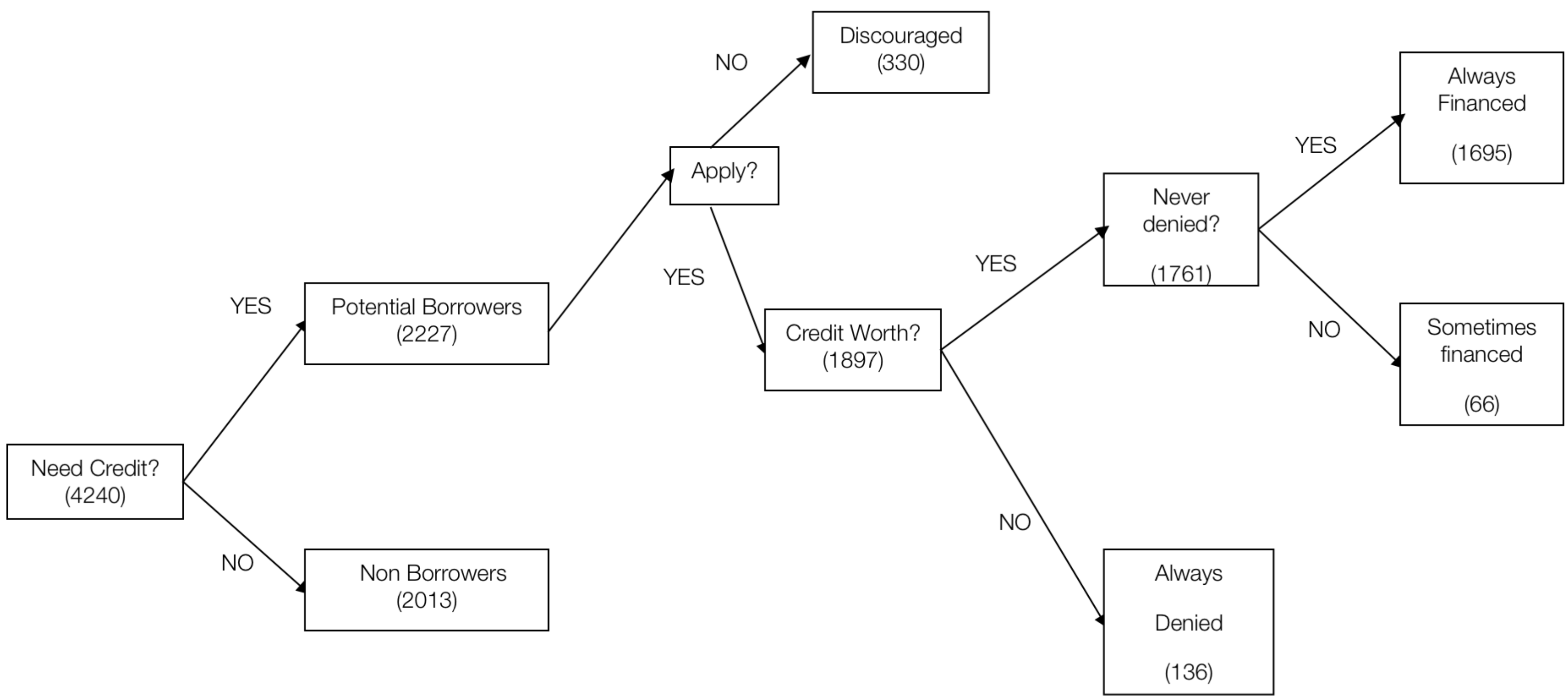

Table 1: Loan rate and fraction of rationed firms by exemption, collateral and assets above and below median

		Any asset			Low asset			High asset		
		WS	LEX	HEX	WS	LEX	HEX	WS	LEX	HEX
C=1	Loan rate	5.49	5.53	5.37	6.06	6.13	5.89	5.08	5.12	4.96
C=0	Loan rate	6.19	6.06	6.57	6.95	6.8	7.42	5.04	5.02	5.12
C=1	Rationed firms	0.045	0.047	0.038	0.081	0.086	0.067	0.019	0.020	0.015
C=0	Rationed firms	0.030	0.019	0.065	0.037	0.023	0.077	0.019	0.012	0.044
Any firm	Loan rate	5.81	5.78	5.9	6.55	6.50	6.70	5.06	5.08	5.02
Any firm	Fract. of rationed firms	0.038	0.034	0.050	0.057	0.051	0.072	0.019	0.017	0.025

WS: Whole sample; LEX: Low exemption (below median); HEX: High exemption (above median).

Low asset: asset below median value; High asset: asset above the median value.

Table 2: Estimation results : Cost of credit

Variable	Coefficient	(Std. Err.)
Homestead and personal property exemption ($)	0.0000000826*	(0.000000048)
Dummy=1 if firm posted collateral	-0.31451***	(0.06500)
Interaction term between dummy collateral and exemption levels	-0.000000245***	(0.000000065)
Dummy=1 if Fixed interest rate	0.94483***	(0.05912)
Dummy=1 if lending was a new line of credit	-0.16645**	(0.07423)
Dummy=1 if lending was a mortgage	0.16972*	(0.09571)
Banking market concentration: Dummy=1 if Herfindahl index> 1800	0.23343***	(0.05356)
Dummy=1 if firm's Credit score is top 25%	-0.11390**	(0.05567)
Owner managing experience (n. of years)	-0.01521***	(0.00268)
Dummy=1 if Owner is black	0.89743***	(0.22360)
Dummy=1 if Owner belongs to an ethnic minority other than black	0.83107***	(0.10711)
Dummy=1 if Owner is female	-0.03713	(0.07517)
Dummy=1 if firm is family owned	-0.27228***	(0.06694)
Number of credit applications	0.01481	(0.01067)
Years of firm bank relationship	-0.00875***	(0.00258)
Distance of firm from bank (miles)	0.00115***	(0.00039)
Natural log of total sales	-0.32263***	(0.01659)
Debt on total asset	0.02518***	(0.00703)
Total Asset - thousands of $	-0.00001**	(0.0000025)
Intercept	10.73249***	(0.25748)
N	1691	
R^2	0.17366	
$F_{(19,1671)}$	93.29868	

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 3: Probability to access credit

Variable	**Coefficient**	**(Std. Err.)**
Homestead and personal property exemption ($)	-0.0000000097***	(0.0000000019)
Dummy=1 if firm posted collateral	-0.01407***	(0.00296)
Interest rate on loan (%)	-0.00236***	(0.00042)
Interaction term between dummy collateral and exemption levels	0.0000000096***	(0.0000000025)
Loan original maturity (n. of months)	-0.00005***	(0.00002)
Amount granted over total applied	0.03178***	(0.00555)
Years of firm bank relationship	0.00056***	(0.00013)
Dummy=1 if firm's Credit score is top 25%	0.01182***	(0.00257)
Dummy=1 if firm has delinquency records	-0.00721***	(0.00101)
Debts on equity	-0.00019**	(0.00008)
Dummy=1 if firm has limited liability	0.01099***	(0.00363)
Total Asset - thousands of $	0.00000017***	(0.00000031)
N	1654	
$\chi^2_{(12)}$	361.94318	

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 4: Simultaneous model: dependent variable R^b

	Whole sample	Low exemption	High exemption
Collateral	-0.4686***	-0.3463***	-0.9314***
	(.1124)	(.1267)	(.3478)
Exemption	0.0000000715***	-	-
	(0.0000000335)	-	-
N. obs	1596	1199	397
R^2	0.16	0.14	0.24
F	81.12	60.02	37.80

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Two stages probit least square estimation (Maddala, Lee, 1976; Keshk, 2003)
List of controls: Dummy Fixed interest rate; Dummy new line of credit; Dummy mortgage; Dummy Credit score top 25%; Number of credit applications; Total sales; Banking market concentration; Owner managing experience (n. of years); Dummy female owner; Dummy black owner; Dummy other minority owner; Years of firm bank relationship; Distance of firm from bank; Debt on total asset; Dummy family owned, Firm Asset.

Table 5: Simultaneous model: dependent variable C

	Whole sample	Low exemption	High exemption
Loan Rate	-0.2336***	-0.2409***	-0.1855***
	(.0170)	(.0209)	(.0279)
N. obs	1596	1119	397
LR chi2 (9)	636.02	458.22	195.30

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Two stages probit least square estimation (Maddala, Lee, 1976; Keshk, 2003)
List of controls: Dummy new line of credit; Dummy Credit score top 25%; Loan Maturity; Amount granted over applied; Banking market concentration; Dummy limited liability; Dummy rationing sometimes; Dummy female owner; Years of firm bank relationship; Dummy family owned.

Table 6: Estimation results : Cost of credit with selection

Variable	**Coefficient**	**(Std. Err.)**
Homestead and personal property exemption ($)	0.00000007	(0.00000005)
Dummy=1 if firm posted collateral	-0.32885***	(0.06522)
Interaction term between dummy collateral and exemption levels	-0.000000232***	(0.000000065)
Dummy=1 if Fixed interest rate	0.93518***	(0.05925)
Dummy=1 if lending was a new line of credit	-0.15930**	(0.07426)
Dummy=1 if lending was a mortgage	0.19223**	(0.09589)
Banking market concentration: Dummy=1 if Herfindahl index> 1800	0.23301***	(0.05364)
Dummy=1 if firm's Credit score is top 25%	0.06640	(0.06296)
Owner managing experience (n. of years)	-0.01368***	(0.00271)
Dummy=1 if Owner is black	0.89639***	(0.22334)
Dummy=1 if Owner belongs to an ethnic minority other than black	0.84714***	(0.10790)
Dummy=1 if Owner is female	-0.05663	(0.07532)
Dummy=1 if firm is family owned	-0.35604***	(0.06819)
Number of credit applications	0.01173	(0.01067)
Years of firm bank relationship	-0.00884***	(0.00258)
Distance of firm from bank (miles)	0.00127***	(0.00039)
Natural log of total sales	-0.28923***	(0.01740)
Debt on total asset	0.01336*	(0.00728)
Total Asset - thousands of $	-0.00001**	(0.00000)
Non selection hazard from Creditworth — Apply	1.30760***	(0.21305)
Intercept	9.99782***	(0.28273)

N	1683
R^2	0.17734
$F_{(20,1683)}$	90.49614

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 7: Marginal effects for probability to be financed with selection (selection equation: prob. to apply for a loan)

Variable	dy/dx	Std. Err.
Homestead and personal property exemption	-0.0000000086***	0.0000000
Dummy=1 if firm posted collateral	-0.0136461000***	0.00253
Interest rate on loan (%)	-0.0021316000***	0.00033
Interaction term between dummy collateral and exemption levels	0.0000000087***	0.0000000
Loan original maturity (n. of months)	-0.0000430000***	0.00001
Amount granted over total applied	0.0387604000***	0.00479
Years of firm bank relationship	0.0004486000***	0.00011
Dummy=1 if firm's Credit score is top 25%	0.0093832000***	0.00206
Dummy=1 if firm has delinquency records	-0.0052853000***	0.00081
Debts on equity	-0.0001288000**	0.00006
Dummy=1 if firm has limited liability	0.0057505000**	0.00265
Total Asset - thousands of $	0.0000011500***	0.0000000
N	1776	
Censored observations	130	
Uncensored observations	1646	
ρ	-0.676	
LR test of indep. eqns. ($\rho = 0$)	$\chi^2(1) = 11.99$	$Prob > \chi^2 = 0.0005$

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 8: Simultaneous model with selection: dependent variable R^b

	Whole sample	Low exemption	High exemption
Guarantees	-0.4758***	-0.3770***	-0.9210***
	(0.1230)	(0.1308)	(.3544)
Exemption	-0.0000000801***	-	-
	(0.0000000335)	-	-
Inverse Mills ratio	1.4074***	1.7717***	-0.1424
	(0.2125)	(0.2585)	(0.5644)
N. obs	1589	1193	396
R^2	0.16	0.15	0.24
F	79.17	58.98	35.98

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Two stages probit least square estimation (Maddala, Lee, 1976; Keshk, 2003)
List of controls: Dummy Fixed interest rate; Dummy new line of credit; Dummy mortgage; Dummy Credit score top 25%; Number of credit applications; Total sales; Banking market concentration; Owner managing experience (n. of years); Dummy female owner; Dummy black owner; Dummy other minority owner; Years of firm bank relationship; Distance of firm from bank; Debt on total asset; Dummy family owned, Firm Asset.

Table 9: Simultaneous model with selection: dependent variable G

	Whole sample	Low exemption	High exemption
Loan Rate	-0.2247***	-0.2155***	-0.1973***
	(.0166)	(.0201)	(.0282)
N. obs	1589	1193	396
LR $\chi^2(9)$	624.79	432.60	199.99

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Two stages probit least square estimation (Maddala, Lee, 1976; Keshk, 2003)
List of controls: Dummy new line of credit; Dummy Credit score top 25%; Loan Maturity; Amount granted over applied; Banking market concentration; Dummy limited liability; Dummy rationing sometimes; Dummy female owner; Years of firm bank relationship; Dummy family owned.