

## Mathematics (MATH)

School of Mathematics and Statistics  
Faculty of Science

### Note

- Consult the regulations concerning Deferred Examinations in the Academic Regulations section of this Calendar.
- See also the course listings under Statistics (STAT) in this Calendar.

### Prerequisites for first-year Calculus Courses

Courses included:

MATH 1002, MATH 1004, MATH 1007, MATH 1009

Pre-university Calculus:

Ontario Grade 12 Mathematics: Advanced Functions and Introductory Calculus, or an OAC in Calculus, or MATH 0007, or equivalent.

Students must successfully complete one of: Grade 12 Mathematics: Advanced Functions and Introductory Calculus, or an OAC in Calculus, or MATH 0005 and MATH 0007, or equivalent, prior to taking a 1000-level Calculus course.

For students in any program offered by the School of Mathematics and Statistics, the above requirement of MATH 0005 and MATH 0007 is in addition to the minimum 15.0 credits in General programs, or 20.0 credits in Honours programs.

### Prerequisites for most first-year Algebra Courses

Courses included:

MATH 1102, MATH 1104, MATH 1107

Pre-university Algebra:

Ontario Grade 12 Mathematics: Geometry and Discrete Mathematics, or an OAC in Algebra and Geometry, or MATH 0107, or equivalent.

**Note:** an OAC in Finite Mathematics is **not** an equivalent.

Students must successfully complete one of: Grade 12 Geometry and Discrete Mathematics, or an OAC in Algebra and Geometry, or MATH 0107, or equivalent, prior to taking any one of the three 1000-level algebra courses listed above.

For students in any program offered by the School of Mathematics and Statistics, the above requirement of MATH 0107 is in addition to the minimum 15.0 credits in General programs, or 20.0 credits in Honours programs.

MATH 0005 [0.5 credit]

### Pre-calculus: Functions and Graphs

Review of algebraic manipulations. Polynomials: the remainder theorem, and the factor theorem; graphing. Real and Complex roots. Absolute values. Inequalities. Functions, including composition of functions, and Inverse functions. Logarithmic and exponential functions.

Not available for degree credit for students who have successfully completed: Grade 12 University Preparation Mathematics – Advanced Functions and Introductory Calculus (MCB4U), or an equivalent High School functions course.

Prerequisite: Grade 11 Functions and Relations (University Preparation), or Grade 11 Functions (University/College Preparation), or equivalent.

Lectures three hours a week, tutorial one hour a week.

MATH 0007 [0.5 credit] (formerly 69.007\*)

### Introductory Calculus

Limits and continuity. Differentiation. Logarithmic and exponential functions and their derivatives; curve sketching; applied problems in maxima and minima, and related rates. Not available for degree credit for students who have

successfully completed Ontario Grade 12 Mathematics (Advanced Functions and Introductory Calculus) or an OAC in Calculus, or an equivalent High School Calculus course.

Precludes additional credit for BUSI 1703 or BUSI 1705.

Prerequisite: MATH 0005; or Grade 12 Mathematics (Advanced/Academic Level), taken before September 2002, or equivalent.

Lectures three hours a week, tutorial one hour a week.

MATH 0107 [0.5 credit] (formerly 69.017\*)

### Algebra and Geometry

Vectors in the plane and in 3-space. Linear combinations and linear independence. Equations of lines and planes in space. Solution of systems of linear equations. Proofs by induction. Binomial Theorem. Logic.

Not available for degree credit for students who have successfully completed Ontario Grade 12 Geometry and Discrete Mathematics, or an OAC in Algebra and Geometry, or an equivalent high school Algebra course.

Prerequisite: Ontario Grade 11 Functions and Relations (University Preparation), or Grade 11 Functions (University/College Preparation); or Ontario Grade 12 Mathematics (Advanced/Academic Level), taken before September 2002; or equivalent.

Lectures three hours a week, tutorial one hour a week.

MATH 1002 [1.0 credit] (formerly 69.102)

### Calculus

Limits, differentiation, the definite integral, elementary functions, trigonometric functions (identities, limits, derivatives), techniques of integration, parametric equations, polar coordinates. Improper integrals, L'Hôpital's rules, sequences and series, Taylor's formulae. Introduction to differential equations.

Strongly recommended for students intending to specialize in mathematics, statistics, physics, or related areas.

Precludes additional credit for MATH 1004, MATH 1005, MATH 1007, MATH 1009, MATH 2007, and for MATH 2001, MATH 2002.

Prerequisites: i) a pre-university calculus course with a grade of 65 percent or better; and ii) Ontario Grade 12 Mathematics: Geometry and Discrete Mathematics, or an OAC in Algebra and Geometry, or MATH 0107, or permission of the School. (See prerequisites for first-year calculus courses and for first-year algebra courses at the beginning of this section.)

Lectures three hours a week and one hour tutorial.

**Note:** Although the main prerequisite for MATH 1002 is a grade of 65 percent or better in a pre-university calculus course, past experience indicates that students with less than 75 percent in their prerequisite calculus have only a small chance of success in MATH 1002.

MATH 1004 [0.5 credit] (formerly 69.104\*)

### Calculus for Engineering Students

Definite and indefinite integrals; numerical approximation. Transcendental functions (trigonometric and inverse trigonometric, logarithm and exponential), their derivatives and integrals. Applications: area, volume, average value. Further techniques of integration: integration by parts, partial fractions, and substitutions.

Restricted to students in the Faculty of Engineering, or in B.Sc. (Honours) in Applied Physics.

Precludes additional credit for MATH 1002, MATH 1007, MATH 1009.

Prerequisite: Ontario Grade 12 Mathematics: Advanced Functions and Introductory Calculus; or an OAC in Calculus, or MATH 0007 (or equivalent).

Lectures three hours a week and one hour tutorial.

MATH 1005 [0.5 credit] (formerly 69.105\*)

### Differential Equations and Infinite Series for Engineering Students

First-order differential equations. Linear differential equations with constant coefficients; undetermined coefficients; annihilator operators. Variation of parameters. Euler-Cauchy differential equation. Indeterminate forms. Sequences and

series; convergence tests; estimation of sums. Power series, Taylor series, remainders. Use of power series to solve differential equations.

Restricted to students in the Faculty of Engineering, or in B.Sc.(Honours) in Applied Physics.

Precludes additional credit for MATH 1002, MATH 2001, MATH 2002, MATH 2007, MATH 2454, MATH 2404 and MATH 2600.

Prerequisites: i) MATH 1004, or a grade of C- or better in MATH 1007; and ii) either: successful completion of an OAC in Algebra and Geometry, or MATH 0107 [prior to Fall 2003]; or: concurrent registration in MATH 1104 (or MATH 1107); or permission of the School.

Lectures three hours a week, tutorial one hour a week.

MATH 1007 [0.5 credit] (formerly 69.107\*)

### Elementary Calculus I

Definite and indefinite integrals, differentiation and integration of the elementary functions and transcendental functions, techniques and applications of integration.

Precludes additional credit for MATH 1002, MATH 1004, MATH 1009.

Prerequisite: Ontario Grade 12 Mathematics: Advanced Functions and Introductory Calculus; or an OAC in Calculus, or MATH 0007; or equivalent.

Lectures three hours a week and one hour tutorial.

MATH 1009 [0.5 credit] (formerly 69.109\*)

### Calculus: with Applications to Business and Economics

Definite and indefinite integrals; integration of the elementary functions; some techniques of integration. Functions of several variables; partial differentiation; constrained optimization. Selected applications in business and economics.

Precludes additional credit for MATH 1002, MATH 1004, and MATH 1007.

Prerequisite: Ontario Grade 12 Mathematics: Advanced Functions and Introductory Calculus; or an OAC in Calculus; or MATH 0007; or equivalent.

Lectures three hours a week and one hour tutorial.

MATH 1102 [1.0 credit] (formerly 69.112)

### Algebra

Fields, complex numbers, vector algebra and geometry in 2 and 3 dimensions, matrix algebra, linear dependence, bases, linear transformations, bilinear and quadratic forms, inner products, eigenvalues, principal axis theorem. Strongly recommended for students intending to specialize in mathematics, statistics, physics, or related areas.

Precludes additional credit for MATH 1104, MATH 1107, MATH 1109, MATH 1119, MATH 2107. Note: MATH 1119 is **not** an acceptable substitute for half of MATH 1102.

Prerequisites: i) a pre-university algebra course with a grade of 65 percent or better, and ii) Grade 12 Mathematics: Advanced Functions and Introductory Calculus; or an OAC in Calculus; or MATH 0007; or equivalent; or permission of the School of Mathematics and Statistics. (See Prerequisites for first-year Calculus and Algebra Courses at the beginning of this section.)

Lectures three hours a week and one hour tutorial.

**Note:** Although the main prerequisite for MATH 1102 is a grade of 65 percent or better in a pre-university calculus course, past experience indicates that students with less than 75 percent in their prerequisite calculus have only a small chance of success in MATH 1102.

MATH 1104 [0.5 credit] (formerly 69.114\*)

### Linear Algebra for Engineering and Computer Science Students

Systems of linear equations. Matrix algebra. Determinants. Complex numbers. Eigenvalues. Diagonalization and applications.

Restricted to students in the Faculty of Engineering, in the School of Computer Science, or in B.Sc.(Honours) in Applied Physics.

Precludes additional credit for MATH 1102, MATH 1107, MATH 1109, MATH 1119.

Note: MATH 1119 is not an acceptable substitute for

MATH 1104.

Prerequisite: Ontario Grade 12 Mathematics: Geometry and Discrete Mathematics; or an OAC in Algebra and Geometry; or MATH 0107; or equivalent.

Lectures three hours a week and one hour tutorial.

MATH 1107 [0.5 credit] (formerly 69.117\*)

### Linear Algebra I

Systems of linear equations; vector space of n-tuples, subspaces and bases; matrix transformations, kernel, range; matrix algebra and determinants. Dot product. Complex numbers (including de Moivre's Theorem, and n-th roots). Eigenvalues, diagonalization and applications.

Precludes additional credit for MATH 1102, MATH 1104, MATH 1109 and MATH 1119.

Note: MATH 1119 is not an acceptable substitute for MATH 1107.

Prerequisite: Ontario Grade 12 Mathematics: Geometry and Discrete Mathematics; or an OAC in Algebra and Geometry; or MATH 0107 (or equivalent).

Lectures three hours a week and one hour tutorial.

MATH 1119 [0.5 credit]

### Linear Algebra: with Applications to Business and Economics

Introduction to systems of linear equations, geometric interpretation in two and three dimensions, introduction to matrices, vector addition and scalar multiplication, linear dependence, matrix operations, rank, inversion, invertible matrix theorem, determinants. Use of illustrative examples related to business and economics.

Precludes additional credit for, but is **not** an acceptable substitute for: MATH 1102, MATH 1104, MATH 1107.

Precludes additional credit for MATH 1109.

This course is not acceptable for (substitute) credit in any of the following degree programs: B.Math., and also B.Sc., B.C.S., B.Eng., B.I.D.

Prerequisite: Ontario Grade 12 Mathematics of Data Management (MDM4U); or one of: Grade 12 Mathematics: Geometry and Discrete Mathematics (MBA4U), or an OAC in Algebra and Geometry, or an OAC in Finite Mathematics, or MATH 0107, or equivalent, or permission of the School of Mathematics and Statistics.

Lectures three hours a week, tutorial one hour a week.

MATH 1805 [0.5 credit] (formerly 69.185\*)

### Discrete Structures I

An introduction to discrete mathematics and discrete structures. Topics include: propositional and predicate calculus, Boolean algebra, introduction to complexity of algorithms, mathematical reasoning, counting, recurrences, relations, introduction to graphs. (Also listed as COMP 1805.) This course is available to Science students only as a free option.

Prerequisites: two OACs in Mathematics, or two Grade 12 university-preparation Mathematics courses (after Summer 2002); and one of: COMP 1005 or COMP 1007 or COMP 1405 or SYSC 1100 (which may be taken concurrently).

Lectures three hours a week.

MATH 2000 [1.0 credit] (formerly 70.200)

### Calculus and Introductory Analysis

Higher dimensional calculus, chain rule, gradient, line and multiple integrals with applications. Use of implicit and inverse function theorems. Real number axioms, limits, continuous functions, differentiability, infinite series, uniform convergence, the Riemann integral.

Precludes additional credit for MATH 2004, MATH 2008, MATH 2009, MATH 3009, and for MATH 2001, MATH 2002.

Prerequisites: i) MATH 1002 or MATH 2007 with a grade of C+ or better; and ii) MATH 1102 or MATH 1107 with a grade of C+ or better; or permission of the School.

Lectures three hours a week and one hour tutorial.

MATH 2004 [0.5 credit] (formerly 69.204\*)

### Multivariable Calculus for Engineering Students

Fourier series; expansions for even and odd functions; half-range expansions. Surfaces in R3. Differential calculus of

functions of several variables. Extrema and Lagrange multipliers. Exact differentials. Line integrals. Double integrals; polar coordinates; applications. Triple integrals; cylindrical and spherical coordinates; applications. Restricted to students in the Faculty of Engineering, or in B.Sc.(Honours) in Applied Physics.  
 Precludes additional credit for: MATH 2001, MATH 2002, MATH 2008, MATH 2009 and MATH 2000.  
 Prerequisites: i) MATH 1005 or MATH 2007; and ii) MATH 1104 or MATH 1107; or permission of the School.  
 Lectures three hours a week, tutorial one hour a week.

MATH 2007 [0.5 credit] (formerly 69.207\*)

### Elementary Calculus II

Further techniques of integration, improper integrals, polar coordinates, parametric equations, indeterminate forms, sequences and series, Taylor's formula and series, first order and linear differential equations.

Precludes additional credit for MATH 1002, MATH 1005, and for MATH 2001, MATH 2002.

Prerequisites: i) MATH 1004, or a grade of C- or better in MATH 1007 or MATH 1009; and ii) an OAC in Algebra and Geometry, or MATH 0107, or permission of the School.

Lectures three hours a week and one hour tutorial.

MATH 2008 [0.5 credit] (formerly 69.208\*)

### Intermediate Calculus

Partial differentiation, chain rule, gradient, line and multiple integrals with applications, transformations, implicit and inverse function theorems.

Precludes additional credit for MATH 2004, MATH 2009, MATH 2000, and for MATH 2001, MATH 2002.

Prerequisites: MATH 1002 or MATH 2007, and MATH 1102 or MATH 1107.

Lectures three hours a week and one hour tutorial.

MATH 2009 [0.5 credit] (formerly 69.209\*)

### Intermediate Calculus for Science Students

Differential equations; differential calculus of functions of several variables; multiple integration; introduction to Fourier series.

Precludes additional credit for MATH 2001, MATH 2002, MATH 2004, MATH 2008 and MATH 2000.

Prerequisites: i) MATH 2007, or MATH 1002; and ii) MATH 1107 or MATH 1104 or MATH 1102; or their equivalents, or permission of the School.

Lectures three hours a week, tutorial one hour a week.

MATH 2100 [1.0 credit] (formerly 70.210)

### Algebra

Set theory, algebraic systems, vector spaces, inner product spaces, linear transformations, determinants, quadratic forms, selected applications.

Precludes additional credit for MATH 2108 or MATH 3101.

Prerequisite: MATH 1102 or MATH 2107 with a grade of C+ or better.

Lectures three hours a week and one hour tutorial.

MATH 2107 [0.5 credit] (formerly 69.217\*)

### Linear Algebra II

Finite-dimensional vector spaces (over  $\mathbb{R}$  and  $\mathbb{C}$ ), subspaces, linear independence and bases. Linear transformations and matrices. Inner product spaces (over  $\mathbb{R}$  and  $\mathbb{C}$ ); Orthonormal bases. Eigenvalues and diagonalization. Bilinear and quadratic forms; principal axis theorem.

Precludes additional credit for MATH 1102.

Prerequisites: i) MATH 1104, or a grade of C- or better in MATH 1107 or MATH 1109; and ii) a grade of C- or better in MATH 1007 or equivalent; or permission of the School.

**Note:** in item i), MATH 1119 is NOT acceptable as a substitute for MATH 1109.

Lectures three hours a week and one hour tutorial.

MATH 2108 [0.5 credit] (formerly 69.218\*)

### Abstract Algebra I

Sets and relations, number theory, group theory, ring theory, cardinal numbers.

Precludes additional credit for MATH 3101 and MATH 2100.

Prerequisite: MATH 1102 or MATH 2107.

Lectures three hours a week and one hour tutorial.

MATH 2200 [0.5 credit] (formerly 70.220\*)

### Co-operative Work Term Report 1

On completion of the work term, the student must submit to the School of Mathematics and Statistics a written report on the work performed. Graded Sat or Uns.

Prerequisites: registration in the Co-operative Education Option of an Honours program offered by the School of Mathematics and Statistics, completion of the Co-op preparation classes offered by the Co-op office, and permission of the School.

MATH 2404 [0.5 credit] (formerly 69.244\*)

### Ordinary Differential Equations I

Ordinary differential equations; applications; special first and second order types. Linear homogeneous and non-homogeneous equations with constant coefficients; variation of parameters; simple harmonic motion. Special linear equations with variable coefficients. Series solutions about ordinary points.

Precludes additional credit for MATH 1005, MATH 2001, MATH 2002, MATH 2454 and MATH 2600.

Prerequisites: MATH 1002 and MATH 1102 (or MATH 1107 and MATH 2007).

Lectures three hours a week and one hour tutorial.

MATH 2454 [0.5 credit] (formerly 70.244\*)

### Ordinary Differential Equations

Ordinary differential equations of the first and second order, existence and uniqueness of solutions; numerical solutions, error analysis; linear systems of first order; difference equations; higher order linear equations. Modeling with differential equations. A mathematical computer software package will be used.

Precludes additional credit for MATH 1005, MATH 2001, MATH 2002, MATH 2404, MATH 2600.

Prerequisites: MATH 1002 (or MATH 2007) with a grade of C+ or better and MATH 1102 (or MATH 2107) with a grade of C+ or better.

Lectures three hours a week, tutorial one hour a week.

MATH 2800 [0.5 credit] (formerly 69.280\*)

### Discrete Mathematics and Algorithms

An introduction to discrete mathematics and algorithms in the context of the computational sciences. Basic number theory and counting methods, algorithms for strings, trees and sequences. Applications to DNA and protein sequencing problems. Analysis and complexity of algorithms. (Also listed as CMPS 2800.)

Only one of MATH 1805/COMP 1805 or MATH 2800/CMPS 2800 may count for credit in a B.Math. program.

Prerequisites: COMP 1006 and at least one of MATH 1007, MATH 1107, or STAT 2507.

Lectures three hours a week.

MATH 2907 [0.5 credit] (formerly 70.297\*)

### Directed Studies

Available only to Honours students whose program requires a 0.5 credit not offered by the School of Mathematics and Statistics.

MATH 3001 [0.5 credit] (formerly 70.301\*)

### Real Analysis

Metric spaces; limits, continuity, open and closed sets, compactness, connectedness and completeness. Uniform convergence of sequences of functions, as convergence in metric spaces of bounded and continuous functions. Weierstrass approximation theorem. Contraction mappings and applications to integral and differential equations.

Prerequisite: MATH 2000 or permission of the School.

Lectures three hours a week and one hour tutorial.

MATH 3002 [0.5 credit] (formerly 70.302\*)

### Advanced Calculus

Vector fields on surfaces. The functions div, curl and grad. Line and surface integrals. The divergence theorem and Stokes' theorem. Exterior algebra. Stokes' formula. Functions of bounded variation. The Riemann-Stieltjes integral.

Prerequisite: MATH 2000 or permission of the School.

Lectures three hours a week and one hour tutorial.

MATH 3007 [0.5 credit] (formerly 69.307\*)

**Functions of a Complex Variable**

Analytic functions, contour integration, residue calculus, conformal mapping. Intended for non-engineering students.

Precludes additional credit for MATH 3706, MATH 3057, PHYS 3807 and PHYS 3806.

Prerequisite: one of MATH 2001, MATH 2002, MATH 2004, MATH 2008 or MATH 2009, or permission of the School. Lectures three hours a week and one hour tutorial.

MATH 3008 [0.5 credit] (formerly 70.308\*)

**Ordinary Differential Equations**

Analytic ordinary differential equations: series solutions of ordinary differential equations about ordinary and regular singular points. Asymptotic solutions. Sturm-Liouville theory. Bessel and Legendre functions. Fourier series.

Precludes additional credit for MATH 3404, PHYS 3808, and PHYS 3806.

Prerequisites: MATH 2000; and MATH 2454 or MATH 2600. Lectures three hours a week and one hour tutorial.

MATH 3009 [0.5 credit] (formerly 69.309\*)

**Introductory Analysis**

The real number system, sequences and series, functions of a single real variable, derivatives, the definite integral, uniform convergence.

Precludes additional credit for MATH 2000.

Prerequisite: one of MATH 2001, MATH 2002, MATH 2004, MATH 2008, MATH 2009, or permission of the School. Lectures three hours a week and one hour tutorial.

MATH 3057 [0.5 credit] (formerly 70.307\*)

**Functions of a Complex Variable**

Analytic functions, contour integration, residue calculus, conformal mapping.

Precludes additional credit for MATH 3007, MATH 3706, PHYS 3807, and PHYS 3806.

Prerequisite: MATH 2000 or permission of the School. Lectures three hours a week and one hour tutorial.

MATH 3101 [0.5 credit] (formerly 69.311\*)

**Algebraic Structures with Computer Applications**

Introduction to algebraic structures: groups, rings, fields, lattices, and Boolean algebras; with applications of interest to students in Computer Science.

This course may not be used to meet the 3000-level course requirements in any Major or Honours program in Mathematics and Statistics.

Precludes additional credit for MATH 2108 and MATH 2100.

Prerequisite: MATH 2107, or permission of the School. Lectures three hours a week and one hour tutorial.

MATH 3106 [0.5 credit] (formerly 70.316\*)

**Introduction to Group Theory**

Homomorphism theorems; groups acting on sets; permutation groups and groups of matrices; Sylow theory for finite groups; finitely generated abelian groups; generators and relations; applications.

Precludes additional credit for MATH 3100 and MATH 3108.

Prerequisite: MATH 2100, or permission of the School. Lectures three hours a week, tutorial one hour a week.

MATH 3107 [0.5 credit] (formerly 69.317\*)

**Linear Algebra III**

Similarity and unitary triangularization of matrices. Direct methods of solving a system of linear equations. Iterative techniques. Bounds for eigenvalues. Power method and de- ation techniques of approximation. Emphasis is primarily on computational aspects.

Prerequisites: i) a grade of C- or better in MATH 1102 or MATH 2107; and ii) credit in MATH 1002 or MATH 2007; or permission of the School.

Lectures three hours a week and one hour tutorial.

MATH 3108 [0.5 credit] (formerly 69.318\*)

**Abstract Algebra II**

Groups and rings. Permutations. Finite symmetry groups. Polynomials, unique factorization domains. Quotient rings, ideals. Field extensions, finite fields. Polynomial equations. Geometric constructions - three famous problems: duplication of the cube, trisection of an arbitrary angle, quadrature of the circle.

Precludes additional credit for MATH 3106, MATH 3158, and MATH 3100.

Prerequisite: MATH 2108, or permission of the School.

Lectures three hours a week and one hour tutorial.

MATH 3158 [0.5 credit] (formerly 70.318\*)

**Rings and Fields**

Rings; integral domains; Euclidean and principal ideal domains; polynomial rings over a field; modules over principal ideal domains and applications; fields; algebraic extensions of fields; finite fields; applications.

Precludes additional credit for MATH 3100 and MATH 3108.

Prerequisite: MATH 2100, or permission of the School.

Lectures three hours a week, tutorial one hour a week.

MATH 3200 [0.5 credit] (formerly 70.320\*)

**Co-operative Work Term Report 2**

On completion of the work term, the student must submit to the School of Mathematics and Statistics a written report on the work performed. Graded Sat or Uns.

Prerequisites: Registration in the Co-operative Education Option of an Honours program offered by the School of Mathematics and Statistics, and permission of the School.

MATH 3201 [0.5 credit] (formerly 70.321\*)

**Co-operative Work Term Report 3**

On completion of the work term, the student must submit to the School of Mathematics and Statistics a written report on the work performed. Graded Sat or Uns.

Prerequisites: Registration in the Co-operative Education Option of an Honours program offered by the School of Mathematics and Statistics, and permission of the School.

MATH 3205 [0.5 credit] (formerly 69.325\*)

**Euclidean Geometry and its Groups**

Transformations of the Euclidean plane (isometries, similarities); solutions of geometric problems using these transformations; groups of symmetries of finite plane figures, frieze patterns, and regular polyhedra; inversion and the extension to the inversive plane; problems solved using inversion; orthogonal circles and pencils of coaxial circles. Prerequisite: MATH 2108.

Lectures three hours a week and one hour tutorial.

MATH 3206 [0.5 credit] (formerly 69.326\*)

**Plane Projective Geometry**

Axioms of Desarguesian geometry, principle of duality; projectivities, perspectivities, and the fundamental theorem; collineations (homologies and elations); correlations (polarities and conics); algebraic model; introduction to finite projective planes.

Precludes additional credit for MATH 3256.

Prerequisite: MATH 2108.

Lectures three hours a week and one hour tutorial.

MATH 3256 [0.5 credit] (formerly 70.326\*)

**Foundations of Projective Geometry**

Definition of a general projective plane and immediate consequences; finite planes (combinatorial results, sub-planes, incidence matrices) and planar ternary rings; collineations, role of Desargues' configuration, examples of types of planes.

Precludes additional credit for MATH 3206.

Prerequisite: MATH 2100.

Lectures three hours a week and one hour tutorial.

MATH 3306 [0.5 credit] (formerly 70.336\*)

**Elements of Set Theory**

Axioms of set theory. Development of the systems of natural numbers and the real numbers. Axiom of choice, Zorn's lemma, well-ordering. The Schröder-Bernstein theorem,

cardinal numbers, ordinal numbers, transfinite induction, cardinal and ordinal arithmetics.

Prerequisite: MATH 2100 or permission of the School.

Lectures three hours a week and one hour tutorial.

MATH 3404 [0.5 credit] (formerly 69.344\*)

### Ordinary Differential Equations II

Series solutions of ordinary differential equations of second order about regular singular points; asymptotic solutions. Systems of ordinary differential equations of first order; matrix methods. Existence and uniqueness theorems. Nonlinear autonomous systems of order 2; qualitative theory. Numerical solutions of ordinary differential equations.

Precludes additional credit for MATH 3008.

Prerequisites: MATH 2404, MATH 2008; and MATH 1102 or MATH 2107.

Lectures three hours a week and one hour tutorial.

MATH 3406 [0.5 credit] (formerly 70.346\*)

### Autonomous Dynamical Systems

Basic concepts of dynamical systems. Stability; limit cycles; Lyapunov's direct method. Theory of autonomous dynamical systems. Volterra systems; principle of competitive exclusion in population biology. The threshold theorem of epidemiology. Basic concepts of nonequilibrium statistical mechanics.

Prerequisites: MATH 2000; and MATH 2454 or MATH 2600.

Lectures three hours a week and one hour tutorial.

MATH 3705 [0.5 credit] (formerly 69.375\*)

### Mathematical Methods I

Laplace transforms, Fourier series and Fourier transforms, solutions of partial differential equations of mathematical physics, boundary value problems, applications.

This course may be taken for credit as a 3000-level Honours Mathematics course, by students in any Honours program in the School of Mathematics and Statistics.

Precludes additional credit for MATH 3004, PHYS 3808, and PHYS 3806.

Prerequisite: MATH 2001 or MATH 2002; or i) MATH 2404 or MATH 1005, and ii) MATH 2004 or MATH 2008 or MATH 2009; or permission of the School.

Lectures three hours a week and one hour tutorial.

MATH 3800 [0.5 credit] (formerly 69.380\*)

### Modeling and Computational Methods for Experimental Science

Mathematical modeling in the experimental sciences: design, analysis and pitfalls. Computational methods directly applicable to problems in science will be described, including: function evaluation, interpolation, solution of linear equations, root finding, integration, solution of differential equations, Fourier series and Monte Carlo methods. (Also listed as CMPS 3800.)

Only one of MATH 3806/COMP 3806 or MATH 3800/CMPS 3800 may count for credit in a B.Math. program.

Prerequisites: MATH 1107; MATH 2007 or MATH 2009; and COMP 1006.

Lectures three hours a week.

MATH 3801 [0.5 credit] (formerly 69.381\*)

### Linear Programming

Formulation of linear programming problems, the simplex method, duality theory, implementations, extensions and applications. Network flow problems and the network simplex method.

Precludes additional credit for ECON 4004, SYSC 3200.

Prerequisite: MATH 1102 or MATH 2107, or permission of the School.

Lectures three hours a week and one hour tutorial.

MATH 3802 [0.5 credit] (formerly 69.382\*)

### Combinatorial Optimization

Dijkstra's algorithm and Bellman-Ford algorithm for the minimum weight dipath problem, the minimum weight spanning tree problem, augmenting path algorithm and pre ow-push algorithm for the max- ow min-cut problem, connections to linear programming, matchings in bipartite

graphs and the assignment problem, the transportation problem, and the general minimum-cost ow problem.

Prerequisite: MATH 3801, or permission of the School.

MATH 3804 [0.5 credit] (formerly 69.384\*)

### Design and Analysis of Algorithms I

An introduction to the design and analysis of algorithms. Topics include: recurrence relations, sorting and searching, divide-and-conquer, dynamic programming, greedy algorithms, amortized analysis. (Also listed as COMP 3804.)

Prerequisites: COMP 2002, and either COMP 2805 or both of MATH 2007 and MATH 2108 or equivalents.

Lectures three hours a week.

MATH 3805 [0.5 credit] (formerly 70.385\*)

### Discrete Structures and Applications

Enumeration: elementary methods, inclusion and exclusion, recurrence relations, generating functions and applications. Graph theory and algorithms: connectivity, planarity, Hamilton and Euler paths. Error-correcting codes. (Also listed as COMP 3805.)

Prerequisite: one of MATH 2108, MATH 3101, or MATH 2100.

Lectures three hours a week and one hour tutorial.

MATH 3806 [0.5 credit] (formerly 69.386\*)

### Numerical Analysis

Elementary discussion of error, polynomial interpolation, quadrature, linear systems of equations and matrix inversion, non-linear equations, difference equations and ordinary differential equations. (Also listed as COMP 3806.)

Prerequisites: i) MATH 1002, MATH 1005 or MATH 2007 (or MATH 2001 or MATH 2002); and ii) MATH 1102 or MATH 2107; and (iii) knowledge of a computer language.

Lectures three hours a week and one hour tutorial.

MATH 3807 [0.5 credit] (formerly 69.387\*)

### Mathematical Software

Incorporation of basic numerical methods into efficient, reliable software. The course includes examination of existing software systems, e.g., linear systems, non-linear systems, optimization, or differential equations. (Also listed as COMP 3807.)

Prerequisite: MATH 3806.

Lectures three hours a week and one hour tutorial.

MATH 3809 [0.5 credit] (formerly 69.389\*)

### Introduction to Number Theory and Cryptography

Congruences, distribution of primes, general cryptographic systems, public key cryptographic systems and authentication using number theory, primality testing and factoring in relation to cryptography, continued fractions and diophantine equations.

Prerequisites: MATH 2108 or MATH 3101 or MATH 2100; knowledge of a computer language.

Lectures three hours a week and one hour tutorial.

MATH 3819 [0.5 credit]

### Modern Computer Algebra

Algorithms for multiplication, division, greatest common divisors and factorization over the integers, finite fields and polynomial rings. Basic tools include modular arithmetic, discrete Fourier transform, Chinese remainder theorem, Newton iteration, and Hensel techniques. Some properties of finite fields and applications to cryptography.

Prerequisite: MATH 2108 or MATH 3101 or MATH 2100, or permission of the School.

Lectures three hours a week, tutorial/laboratory one hour a week.

MATH 3907 [0.5 credit] (formerly 69.397\*)

### Directed Studies

Available only to students whose program requires a 0.5 credit not offered by the School of Mathematics and Statistics.

*A selection of courses in the 4000-series will be offered.*

MATH 4003 [0.5 credit] (formerly 70.403\*)

### Functional Analysis

Banach spaces and bounded linear operators, Hahn-Banach

extension and separation, dual spaces, bounded inverse theorems, uniform boundedness principle, applications. Compact operators. Differential calculus in Banach spaces, inverse and implicit function theorems and their application to differential equations. Also offered at the graduate level, with additional or different requirements, as MATH 5008, for which additional credit is precluded.  
Prerequisite: MATH 3001 or permission of the School.  
Lectures three hours a week.

MATH 4007 [0.5 credit] (formerly 70.407\*)

#### Measure and Integration Theory

Lebesgue measure and integration on the real line; sigma algebras and measures; integration theory;  $L_p$  spaces; Fubini's theorem; decomposition theorems and Radon-Nikodym derivatives. Also offered at the graduate level, with additional or different requirements, as MATH 5007, for which additional credit is precluded.  
Prerequisite: MATH 3001 or MATH 3002 or permission of the School.  
Lectures three hours a week.

MATH 4105 [0.5 credit] (formerly 70.415\*)

#### Rings and Modules

Fundamental concepts in rings and modules, structure theorems, applications.  
Prerequisite: MATH 3158 or MATH 3100 or permission of the School.  
Lectures three hours a week.

MATH 4106 [0.5 credit] (formerly 70.416\*)

#### Group Theory

Fundamental principles as applied to abelian, nilpotent, solvable, free and finite groups; representations. Also offered at the graduate level, with additional or different requirements, as MATH 5106, for which additional credit is precluded.  
Prerequisite: MATH 3106 or MATH 3100 or permission of the School.  
Lectures three hours a week.

MATH 4107 [0.5 credit] (formerly 70.417\*)

#### Commutative Algebra

Fields, including algebraic and transcendental extensions, Galois theory, valuation theory; Noetherian commutative rings, including Noether decomposition theorem and localization.  
Prerequisite: MATH 3158 or MATH 3100 or permission of the School.  
Lectures three hours a week.

MATH 4108 [0.5 credit] (formerly 70.418\*)

#### Homological Algebra and Category Theory

Axioms of set theory; categories, functors, natural transformations; free, projective, injective and flat modules; tensor products and homology functors, derived functors; dimension theory. Also offered at the graduate level, with additional or different requirements, as MATH 5108, for which additional credit is precluded.  
Prerequisite: MATH 3158 or MATH 3100 or permission of the School.  
Lectures three hours a week.

MATH 4109 [0.5 credit] (formerly 70.419\*)

#### Fields and Coding Theory

Introduction to field theory, emphasizing the structure of finite fields, primitive elements and irreducible polynomials. The influence of computational problems will be considered. Theory and applications of error-correcting codes: algebraic codes, convolution codes, decoding algorithms, and analysis of code performance.  
Prerequisite: MATH 2100, or MATH 3101 or MATH 2108 or equivalent; or permission of the School.  
Lectures three hours a week.

MATH 4200 [0.5 credit] (formerly 70.420\*)

#### Co-operative Work Term Report 4

On completion of the work term, the student must submit to the School of Mathematics and Statistics a written report on the work performed. Graded Sat or Uns.  
Prerequisites: Registration in the Co-operative Education

Option of an Honours program offered by the School of Mathematics and Statistics, and permission of the School.

MATH 4201 [0.5 credit] (formerly 70.421\*)

#### Co-operative Work Term Report 5

On completion of the work term, the student must submit to the School of Mathematics and Statistics a written report on the work performed. Graded Sat or Uns.  
Prerequisites: Registration in the Co-operative Education  
Option of an Honours program offered by the School of Mathematics and Statistics, and permission of the School.

MATH 4205 [0.5 credit] (formerly 70.425\*)

#### Introduction to General Topology

Topological spaces, maps, subspaces, product and identification topologies, separation axioms, compactness, connectedness. Also offered at the graduate level, with additional or different requirements, as MATH 5205, for which additional credit is precluded.  
Prerequisite: MATH 3001 or permission of the School.  
Lectures three hours a week.

MATH 4206 [0.5 credit] (formerly 70.426\*)

#### Introduction to Algebraic Topology

An introduction to homotopy theory. Topics include the fundamental group, covering spaces and the classification of two-dimensional manifolds. Also offered at the graduate level, with additional or different requirements, as MATH 5206, for which additional credit is precluded.  
Prerequisites: MATH 3106 (or MATH 3100) and MATH 4205; or permission of the School.  
Lectures three hours a week.

MATH 4207 [0.5 credit] (formerly 70.427\*)

#### Foundations of Geometry

A study of at least one modern axiom system of Euclidean and non-Euclidean geometry, embedding of hyperbolic and Euclidean geometries in the projective plane, groups of motions, models of non-Euclidean geometry.  
Prerequisite: MATH 3106 (may be taken concurrently) or MATH 3100, or permission of the School.  
Lectures three hours a week.

MATH 4208 [0.5 credit] (formerly 70.428\*)

#### Introduction to Differentiable Manifolds

A study of differentiable manifolds from the point of view of either differential topology or differential geometry. Topics such as smooth mappings, transversality, intersection theory, vector fields on manifolds, Gaussian curvature, Riemannian manifolds, differential forms, tensors and connections are included.  
Prerequisite: MATH 3001 or permission of the School.  
Lectures three hours a week.

MATH 4305 [0.5 credit] (formerly 70.435\*)

#### Analytic Number Theory

Dirichlet series, characters, Zeta-functions, prime number theorem, Dirichlet's theorem on primes in arithmetic progressions, binary quadratic forms. Also offered at the graduate level, with additional or different requirements, as MATH 5305, for which additional credit is precluded.  
Prerequisite: MATH 3057 or permission of the School.  
Lectures three hours a week.

MATH 4306 [0.5 credit] (formerly 70.436\*)

#### Algebraic Number Theory

Algebraic number fields, bases, algebraic integers, integral bases, arithmetic in algebraic number fields, ideal theory, class number. Also offered at the graduate level, with additional or different requirements, as MATH 5306, for which additional credit is precluded.  
Prerequisite: MATH 3158 (may be taken concurrently) or MATH 3100, or permission of the School.  
Lectures three hours a week.

MATH 4407 [0.5 credit] (formerly 70.447\*)

#### Tensor Analysis and Relativity Theory

Development of tensor analysis, application to Riemannian spaces and relativity theory.  
Prerequisites: MATH 3405 and MATH 3406 or permission of the School.  
Lectures three hours a week.

MATH 4600 [0.5 credit] (formerly 70.460\*)

### Case Studies in Operations Research

Applications of the principles of Operations Research to practical problems in business, management, and science. Students present at least one case and analyze cases in the published literature. Cases may also be presented by visiting practitioners.

Note: students in Honours Mathematics/Statistics programs may only take this course as a free option.

Prerequisites: STAT 2509 (or STAT 2559) and MATH 3801; or permission of the School.

Seminars three hours a week.

MATH 4700 [0.5 credit] (formerly 70.470\*)

### Partial Differential Equations

First order linear, quasi-linear, and non-linear equations; second order equations in two and more variables; systems of equations; the wave equation; Laplace and Poisson equations, Dirichlet and Neumann problems; Green's functions. Also offered at the graduate level, with additional or different requirements, as MATH 5406, for which additional credit is precluded.

Prerequisites: MATH 3008 and one of MATH 3002 or MATH 3057 or permission of the School.

Lectures three hours a week.

MATH 4701 [0.5 credit] (formerly 70.471\*)

### Topics in Partial Differential Equations

Theory of distributions, initial-value problems based on 2-dimensions wave equations, Laplace transform, Fourier integral transform, diffusion problems, Helmholtz equation with application to boundary and initial-value problems in cylindrical and spherical coordinates. Also offered at the graduate level, with additional or different requirements, as MATH 5407, for which additional credit is precluded.

Prerequisites: MATH 3008 and one of MATH 3002 or MATH 3057 or permission of the School.

Lectures three hours a week.

MATH 4702 [0.5 credit] (formerly 70.472\*)

### Integral Transforms

Laplace, Fourier, Hankel and Mellin transforms, selection of a suitable transform for a given partial differential equation boundary value problem. Operational properties of transforms. Inversion theorems. Approximate evaluation of inversion integrals for small and large values of parameter. Application to the solution of integral equations.

Prerequisite: MATH 3057 or permission of the School.

Lectures three hours a week.

MATH 4703 [0.5 credit] (formerly 70.473\*)

### Qualitative Theory of Ordinary Differential Equations

Ordinary differential equations: existence-uniqueness theorems, vector formulation for systems; stability theory, Lyapunov theorems, perturbation theorems and structural stability; Poincaré-Bendixon theory.

Prerequisites: MATH 3001, MATH 3008, MATH 3406.

Lectures three hours a week.

MATH 4801 [0.5 credit] (formerly 70.481\*)

### Topics in Combinatorics

An in-depth study of one or more topics from: generating functions, Polya's theory of counting, block designs, coding theory, partially ordered sets and Ramsey theory.

Prerequisites: MATH 2100 and MATH 3805 or permission of the School.

Lectures three hours a week.

MATH 4802 [0.5 credit] (formerly 70.482\*)

### Introduction to Mathematical Logic

Symbolic logic, propositional and predicate calculi, set theory and model theory, completeness.

Prerequisite: MATH 2100 or permission of the School.

Lectures three hours a week.

MATH 4803 [0.5 credit] (formerly 70.483\*)

### Computable Functions

Recursive functions and computability, algorithms, Church's thesis, Turing machines, computational logic,

NP-completeness. (Also listed as COMP 4803.)

Prerequisite: MATH 2100 or MATH 3805 or permission of the School.

Lectures three hours a week.

MATH 4805 [0.5 credit] (formerly 70.485\*)

### Theory of Automata

Finite automata and regular expressions, properties of regular sets, context-free grammars, pushdown automata, deterministic context-free languages. Turing machines, the Chomsky hierarchy. Undecidability, intractable problems. Also offered at the graduate level, with additional or different requirements, as MATH 5605, for which additional credit is precluded. (Also listed as COMP 4805.)

Prerequisite: MATH 3805 or MATH 3106 or MATH 3158 (or MATH 3100) or permission of the School.

Lectures three hours a week.

MATH 4806 [0.5 credit] (formerly 70.486\*)

### Numerical Linear Algebra

Study of matrix inversion techniques; techniques of finding eigenvalues and eigenvectors, solution of systems of linear equations; direct and indirect methods, their comparison and error analysis; applications in optimization and other areas. (Also listed as COMP 4806.)

Prerequisites: MATH 1102 or MATH 2107; and MATH 2000 or MATH 3009, or permission of the School.

Lectures three hours a week.

MATH 4807 [0.5 credit] (formerly 70.487\*)

### Game Theory

Two-person zero-sum games; infinite games; multistage games; differential games; utility theory; two-person general-sum games; bargaining problem; n-person games; games with a continuum of players. Also offered at the graduate level, with additional or different requirements, as MATH 5607, for which additional credit is precluded.

Prerequisite: MATH 3001 or permission of the School.

Lectures three hours a week.

MATH 4808 [0.5 credit] (formerly 70.488\*)

### Graph Theory and Algorithms

Paths, circuits, Eulerian and Hamiltonian graphs, connectivity, colouring problems, matching, Ramsey theory, network flows.

Prerequisites: MATH 3805 or MATH 3106 or MATH 3158 (or MATH 3100) or permission of the School.

Lectures three hours a week.

MATH 4809 [0.5 credit] (formerly 70.489\*)

### Mathematical Cryptography

Topics covered include: a general survey of public key cryptography; classical applications of finite fields and number theory; relevant background in geometry and algebraic curves; computational issues concerning elliptic curves; elliptic curve cryptosystems; security issues.

Prerequisite: MATH 3809, or permission of the School.

Lectures three hours a week.

MATH 4811 [0.5 credit]

### Combinatorial Design Theory

Existence and construction of combinatorial designs: finite geometries, pairwise balanced designs, balanced incomplete block designs, Steiner triple systems, symmetric designs, PBD closure, latin squares, transversal designs, and applications to information theory.

Prerequisite: MATH 3805, or permission of the School.

Lectures three hours a week.

MATH 4905 [0.5 credit] (formerly 70.495\*)

### Honours Project

Consists of a written report on some approved topic or topics in the field of mathematics, together with a short lecture on the report.

Prerequisite: B.Math.(Honours) students only.

MATH 4906 [0.5 credit] (formerly 70.496\*)

### Directed Studies

Prerequisite: B.Math.(Honours) students only.

MATH 4907 [0.5 credit] (formerly 70.497\*)

**Directed Studies**

Prerequisite: B.Math.(Honours) students only.

*The following courses have not been offered in recent years, but may be offered sometime in the future:*

MATH 3405 Classical Mechanics

MATH 3706 Mathematical Methods II

MATH 3900 Mathematical Problem Solving

MATH 4001 Vector Calculus

MATH 4405 Analytical Dynamics

MATH 4406 Hydrodynamics and Elasticity

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## Mechanical Engineering (MECH)

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Department of Mechanical and Aerospace  
Engineering  
Faculty of Engineering

MECH 3002 [0.5 credit] (formerly 88.302\*)

**Machine Design and Practice**

The design of mechanical machine elements is studied from theoretical and practical points of view. Topics covered include: design factors, fatigue, and discrete machine elements. Problem analysis emphasizes the application to practical mechanical engineering problems.

Prerequisites: MAAE 2001, MAAE 3202.

Lectures three hours a week, problem analysis three hours a week.

MECH 3700 [0.5 credit] (formerly 88.370\*)

**Principles of Manufacturing**

Manufacturing processes, materials. Casting: solidification and heat flow theory, defect formation, casting design. Metal forming: elementary plasticity theory, plastic failure criteria, force and work calculations. Bulk and sheet forming. Joining: heat flow and defect formation theory, residual stresses. Machining theory and practice. Hardening: diffusion, wear resistance.

Prerequisite: MAAE 2700.

Lectures and tutorials three hours a week; problem analysis and laboratories one hour a week.

MECH 4003 [0.5 credit] (formerly 88.403\*)

**Mechanical Systems Design**

Design of mechanical systems: establishing design criteria, conceptual design, design economics, value analysis, synthesis and optimization. Mechanical elements/systems: gear and flexible drive systems, fluid power systems. These elements are utilized in group design projects.

Prerequisite: MECH 3002.

Lectures three hours a week, problem analysis three hours a week.

MECH 4006 [0.5 credit] (formerly 88.406\*)

**Vehicle Engineering I**

The course emphasizes the engineering and design principles of road transport vehicles. Topics to be covered include: performance characteristics, handling behaviour and ride quality of road vehicles.

Prerequisites: MAAE 2101, MAAE 3004 (Dynamics of Machinery) and third- or fourth-year registration.

Lectures three hours a week.

MECH 4007 [0.5 credit] (formerly 88.407\*)

**Vehicle Engineering II**

Engineering and design principles of off-road vehicles and air cushion technology. Topics include: mechanics of vehicle-terrain interaction - terramechanics, performance characteristics of off-road vehicles, steering of tracked vehicles, air cushion systems and their performance, applications of air cushion technology to transportation.

Prerequisites: MAAE 2101, MAAE 3004 (Dynamics of Machinery) and third-or fourth-year registration.

Lectures three hours a week.

MECH 4101 [0.5 credit] (formerly 88.411\*)

**Mechanics of Deformable Solids**

Course extends the student's ability in design and stress analysis. Topics include: introductory continuum mechanics, theory of elasticity, stress function approach, Lamé and Mitchell problems, stress concentrations, thermoelasticity and plasticity.

Prerequisite: MAAE 3202.

Lectures three hours a week.

MECH 4103 [0.5 credit] (formerly 88.413\*)

**Fatigue and Fracture Analysis**

Elastic and elasto-plastic fracture mechanics. Fatigue design methods, fatigue crack initiation and growth Paris law and